IMPLICIT NUMERICAL INTEGRATION SCHEMES IN THE PROBLEMS OF CREEP AND CYCLIC PLASTICITY

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Long-term strength and low-cycle fatigue calculations are associated with the necessity to determine the stress and inelastic strains fields in structural components of complicated geometry. To take into account actual service conditions of structures in calculations of the stress-strain state, the models of inelastic deformation should be used, which make it possible to consider the history of thermomechanical loading. The theories of thermoplasticity and thermoviscoplasticity of the incremental type are referred to such models.

The statement of the corresponding boundary problems yields the systems of non-linear partial differential equations. For the majority of the problems considered the above systems of equations are "stiff". The problems of this type are generally solved by step-by-step methods and the differential time approximation is combined with the finite element discretization with respect to space coordinates.

For the numerical time integration of the constitutive equations of the theories of thermoplasticity and thermoviscoplasticity explicit and implicit differential schemes are often used. Relative simplicity of the numerical implementation has determined the priority development of the explicit numerical integration schemes. However, practical application of the explicit integration methods for "stiff" systems of differential equations has been found difficult due to the necessity of using the inadmissibly small time integration step determined from the condition of the stability and convergence of the calculation process. In the case of long-term strength and low-cycle fatigue problems the use of the explicit time integration schemes for the calculations is time consuming and expensive.

Thus, algorithms are needed in which the condition of stability does not impose any limitation on the value of a time step. The implicit Euler's scheme belongs to this type of algorithms.

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To solve the problems considered in the paper presented, a reduced mixed FEM scheme was used combined with time integration based on the unconditionally stable step-iteration process.

In the FEM scheme mentioned the displacement and stresses are approximated with the help of a set of basis functions independent of each other. In addition, variational equations describing the problem under consideration are reduced by the approximate calculation of some terms with cubature formulae whose points of weighing coincide with the nodal points of the finite element mesh. Owing to this fact the solution of the problem is fully determined by the magnitude of the nodal displacements and stress (strains). As a result of numerical integration, the matrixes composed of the convolutions of the basis functions (rather than their derivatives) turn to be diagonal which is the reason for a high efficiency of the numerical realization of the approach proposed (Umansky (1)).

The use of Euler's method implicit schemes is generally associated with the necessity of solving the systems of nonlinear equations. A more efficient method has been suggested in the present paper.

The relationships of the plastic flow theory and the theory of viscoplasticity for materials with the nonlinear isotropic and kinematic hardening were used as equations of state. Here the relation between the stress deviators and inelastic strain rates has the form respectively (Chaboche and Rousselier (2), Perzyna (3)).

$$\dot{\xi}_{ij} = \frac{3}{2} d\lambda \frac{S_{ij} - \rho_{ij}}{I_2(S_{ij} - \rho_{ij})} \quad \text{or} \quad \dot{\xi}_{ij} = \frac{3}{2} \gamma \Phi(\mathcal{F}) \frac{S_{ij} - \rho_{ij}}{I_2(S_{ij} - \rho_{ij})};$$

$$\mathcal{F} = I_2(S_{ij} - \rho_{ij}) - R(J_2(\dot{\xi}_{kl})) = 0;$$

$$\rho_{ij} = c\left(\frac{2}{3} a \dot{\xi}_{ij} - J_2(\dot{\xi}_{kl}) \rho_{ij}\right); \quad \dot{R} = b(Q - R) J_2(\dot{\xi}_{kl})$$

where $\mathcal{F}$ and $\Phi(\mathcal{F})$ are the functions of the plastic and viscoplastic potentials; $S_{ij} - \rho_{ij}$ are the components of the active stress deviator; γ is the viscosity parameter; $d\lambda$ is the non-negative scalar function of the loading history; I_2 and J_2 are the second invariants of the stress deviators and inelastic strain rates; a, b, c, Q are the parameters of materials.

The use of the implicit Euler's scheme for the constitutive equations of the theories mentioned is shown to reduce them to a single nonlinear equation in which the intensity of inelastic strain increment or the function of the viscoplastic potential are unknown. In each loading step the nonlinear equations mentioned were solved by Newton's method

The method proposed makes possible to obtain a stable solution for a sufficiently large reloading step, as well as to consider the processes of creep and low-cycle fatigue.

The procedures which realize the proposed algorythms have been included into the engineering software systems SAFE-2D developed at the Institute for Problems of Strength. Solution of a number of test and practical problems revealed a high efficiency of the method suggested.

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