GENERALIZATION OF THE IMPROVED MODIFIED CRACK CLOSURE INTEGRAL METHOD TO SURFACE CRACK PROBLEMS

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The improved modified crack closure integral method has proved to be a numerically highly effective method for the fracture analysis of plane, linear elastic crack problems. It is shown that this method can be generalized for covering 3D-problems involving locally defined energy release rates varying along a curved crack front. This is demonstrated by the fracture analysis of a semi-elliptical surface crack in a plate in tension. The calculated stress-intensity factor distributions along the crack front show reasonable to good agreement with reference solutions with respect to the constant strain - or the coarse linear strain element discretisation under consideration, respectively.

INTRODUCTION

From the various FE-procedures available for the linear elastic fracture analysis of plane crack problems the modified crack closure integral method given by RYBICKI and KANNINEN (1) (MCCI-method) has proved to be a numerically highly effective procedure. The following investigation will show that by defining an effective width for each nodal point force at the crack front, corresponding to the unit thickness inherent in plane problems, the CCI-methods can be generalized to 3D-crack problems. Thereby their main advantage of delivering simultaneously the separated energy release rates $G_i(a)$, i=I,II,III in case of mixed-mode conditions at the crack tip, remains unaffected.

CRACK CLOSURE-AND IMPROVED MODIFIED CRACK CLOSURE METHOD

For the FE-analysis of crack problems IRWIN's well known analytical crack closure integral relation can be written in the following FE-representation

$$G_I(a + \Delta a/2) = \frac{1}{t}\frac{1}{2\Delta a}(F_{y,i}(a)\Delta u_{y,i-1}(a + \Delta a)) , \tag{1}$$

holding for the constant strain element (CSE)-discretisation as given in Fig. 1a. Equation (1)

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delivers the energy release rate $G_I(a+\Delta a/2)$ on the basis of the work to be done by the nodal point force $F_{y,i}(a)$ against the relative nodal point displacement $\Delta u_{y,i-1}(a+\Delta a)$ in order to close the crack by Δa again (Fig. 1a). For plane problems t is denoting the unit thickness of the specimen. It is emphasized that Eq. (1) is holding numerically exact for the actual CSE-discretisation under consideration for $\Delta a \gg 0$. Still the disadvantage of this CCI-method is the requirement of two FE-calculations with different crack lengths a and $a+\Delta a$ to get one $G_I(a+\Delta a/2)$-value. This disadvantage can be avoided by referring to the following formula of the modified CCI-method (MCCI- meth.)

$$G_I(a) = \frac{1}{t} \lim_{\Delta a \to 0} \frac{1}{2\Delta a}(F_{y,i}(a)\Delta u_{y,i-1}(a)) \tag{2}$$

given by RYBICKI and KANNINEN (1) for plane CSE-discretisations as shown in Fig. 1a. By this method the numerical effort is reduced to one half, because only one FE-calculation is required for obtaining one $G_I(a)$-value for a given crack length a. Both methods have been improved by extensions in order to be applicable in connection with the numerically more effective linear strain element (LSE)-discretisations corresponding to Fig. 1b by BUCHHOLZ (2), KRISHNAMURTY, DATTAGURU et al (3) and RAJU (4). If Eqs. (1) and (2) should apply to the surface-cracked plate of Fig. 2, the plane unit thickness t has to be replaced by an effective width $\Delta w_i(a,\varphi_j)$ correlated to each nodal point force $F_{z,i}(a,\varphi_j)$ at the crack front. The effective widths are governed by the shape functions of the elements in use, and for the CSE-discretisation of the surface-cracked plate (Fig. 3a, b) one obtaines $\Delta w_i(a,\varphi_j) = (w_i^{le}() + w_i^{re}())/2$, with w_i^{le} and w_i^{re} being the relevant widths of the adjacent left- and right-hand side elements, respectively. In extending this approach to LSE- discretisations as given in Figs. 1b and 5a, b the follwing formula can be derived for the improved CCI-method

$$G_I(a+\Delta a/2,\varphi_j) = \frac{1}{2\Delta a}\left(\frac{F_{z,i}(a,\varphi_j)}{\Delta w_i(a,\varphi_j)}\Delta u_{z,i-2}(a+\Delta a,\varphi_j) + \right.$$
$$\left. + \frac{F_{z,i+1}(a,\varphi_j)}{\Delta w_{i+1}(a,\varphi_j)}\Delta u_{z,i-1}(a+\Delta a,\varphi_j)\right) . \tag{3}$$

In Eq. (3) $\Delta w_k(a,\varphi_j)$, $k = i, i+1$ is given by $(w_k^{le}() + w_k^{re}())/6$ or by $2w_k^e/3$, if φ_j is indicating a connection line between adjacent LS-elements or a centre line through the elements, respectively.

SEMI-ELLIPTICAL SURFACE CRACK IN A PLATE

For a ratio $a/t = 0.4$ of the surface-cracked plate (Fig.2) the normalized stress intensity factor distribution along the crack front is plotted in Fig. 4. The results obtained by the CCI- and MCCI-methods respectively (6) (K1N-2C and -1C graphs, Eqs. (1), (2)) are in reasonable agreement with RAJU and NEWMAN's reference solution (5) (K1N-REF graph), with respect to the CSE-discretisation as given in Figs. 3a, b.

In Fig. 6 corresponding results are plotted (K1N-2C graphs), calculated by the improved CCI-method for the coarse isoparametric LSE-discretisation of Figs. 5a, b. The final result is given by the dotted line, because Eq. (3) delivers $G_I(a,\varphi)$-values which have to be converted into $K_I(a,\varphi)$, considering that for $0 \leq 2\varphi/\pi \leq 0.3$ plane strain conditions (-EVZ) and for $0.7 \leq 2\varphi/\Pi \leq 1.0$ plane stress conditions (-ESZ) will apply approximately. For the isoparametric LSE-discretisation of Figs. 5a, b the effective widths $\Delta w_k(a,\varphi_j), k = i, i+1$, required for Eq. (3), have to be identified by a special analysis (6) because they are affected by the distorsions of the unstressed isoparametric LS-elements. But the results given in Figs. 4 and 6 and for a related problem in (7) prove that the presented crack closure methods can be generalized for covering this kind of problems.

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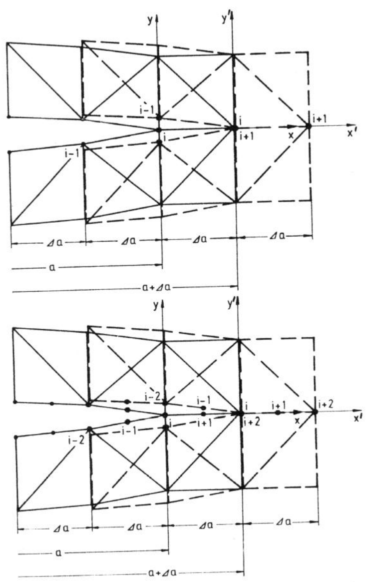

Fig.1a,b Num. crack closure integral methods (orig. and impr. CCI-and MCCI-meth.)

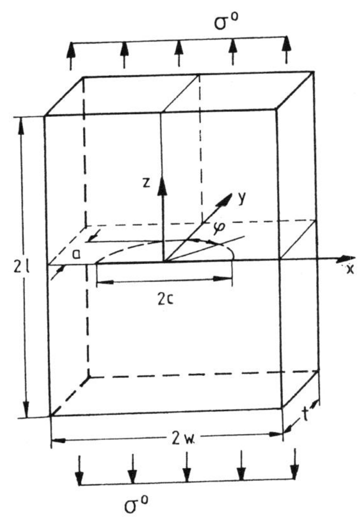

Fig.2 Surface-cracked plate in tension ($c/w \leq 0.2$, $c/l \leq 0.4$, $a/c = 0.4$)

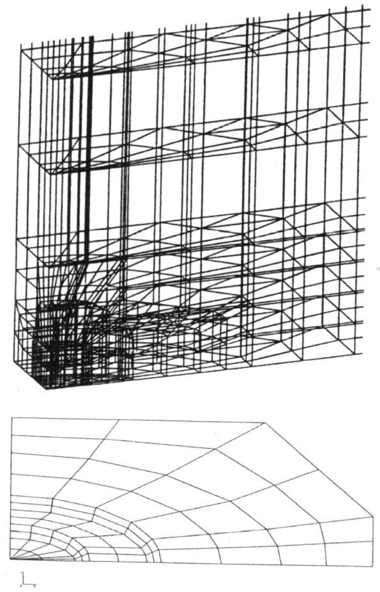

Fig.3a,b CSE-discretisation of the plate (espec. modelled for $a/t = 0.2$ and 0.4)

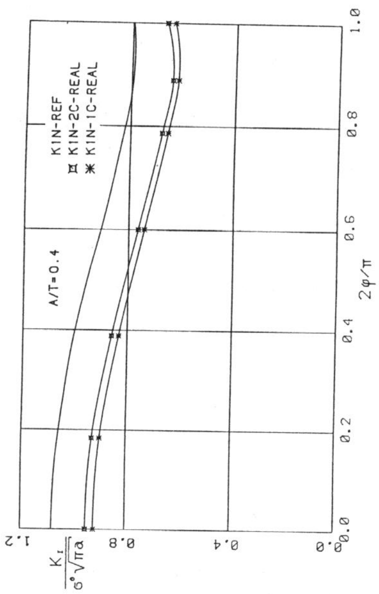

Fig.4 Normalized stress intensity factor distribution ($a/t = 0.4$)

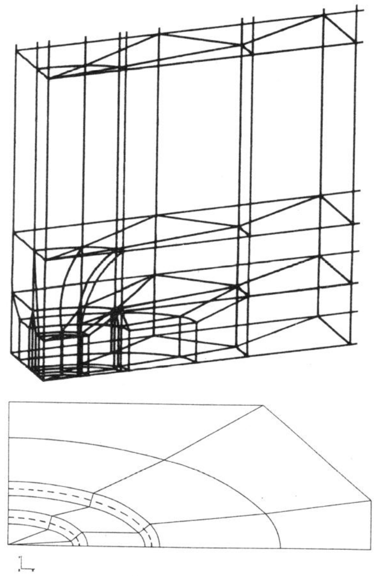

Fig.5a,b LSE-discretisation of the plate (espec. modelled for $a/t = 0.2$ and 0.4)

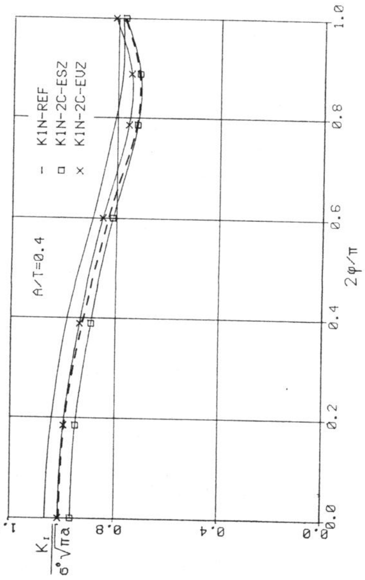

Fig.6 Normalized stress intensity factor distribution ($a/t = 0.4$)