

FUNCTIONS OF STATE IN MATERIALS UNDER CONDITIONS  
OF THERMAL FATIGUE

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To substantiate the critical state parameters when choosing the strength parameters the analysis of the thermodynamic state functions is made for bodies of simple geometry under thermal loading.

Under nonisothermal loading (as compared to isothermal one) a choice of the parameters which describe unambiguously the maximum damage of the material within a cycle is complicated. This can be attributed to different laws of temperature and stress variation in a cycle, different deformation mechanisms at low and high temperatures, the necessity of taking into account the temperature dependence of the material mechanical and thermophysical properties. The above factors should be taken into account when studying thermal fatigue, the nature of which is associated with the exhaustion of the material capability to resist cyclic thermal stresses arising owing to the transition of a part of thermal energy into mechanical one. Therefore, more complete information on the process of loading can be obtained from the analysis of the variation of such thermodynamic state functions as the entropy density  $S$ , internal energy density  $U$ , enthalpy density  $H$ , free energy density  $F$ ,  $G$  Gibbs energy density which take

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into account a total of thermal, mechanical, chemical, electromagnetic and other processes.

Consider the process of the nonisothermal loading of a solid as the variation of the state functions depending on the heat exchange boundary conditions, the character of deformation and the material properties. For this purpose we shall plot thermodynamic cycles, which relate the state functions, making use of the earlier obtained results of the calculation of the disbalance between the thermal and thermal-stress state of the solids with a simple geometry under various heat exchange boundary conditions. The temperature of the material is taken as the argument. To present the results in the general form relative magnitudes have been accepted for temperatures  $\theta$ , strains and stresses  $\sigma^*$ , entropy  $S^*$ , thermodynamic functions  $F^*$ ,  $G^*$

$$\theta = \frac{T - T_0}{T_e - T_0}; \quad \sigma^* = \frac{\sigma(1-\mu)}{E\alpha(T_e - T_0)}; \quad S^* = \frac{(S - c_{\epsilon=0} \ln T/T_0)(1-\mu)}{E\alpha^2(T_e - T_0)} - \theta = \sigma^*;$$

$$F^* = \frac{\left( F + \int_{T_0}^T dT \int_{T_0}^T \frac{c_{\epsilon=0}}{T} dT \right) (1-\mu)}{E\alpha^2(T_e - T_0)^2} = \sigma^* \left( \frac{\sigma^*}{2} - \theta \right);$$

$$G^* = - \frac{\left( G + \int_{T_0}^T dT \int_{T_0}^T \frac{c_{\epsilon=c}}{T} dT \right) (1-\mu)}{E\alpha^2(T_e - T_0)^2} = - \sigma^* \left( \frac{\sigma^*}{2} + \theta \right),$$

where  $E$  - is the elasticity modulus,  $\mu$  - is the Poisson ratio,  $\sigma$  - is the stress tensor,  $c_{\epsilon=0}$  is the specific heat in the absence of deformation,  $T_0$ ,  $T$  are the initial and current temperatures,  $T_e$  - is the temperature of the environment.

Figures 1a and 1b show thermodynamic cycles  $S^* = f(\theta)$   $F^* = -G^* = f(\sigma^*)$  for the volume element on the surface of an infinite cylinder with a uniform initial temperature distribution and heat exchange boundary conditions corresponding to Newton's law ( $Bi = \alpha^*R/\lambda$ ,  $\alpha^*$  - is the surface heat transfer coefficient,  $R$  is the cylinder radius,  $\lambda$  - is the material thermal conductivity coefficient). The thermodynamic cycle  $S^* = f(\theta)$  coincides with the  $\sigma^* = f(\theta)$

cycle to an accuracy of the constant factor  $\alpha^{-1}$  and is related to the variation of the elastic strains with temperature under thermal loading. In heating and cooling the half-cycles are mirror-inverse. If we superimpose the curves representing the thermodynamic cycle with the material yield strength temperature dependence, the intersection points of the respective curves will characterize the loading conditions inducing plastic deformation in the heating and cooling half-cycles.

The thermodynamic cycle in the  $F^* = -G^* = f(\theta)$  coordinates characterizes the work of elastic deformation in tension or compression up to the strain  $\epsilon = \alpha(T - T_0)$ . The difference between thermodynamic functions  $F^*$  and  $G^*$  is the work of deformation for the corresponding temperature. Here the following relationship is satisfied: the change in the function  $F^*$  in heating is equal to the change in the function  $G^*$  in cooling.

In the general case a change in the entropy under nonisothermal loading is written as

$$S = \frac{E}{1-2\mu} \int_0^\epsilon \frac{\sigma d\epsilon}{T} + C_{\epsilon=0} \ln T/T_0 = S_d + S_T,$$

where  $S_d = \int_0^\epsilon (\sigma/T) d\epsilon$  - is a part of the entropy related to the deformation of the body.  $S_T = C_{\epsilon=0} \ln T/T_0$  - is a part of the entropy associated with a change in the thermal state of the body. At the same time if the total strain components are taken into account,  $S_d$  can be represented as the sum of the shape-change induced entropy and volume-change induced entropy,  $S_d^*$  can be written in the relative form as

$$S_d^* = \frac{S_d(1-2\mu)}{E\alpha^2(T_e - T_0)} = \int_0^\theta \frac{\sigma^* \alpha (\theta - \sigma^*)}{\theta} = \int_0^\theta \frac{\sigma^* d\theta}{\theta} - \int_0^\theta \frac{\sigma^* d\sigma^*}{\theta} = S_s^* - S_v^*.$$

Figure 2 presents a thermodynamic cycle  $S_d^* = f(\theta)$  for the material on the surface of a cylinder under various heat exchange boundary conditions. For the most heavily loaded portion of the body this cycle relates the shape- and volume-change induced entropy variation with the temperature in the process of heating and cooling. The data presented show that at  $Bi = \infty$  the loading is characterized first by an adiabatic curve and

then by an isothermal one. In this case at the adiabatic portion of the process thermal deformation is transformed completely into mechanical one and the transition of the thermal energy into mechanical one is also maximal. At  $Bi < \infty$  the share of thermal energy which transforms into mechanical one decreases, as well as the absolute value of the entropy related to the final state of the material in the loading half-cycle. The approach to the adiabatic process at the initial stage of loading and the transition to the isothermal one at the end of the half-cycle is a common feature for different heat exchange boundary conditions. Under similar boundary conditions in the half-cycles the thermodynamic cycle is reversible, i.e. after heating and cooling the state of the material corresponds to the initial one. The areas within the curves represent the elastic deformation work.

The cycle considered gives a general idea of the transition of the thermal energy into mechanical one under the thermal load effect and enables qualitative and quantitative estimate of the loading conditions effect upon the above process.

The damage accumulation in the material represents variation of the shape-change induced entropy  $S_s^* = \int_0^{\theta} (\sigma/\theta) d\theta$ . Figure 3 presents variation in this part of the entropy with temperature for the cylinder surface. The data show that the maximum absolute value of the shape-change induced entropy is reached at  $Bi = \infty$ . With a reduction in the heat exchange intensity its magnitude decreases. At the end of the half-cycle  $S^*$  has the final value due to the nonisothermal character of the process. Under identical heat exchange conditions the entropy variation in the subsequent half-cycle is equal to that in the preceding one and therefore the  $S_s^* = f(\theta)$  cycle is closed. Under other conditions the cycle is open since in this case only a certain part of the entropy was considered instead of the total one.

The character of deformation may change under specific loading conditions. A transition may occur in the material from elastic state into elasto-plastic one. Let us see in what way a change in the deformation character affects the thermodynamic cycle. A nomograph is given in Fig.4 which allows such an estimate to be made. As an example the regularity in  $S$  and  $T$  variation is considered for the 1Kh18N9T steel cylinder surface in heating ( $T_0 = 300$  K,  $T_e = 1300$  K,  $Bi = 1$ ) and in cooling ( $T_0 = 1300$  K,  $T_e = 300$  K,  $Bi = 1$ ). We assume

that plastic deformation is localized in a small region and consequently, after the loading cycle is completed residual stresses are absent.

In the heating half-cycle the portion  $OA_1$  relates to the material elastic behaviour and the ordinate of the point A corresponds to the change in the entropy. With the account taken of the above assumptions, the character of the deformation does not affect the strain state. Therefore, an increase in the entropy will depend on the  $E/E_{300}$  ratio. The magnitude of the increase in the entropy can be obtained from the similarity of the triangles PDK and MLK. The plastic strain increase ceases at the point B. Then follows unloading and the entropy increment is determined by the  $E_c/E_{300}$  ratio for zero strain. The final magnitude of the entropy in the half-cycle is determined by an algebraic summation of the current values and corresponds to the ordinate of the point C. The latter is the initial state for the subsequent half-cycle.

The final magnitude of the entropy differs from the initial one due to plastic deformation. Being a damage accumulation measure this difference is the result of irreversible changes in the material and can be employed as an argument in the thermal fatigue equations.

#### SYMBOLS USED

- $E_c, E_{300}$  - secant modulus, elasticity modulus at  $T = 300$  K (MPa)
- Bi - Bi criterion
- $\epsilon_{0.2} = f(T)$  - temperature dependence of the elasto-plastic strains giving rise to the residual strain 0.2 %

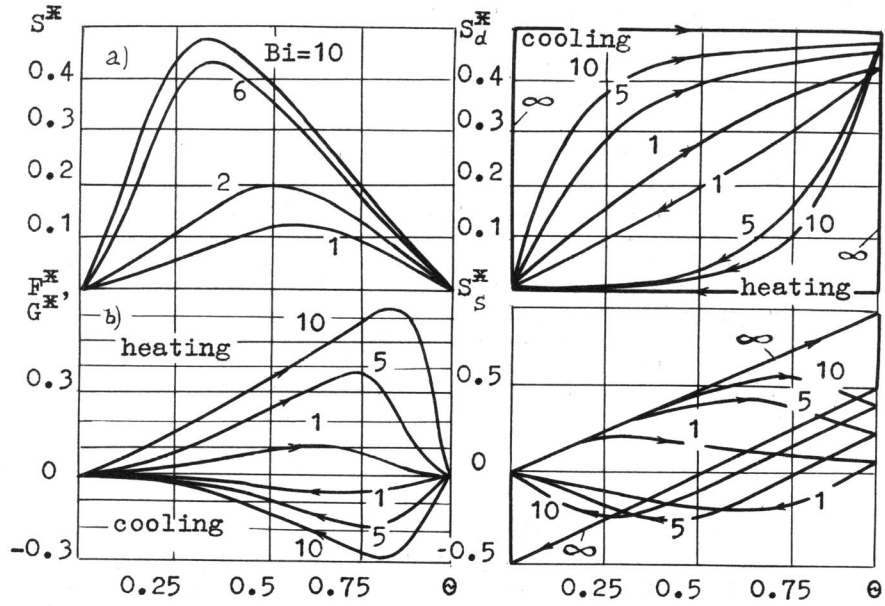


Figure 1

Figures 2, 3

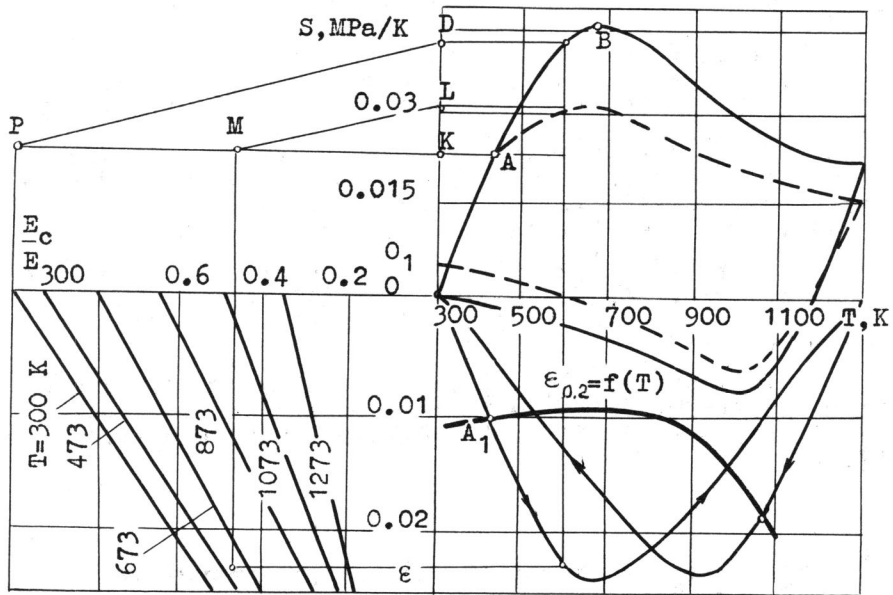


Figure 4