FATIGUE CRACK GROWTH UNDER MIXED MODE AND MODE II CYCLIC LOADING

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The knowledge of the conditions at fatigue crack growth are of great importance for the achievement of measures and concepts for the prevention and prediction of damage. To gain further insight of this problem, fatigue tests were performed on 7075-T3 aluminum alloy (AlZnMgCu 1.5) under Mixed-Mode- and Mode II-cyclic loading. The results of these tests show that compared to Mode I-loading the crack growth rate increases under combined Mode I- and Mode II-loading. The augmentation of the crack growth rate is proportional to the K_{II}/K_I-ratio of the fatigue stress. The so far known concepts for the mixed-mode fatigue crack growth prediction are not sufficient enough to fit the experimental results.

INTRODUCTION

Fatigue crack growth in complex loaded components and structures often causes failure or damage. In order to give an accurate prediction of fatigue lives or inspection intervals, the behavior of fatigue cracks under mode I condition has been object of detailed investigations (Schwalbe (1), Munz (2) and Shijve (3)). On the contrary the influence of overlapping mode I and II on the propagation of fatigue cracks is largely unknown. Some theoretical reflections have been already made (summary in Henn (4,5)) but the experimental results are not sufficient.

Due to the great importance for system safety and maintenance the crack growth under combined mode cyclic loading has been investigated in this examination.

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THEORIES AND CONCEPTS TO PREDICT THE FATIGUE CRACK GROWTH UNDER MIXED-MODE-LOADING

The propagation of fatigue cracks is mainly influenced by cyclic plastic deformations at the crack tip. In case of single mode I crack opening these deformations are directed by the cyclic stress intensity factor ΔK_I for mode I and therefore one obtains the crack growth rate da/dN as function of ΔK_I. On the contrary to mode I almost no fully developed and experimentally confirmed concepts for mixed-mode- and mode II-fatigue crack growth predictions are at disposal. In this case more complicated theories are necessary, since the mixed mode crack does not follow the direction of the originally present crack, but will be more or less deflected. In the following text a summary of the so far known concepts for the prediction of mixed mode fatigue cracks is given.

Strain energy density criterion

In case of isotropic and homogenic material, the originally for monotonic loading developed criterion by G.C. Sih (6) can also be formulated for mixed mode fatigue loading. The crack growth rate da/dN is expressed with the help of the Paris law in dependence of the cyclic energy density factor ΔS_{min}.

$$da/dN = C'(\Delta S_{min})^{m'} \quad \text{.......................................(1)}$$

The parameters C' and m' are experimentally obtained.

Criterion by Tanaka

As a basis for his criterion Tanaka (7) also uses a modified Paris relation as description for crack growth under mixed mode.

$$da/dN = C(\Delta K_v)^m \quad \text{.......................................(2)}$$

The parameters C and m as well as the cyclic comparative stress intensity factor ΔK_v are analytically intended. The estimation is based on the theory by Weertman (8) or Lardner (9).

Criterion by Fischer

In this concept the Paris law also serves as a description for the crack growth rate under combined mode cyclic loading.

$$da/dN = C(\Delta K_v)^m \quad \text{.......................................(3)}$$

Here the material constants C and m are again experimentally obtained, while ΔK_v is determined through the formation of the total differential of one of the known criteria for static mixed mode loading.

$$\Delta K_v = \frac{\partial K_{vstat.}(K_I, K_{II})}{\partial K_I} \Delta K_I + \frac{\partial K_{vstat.}(K_I, K_{II})}{\partial K_{II}} \Delta K_{II} \quad \text{..........(4)}$$

Generalized criterion by Richard/Henn

This criterion for the calculation of a comparative stress intensity factor was initially formulated for static loading by Richard (11):

$$K_v = \frac{1}{2} K_I + \frac{1}{2} \sqrt{K_I^2 + 4\alpha_1^2 K_{II}^2} \quad \text{..........................(5)}$$

whereas α_1 is a material dependent parameter and at the same time it describes the ratio of the fracture toughness K_{Ic}/K_{IIc}.
For fatigue loading the criterion can be expanded as follows:

$$\Delta K_v = \frac{1}{2} \Delta K_I + \frac{1}{2} \sqrt{\Delta K_I^2 + 4\alpha_1^2 \Delta K_{II}^2} \quad \text{.....................(6)}$$

under the assumption that

$$R = \frac{K_{Imin}}{K_{Imax}} = \frac{K_{IImin}}{K_{IImax}} = \text{const.} \quad \text{...........................(7)}$$

For the calculation of the crack speed the Paris law according to eqn. (3) is also quoted. In order to give a prediction of the crack deviation angle in all criteria it is assumed that the crack deflection under cyclic mixed mode loading is the same as in the static case.

STRESS FIELD AT THE TIP OF A KINKED CRACK

A crack subjected to mixed mode stress generally changes its growing direction. The stress field at a straight crack is characterized by the stress intensity factors K_I and K_{II}. As a result of the mixed mode loading a additional crack in the length of a_z appears which is kinked at a deviation angle of φ_0. The stress intensity factors at the tip of the additional crack are $\overline{K}_I$ and $\overline{K}_{II}$.

The knowledge of the stress intensity factors $\overline{K}_I$ and $\overline{K}_{II}$ at the kinked crack is of decisive importance to achieve a relation between the crack growth rate da/dN and the loading parameter. Often $\overline{K}_I$ and $\overline{K}_{II}$ of a infinitely short kinked crack are given in dependence of the K-factors of the initial crack. As Finite-Element-calculations of a kinked crack in CTS-specimens show, is the

$\overline{K}_{II}$-factor very small and can be neglected in case of monotonic loading. In order to calculate the $\overline{K}_I$-factor for finite additional crack lengths in the CTS-specimen Tenhaeff (12) proposes the following equation:

$$\overline{K}_I = \frac{1}{2} K_I^* + \frac{1}{2} \sqrt{K_I^{*2} + 6 K_{II}^{*2}} + 38.1 \frac{F}{w^2 t} (a_z \sin\varphi_o)^{1.5} \quad \text{.........(8)}$$

Here $K^*_{I,II}$ are the K-factors of a straight crack with a length of $a_o + a_z \cos\varphi_o$, φ_o is the crack deviation angle, w the specimen width and t the specimen thickness, figure 1.

As fatigue tests under mixed mode loading show, crack growth rate da/dN augments with a increasing mode II share. If $\overline{K}_I$ at the kinked crack would solely be responsible for crack growth there should be no change of the crack growth rate. The assumption that the growth of a kinked crack is only described by the $\overline{K}_I$-factor can apparently not be maintained for fatigue crack growth. Therefore the curved path of a fatigue crack in a CTS-specimen which was generated under mixed-mode-loading (load angle $\alpha = 75^o$ and $K_{II}/K_I = 1.49$) was simulated with finite elements and the stress intensity factors were computed at different points of the crack, figure 2. The calculations supply the following results:

TABLE 1 - Stress Intensity Factors $\overline{K}_I$ and $\overline{K}_{II}$ at the Tip of Kinked Crack ($\alpha = 75^o$, $a_o = 54.49$ mm)

$\frac{a_o+a_z}{w}$	$\frac{\overline{K}_I}{\sigma\sqrt{\pi a}}$	$\frac{\overline{K}_{II}}{\sigma\sqrt{\pi a}}$	$\overline{K}_{II}$ in % $\overline{K}_I$
0.66	2.918	0.057	1.95
0.71	3.632	0.097	2.67
0.77	4.664	0.284	6.09
0.83	6.103	0.399	6.54

As can be concluded from table 1 the share of $\overline{K}_{II}$ compared to $\overline{K}_I$ increases with growth of the additional crack. Obviously this $\overline{K}_{II}$-factor should no longer be neglected especially considering larger crack lengths that appear during fatigue tests.

EXPERIMENTAL INVESTIGATIONS

With the help of these investigations it should be clarified how fatigue cracks grow under combined tension and shear stresses. After numerous preliminary tests under pure mode I-cyclic loading, figure 3, the propagation rate at constant ΔK_V and different $\Delta K_{II}/\Delta K_I$-ratios will be investigated.

Experimental setup and procedure

Tests were performed on CTS-specimens from 7075-T3 in combination with a special loading device by Richard, figure 4. Proceeding from a mode I fatigue pre-crack with the length of a = 54 mm (a/w = 0.6) the mixed-mode- or mode II-loading will be subjected by turning the device. For better recognition and judgement of the influence of the crack deviation on the crack growth rate the stress intensity and the R-ratio is kept constant during the entire procedure.

$$(\Delta K_I)_{pre\text{-}crack} = (\Delta K_V)_{kinked\ crack} = const. \quad \ldots\ldots\ldots\ldots\ldots (9)$$

The required load ΔF at the straight crack yields from

$$\Delta F = \frac{(\Delta K_I)_{Anri\beta} \cdot w \cdot t}{\sqrt{\pi a} \cdot Y_I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

with Y_I as dimensionless geometric function for the CTS-specimen, Richard (11). At the kinked crack one obtains F from eqn. (8).

During the tests the crack length was constantly measured with the aid of the D.C.-potential drop method and was recorded together with the number of cycles endured.

All tests were carried out with $\Delta K_V = 7$ MPa$\sqrt{m}$ and $R = K_{min}/K_{max} = 0.5$ at a constant frequency of 40 Hz. At this point of the da/dN-K_I curve the crack growth rate for both rolling directions amounts to about $1.6 \cdot 10^{-4}$ mm/cycle, figure 3. Seven mixed mode cases (loading angle $\alpha = 0^o, 15^o, \ldots, 90^o$) have been investigated.

Results and Discussion

The tests on 7075-T3 show the following results. The crack growth rate da/dN obviously increases with growing K_{II}/K_I-ratio, figures 5 and 6.

TABLE 2 - Crack Growth rates da/dN and Crack Deviation Angle in 7075-T3 for Several Mixe Mode Cases (Load Angle α)

load angle α (^{o})	K_{II}/K_I	deviation angle φ_0 (^{o})	crack growth rate da/dN (mm/cycle)
0	0 (Mode I)	0	$1.5 \cdot 10^{-4}$
15	0.1	-14.4 ± 2.8	$1.52 \cdot 10^{-4}$
30	0.23	-23.9 ± 3.5	$1.76 \cdot 10^{-4}$
45	0.39	-37.0 ± 3.5	$2.15 \cdot 10^{-4}$
60	0.68	-45.6 ± 2.0	$2.28 \cdot 10^{-4}$
75	1.49	-55.8 ± 3.2	$2.53 \cdot 10^{-4}$
90	∞ (Mode II)	-62.8 ± 3.2	$2.67 \cdot 10^{-4}$

The crack deviation angles vary slightly from those obtained under static loading. In figure 7 a formula for the calculation of the deflection angles under monotonic stresses (see Richard (11)) is compared to those measured in the fatigue tests.

At the beginning of the mixed mode loading under large load angles ($\alpha = 75^0$ and $\alpha = 90^0$) the crack growth is considerably retarded, figure 6, which can be atributed to friction effects on the surfaces of the mode I pre-crack. Nevertheless these retardations vanish as soon as the kinked crack has slightly grown in the new direction. It is to be assumed that the above mentioned friction effects are also responsible for the differences in crack deviation angle at static and cyclic loading shown in figure 7.

In the tests with ΔK_V = const. the required loads F_{max} and F_{min} which must be set on the traction machine were calculated according to eqn. (8). This equation is a approximation formula through which certain adulteration of test data can appear. In order to exclude these faults, tests with constant $\Delta F = 3.5$ kN and $R = 0.5$ at $\alpha = 75^0$ are executed. Following the path of the fatigue crack was measured and stress intensity factors $\overline{K}_I$ and $\overline{K}_{II}$ were computed with the help of the finite-element-method. From the da/dN-$\Delta\overline{K}_I$-curve, figure 8, one receives at $\Delta\overline{K}_I = 7 MPa\sqrt{m}$ the crack growth rate da/dN = $2.2 \cdot 10^{-4}$ mm/cycle. The latter is slightly lower compared to the one obtained from the tests with constant ΔK_V according to eqn. (8). There the crack growth rate da/dN equals $2.53 \cdot 10^{-4}$ mm/cycle (see table 2), which points to the fact the calculated loads F_{max} and F_{min} were somewhat to high. The crack speed at the kinked crack is still considerably higher than that of the straight crack. The observed effects which were also measured by Dai (13) are obviously characteristic for the propagation of kinked fatigue cracks.

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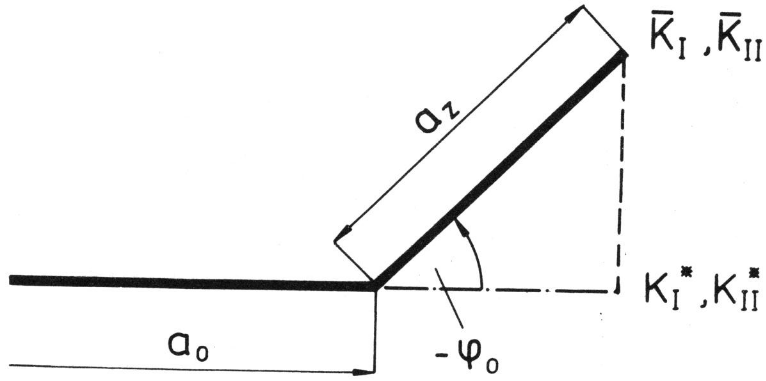


Figure 1 Crack tip geometry

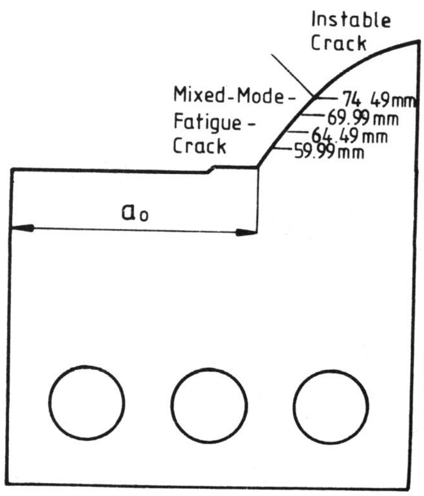


Figure 2 Trajectory of a mixed-mode fatigue crack (α = 75°)

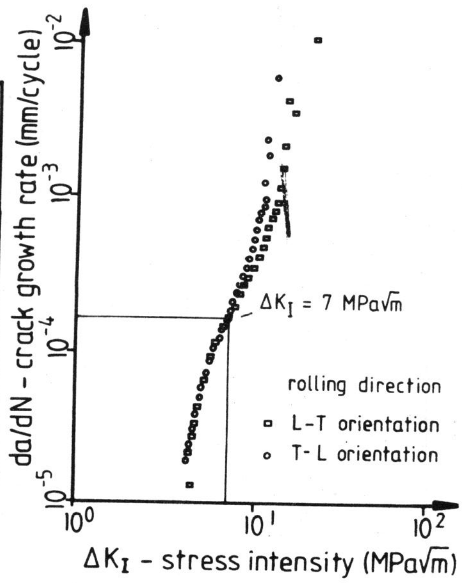


Figure 3 da/dN-ΔK_I curve for different crack orientations

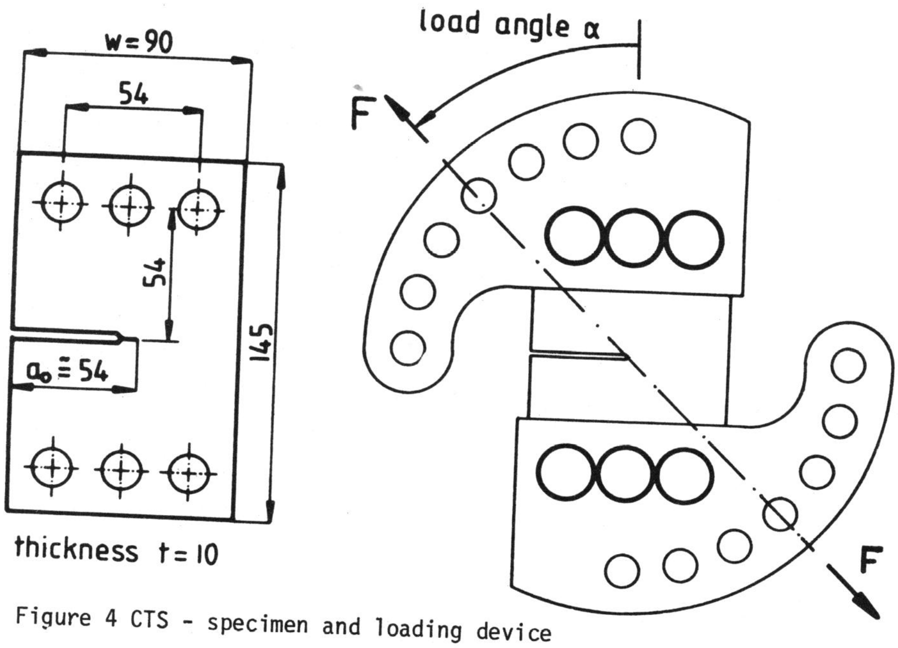
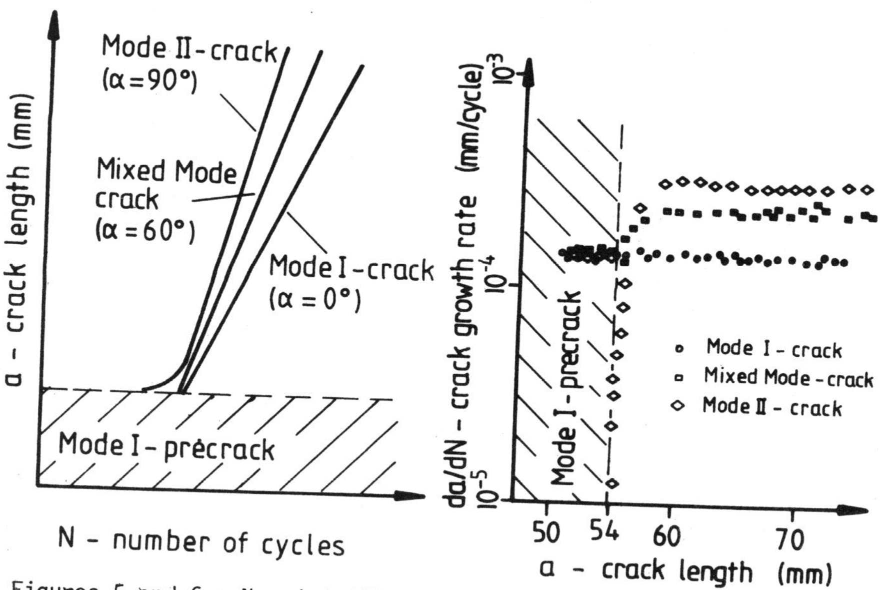


Figure 4 CTS - specimen and loading device



Figures 5 and 6 a-N and da/dN-ΔK_V curves at ΔK_V = 7 MPa$\sqrt{m}$ and R = 0.5 for different loading directions (α = 0°, 60°, 90°)

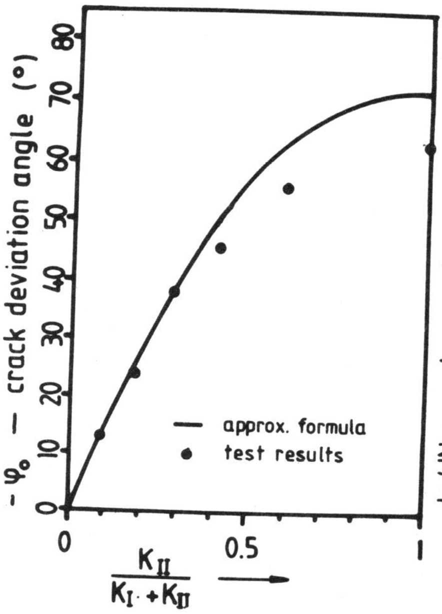


Figure 7 Crack deviation angles for different loading directions

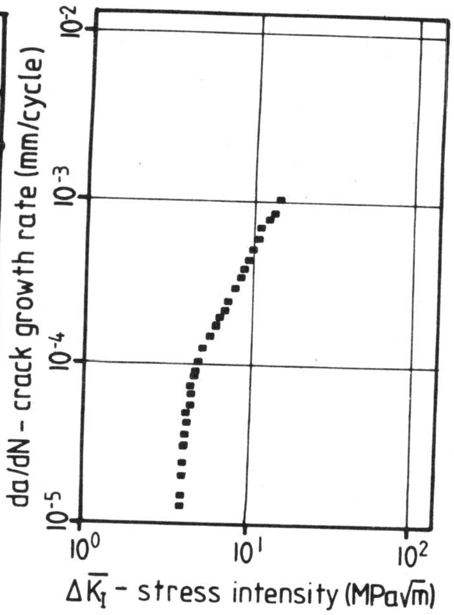


Figure 8 da/dN-$\Delta\bar{K}_I$ curve for ΔF = 3.5 kN and R = 0.5 at α = 75°

SYMBOLS

a = Crack length (mm)

C,C' = Constants of the Paris Law

F = Load (N)

K = Stress intensity factor (MPa√m)

m,m' = Constants of the Paris Law

R = Stress intensity ratio

t = Specimen thickness (mm)

w = Specimen width (mm)

Y = Geometry function for the CTS-specimen

α = Load direction angle (°)

α_1 = Constant

φ_o = Crack deviation angle (°)