FATIGUE ANALYSIS FOR MULTIAXIALLY LOADED STRUCTURES BASED ON FINITE ELEMENT METHOD

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At technical components quite often multiaxial stress/strain states are present. These have a significant effect on the fatigue behaviour. In the present paper a FEM-based fatigue evaluation concept (FEMFAT) is described, which includes a complete three-dimensional elastic-plastic stress/strain analysis module and another module for the evaluation of the crack initiation life using a new incremental damage concept based on plastic work. FEMFAT is applicable to all types of (mechanical) multiaxial fatigue stresses.

INTRODUCTION

Due to the complex shape of technical components and multiple loading inputs stress and strain distributions at the fatigue critical areas can be multiaxial. For a reliable fatigue analysis this complicated stress/strain situation has to be accounted for. If the fatigue life evaluation is performed analytically, normally two steps have to be performed: First, the multiaxial stress/strain path has to be calculated for the given loading history. The calculated stresses and strains are then the basis for the determination of the instantaneous amounts of damage parameters (damage indicators) which are being used to form a bridge between the multiaxially stressed component and the fatigue behaviour in constant amplitude tests on simple uniaxial fatigue specimens. If the loading condition is also variable in time, a cumulative damage hypothesis has additionally to be applied. FEMFAT (1,2) the newly developed fatigue life analysis procedure for specimens/components with complex

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loading and stress/strain conditions fits in general into this line of procedures. However, some shortcomings/limitations of the conventional multiaxial stress/strain and fatigue analysis methods are overcome.

An overview of the FEMFAT program system is given in Figure 1. Those individual parts of FEMFAT where the stresses/strains are calculated and where the fatigue life is evaluated will be briefly described in the present paper.

FEMFAT is a continuum mechanics analysis. That means, that microstructural processes are not explicitly considered. However, the stress/strain analysis part of FEMFAT is able to give a complete record of all stresses and strains in all planes through all points of a test piece. That means, that also those stresses/strains can be calculated which are important for the microstructural processes (as for example the maximum shear stresses and the normal strains, etc.).

In the past several damage parameters (damage indicators) for multiaxial fatigue analyses have been proposed (3,4). Among these are the equivalent stress or the equivalent strain (at higher elastic-plastic deformations) or plastic work, or parameters as those by Brown and Miller (5) or by Kandel et al. (6) or semiempirical as discussed in (7). Especially for the long life range procedures as those by Trost et al. (8), Zenner (9), Simbürger and Grubisic (10) have successfully been applied in various cases. However, regarding the limited life range there exists in general the following limitation: The stress paths in the stress space must be closed or there must exist always defined starting and defined end points of the stress path in the stress space. In-between these points the stress path must form loops which are comparable to the hysteresis loops which are observed under constant amplitude conditions. FEMFAT enables damage calculations for arbitrary stress paths in the stress space and needs convenient constant amplitude plastic work vs. cycle number to crack initiation data for the material under consideration as a basis only. The damage calculation is incremental and considers the individual hardening surfaces of the multiaxial material model being used.

THREE-DIMENSIONAL ELASTIC-PLASTIC STRESS/STRAIN ANALYSIS

FE methods have become predominantly important for stress/strain analyses of technical components. For the consideration of elastic-plastic deformations a suitable material model has to be used in combination with the FE analyses. FEMFAT includes an own FE module with special computer time saving parts/procedures, as for example the utilization of the substructure technique.

The multiaxial material model being used in FEMFAT is a multi surface model as originally proposed by Mroz (11). The yield range of a material is subdivided into several subranges, each with a certain work hardening parameter. If the von Mises criterion is applied to the general three-dimensional case, the transitions from one range with a certain work hardening parameter to another with another work hardening parameter are marked by cylinders in the stress space. The projections of these cylinders onto the π plane are circles. These circles can move when outer stresses are applied to the system. This occurs in an incremental manner until the circle which terminates the next range with another work hardening parameter is reached. A new stress point within one range with a constant work hardening parameter (where the instantaneous moving (Mroz) circle has also to pass through) is always determined according to the following equation:

$$d\sigma_{ij} = D^{e}_{ijkl}\,(d\epsilon_{kl} - d\epsilon^{P}_{kl})$$

with $d\epsilon_{kl}$ as the total strain increment from the FE program, $d\epsilon^{P}_{kl}$ as the plastic strain component perpendicular to the instantaneous (Mroz) circle due to the flow rule after Drucker and D^{e}_{ijkl} as the elastic material matrix. (The rules after which the Mroz circles are shifted and the way how the slight inaccuracies in computations caused by finite stress increments are compensated are described in (1,2)).

In Figure 2 a complete flow diagram of the material model for one integration point at a complex test piece or component is given.

As already mentioned above, the multiaxial material model and the FE program have to interact in a close manner. This is indicated in Figure 3. The output of the FE program (left side) are total strain increments. The individual program modules (notations and functions) for the evaluation of the stresses (strains) in the integration points are shown on the right. The messages from these program modules back to the FE program on the left side are to perform another iteration step or to call a new load increment.

The stress/strain analysis part of the FEMFAT program gives a continuous record of all displacements/strains/stresses in all integration points of the test piece/component and plastic work can also explicitly be determined.

FATIGUE LIFE CALCULATION IN THE CRACK INITIATION STAGE

For the evaluation of fatigue life FEMFAT calculates plastic work and the fatigue damage for all runnings through the ranges with constant work hardening parameters in the material model. This is done as described in the following:

A simple uniaxial $\sigma-\epsilon$ hysteresis loop with one element $l = 1$ in the plastic range shall be considered first (compare Figure 4a). The plastic work per this hysteresis loop is W_{p1}, the hatched area in the figure (this hatched area includes two parts: one above and one below the $\sigma = 0$ axis). For the evaluation of the damage as caused by one hysteresis loop, the plastic work vs. cycle number to crack initiation correlation as derived from constant amplitude tests (compare Figure 4c) is used. First, N_1 is determined corresponding to W_{p1} and then the inverse of N_1 is taken as the damage per one loop (D_1). The specific damage per unit plastic work is now D_1/W_{p1}.

In Figure 4b a hysteresis loop with two elements $l = 1$ and $l = 2$ is shown. The total plastic work per one hysteresis loop is now W_{p2}. W_{p2} includes the hatched areas below the elements $l = 1$ and $l = 2$ and the $\sigma = 0$ axis. (It has to be specially noted that the sum of both hatched areas below $l = 1$ and the $\sigma = 0$ axis in Figure 4b is not identical to W_{p1} in Figure 4a, but smaller!) From Figure 4c the cycle number to crack initiation, N_2, for the hysteresis loop with W_{p2} can be derived and the damage per one loop is: $D_2 = 1/N_2$. The specific damage per one unit work within the hatched areas below $l = 2$ and the $\sigma = 0$ axis of the hysteresis loop in Figure 4b can now be calculated as follows:

$$d_2 = \frac{1/N_2 - W_{p(2,1)} \cdot d_1}{W_{p2}}$$

In this equation $W_{p(2,1)} \cdot d_1$ represents the contribution to the damage by the hatched areas below elements $l = 1$ and the $\sigma = 0$ axis.

If l = n elements in the plastic range are present in a hysteresis loop, the specific damage d_l can be calculated after:

$$d_l = \frac{1/N_l - \sum_{k=1}^{l-1} W_{p(l,k)} \cdot d_l}{W_{p(l,l)}} ,$$

except for a hysteresis loop with only one element. In this case d_l is:

$$d_l = \frac{1/N_l}{W_{pl}}$$

As a basis for the calculations of the amounts of specific damage, d_l, the W_{pSSP} vs. N_i correlation from uniaxial constant amplitude tests has to be available only. The calculated d_l values are then taken for the determination of the damage due to the plastic work which is performed during the transitions through the ranges with constant work hardening parameters in the multiaxial material model. The amounts of plastic work during the transitions are calculated using the results of the FE based stress analysis as described in the previous section and are then multiplied by the corresponding d_l values in order to get the instantaneous state of fatigue damage at every instant in time during the given multiaxial stress history.

It might be of interest that if a history is not multiaxial but only uniaxial, the described procedure is equivalent to the application of Rainflow counting and to calculate the damage based on closed hysteresis loops.

FEMFAT has already been applied to several test pieces: a car axle (2), a part of the rear wheel suspension of a sports car (12), other notched test pieces. The results so far look promising. Regarding further development of the concept it may be worthwhile to consider the introduction of an additional mean stress/mean potential term into the stress/strain analysis.

REFERENCES

(1) Ott, W., "Neues Verfahren zur Betriebsfestigkeitsanalyse komplexer Bauteile auf der Grundlage der FEM", DFVLR, Cologne, DFVLR-FB 87-15, 1987.

(2) Nowack, H., Ott, W., Foth, J., Peeken, H. and Seeger, T., in: ASTM-STP 942 "Low Cycle Fatigue: Directions for the Future". Edited by Solomon, Halford, Kaisand, Leis, Am. Soc. Test. and Mats., 1988, pp. 987-1006.

(3) Garud, Y.S., Trans. of ASME, Vol. 103, 1981, pp. 118-125.

(4) Fash, J.W., "An Evaluation of Damage Development during Multiaxial Fatigue of Smooth and Notched Specimens", College of Engineering, University of Illinois, Urbana-Champaign, USA, Rep. No. 123, 1985.

(5) Brown, M.W. and Miller, K.J., "A Theory for Fatigue Failure under Multiaxial Stress-Strain Conditions", Proceed. of the Institute of Mechanical Engineers, Vol. 187, No. 65, 1973, pp. 745-755.

(6) Kandil, F.A., Brown, M.W. and Miller, K.J., "Biaxial LCF Fracture of 316 Stainless Steel at Elevated Temperatures", Book 280, The Metals Society, London, 1982, pp. 203-210.

(7) Fash, J.W., Socie, D.F. and McDowell, D.L., in: ASTM-STP 853 "Multiaxial Fatigue". Edited by Miller, Brown, Am. Soc. Test. and Mats. 1985, pp. 497-513.

(8) Troost, A. and El-Magd, E., Metall, Vol. 7, No. 31, 1977, pp. 759-764.

(9) Zenner, H. and Richter, I., Konstruktion, Vol. 29, 1977, pp. 11-18.

(10) Simbürger, A., "Festigkeitsverhalten zäher Werkstoffe bei einer mehrachsigen phasenverschobenen Schwingbeanspruchung", Fraunhofer Inst. für Betriebsfestigkeit, Darmstadt, Germany, Report No. FB-121, 1975.

(11) Mroz, Z., Acta Mechanica, Vol. 7, 1969, pp.199-212.

(12) Gerich, F., "Festigkeitsberechnung eines PKW-Querlenkers nach der FE-Methode", Dipl.-Arbeit FH Köln.

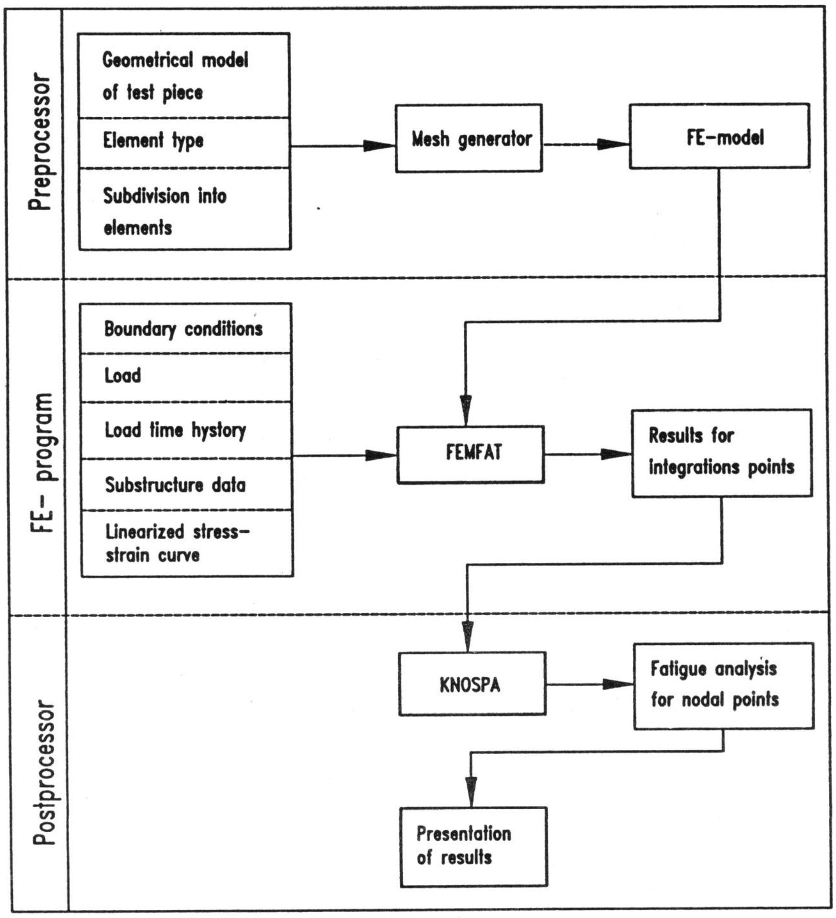

Figure 1 Overview of FEMFAT

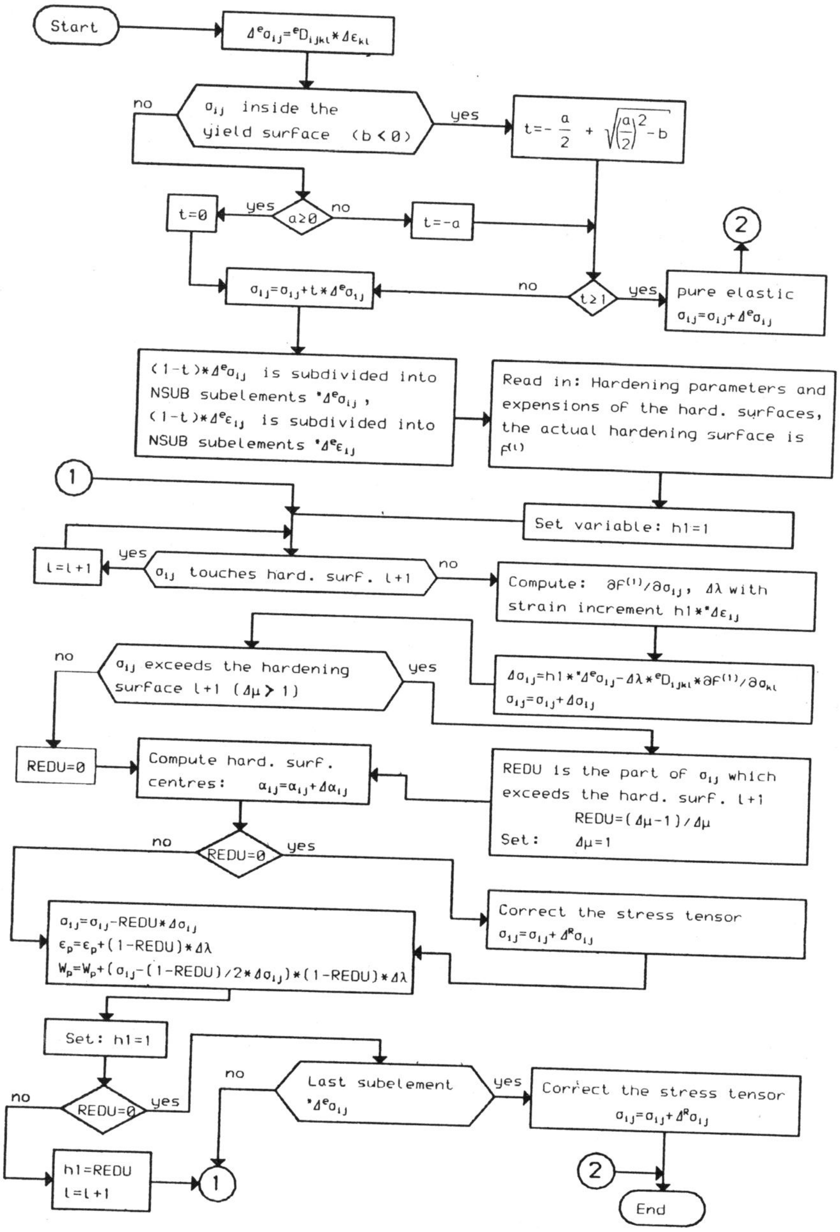

Figure 2 Elastic-plastic material model based on (11), flow diagram

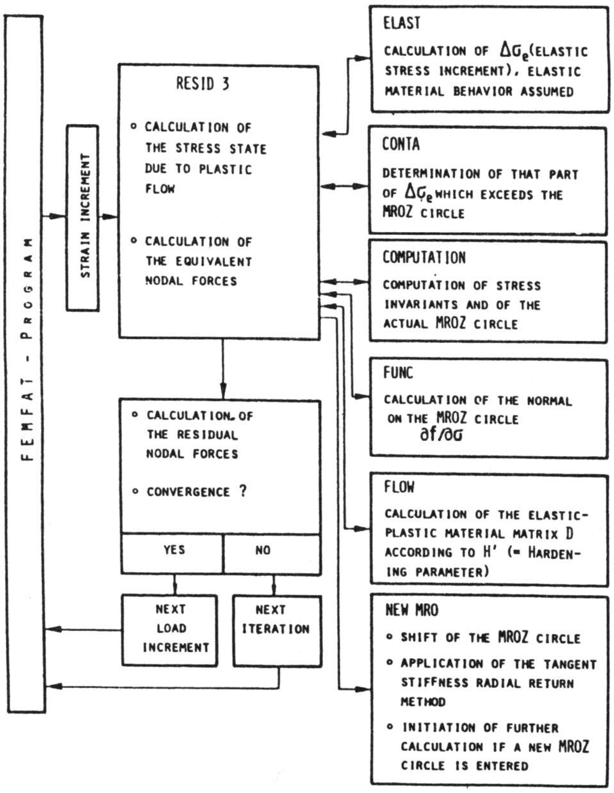

Figure 3 Interaction between the FE program and the elastic-plastic material model in FEMFAT

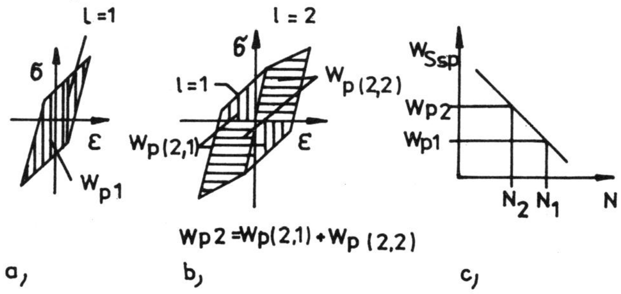

Figure 4 Evaluation of the damage due to elastic-plastic parts of $\sigma-\epsilon$ hysteresis loops