

EXPERIMENTAL DETERMINATION OF J-INTEGRAL BY USING THE  
LASER SPECKLE METHOD

Wei Qun\* , Zhang Jingjian\* , Zhang Zhongyi\*

The J-integral is a rigorous parameter of the stress-strain field in the fracture mechanics . In this paper, the research results of experimental determination of J-integral by using laser speckle method is introduced. The laser speckle can be used to measure the in-plan displacement of the object exactly ,to select a path of integral suitably and keep away from the unapplicable region of the plastic deformation for measuring the displacement . The laser speckle method is simple and reliable, and has considerable accuracy. By means of the structural model test of the horizontal notch in the heel of the gravity dam, it is proved that this method is feasible.

INTRODUCTION

The J-integral proposed by Rice has obvious physical meaning , it can be explained as the decreasing rate of the deformation work and is independent of integral path. The stress intensity toughness  $k_I$  can be obtained by J-integral with small yielding condition . The J-integral can be also regarded as a half-brittle failure criterion with large yielding condition in elasto-plastic fracture mechanics. Thus, J-integral is a rigorous parameter of the stress-strain theory and plays an important part in the fracture mechanics. Because the J-integral can avoid calculating elasto-plastic stress-strain field near the crack tip directly, it provides a convenient fracture criterion for calculating and measuring. At present, studies in fracture analysis for the heel of the gravity dam make some progress but most of them are limited to calculating stress intensity toughness by finite element method ( general element or singular element ). In fact , in the whole course of failure at the heel of the gravity dam, it undergoes elastic period , elasto-plastic period and plastic period, so it is difficult to

\* North China Institute of Hydropower, Handan City, Hebei Prov., P.R.C

determine stress, strain and displacement of the crack tip by means of the mathematical equations only which are derived from some hypotheses. For this reason, the investigation of measuring fracture parameters by using the model test is very signification, The objective and satisfactory results can be obtained if the model materials, boundary conditions, loading conditions and geologic structure are all satisfied with the requirements of the simulation laws. Although some fracture model tests with gypsum materials had been done, the accuracy displacement field was not got due to limited ends and means at that time. (Z.Zhao (1)). Now, the displacement field can be accurately obtained by using the laser speckle technique which has been rapidly developed in recent years. The application of the laser speckle method to calculate J-integral in hydrostructural model test is described in this paper.

THE LASER SPECKLE METHOD

When diffusely reflecting object is illuminated by laser light and imaged by an optical system, random speckles will modulate the image field. If the object surface is well focused, the image speckles will appear to move as if attached to the object surface. Because laser speckle can serve as markers for points on the object surface, so it becomes a new no-touched method to measure displacement and deformation of the object surface. In addition, laser speckle method has many advantages in simplicity setup, in higher sensitivity and accuracy and has got a wider and wider application in various fields. Since 1980, the laser speckle method has been introduced into hydrostructural model test by us and has been developed a set of practical measuring techniques. (Wei Qun, Zhang Jingjian (2),(3),(4))

The experimental arrangement of laser speckle method is schematically shown in Fig. 1. Where the camera is focused on the object surface and the photoplate is put on the back focal plane. The laser speckle information can be recorded by a photoplate with isochronous double exposure before and after the deformation of the object. When the specklegram which has been treated is put on the optical arrangement as shown in Fig.2, the displacement for a point can be calculated according to the measuring intervals of the Young's fringes at that point and can be presented as:

$$D = \lambda \cdot L / ( M \cdot \Delta ) \quad \dots \dots \dots (1)$$

$$d_s = \lambda \cdot L \cdot \cos \alpha / ( M \cdot \Delta ) \quad \dots \dots \dots (2-a)$$

$$d_p = \lambda \cdot L \cdot \sin \alpha / (M \cdot \Delta) \quad \dots \dots \dots (2-b)$$

The direction of displacement is perpendicular to young's fringe

THE TREATMENT OF THE SPECKLE DATA WITH SPLINE SPACE CURVED SURFACE AND THE CALCULATION OF THE J-INTEGRAL

We can regard dispersed displacement data measured point-by-point as a function of coordinates. ( x, y ) and form a spline space curved surface with continuous function and continuous arbitrary order derivativy . It is assumed that spline curved surface is smooth and is equal to a given value at the given point,thus, the spline function can be given by:

$$W(x,y) = a_0 + a_1 x + a_2 y + \sum F_i \cdot r_i^2 \cdot L_\epsilon(r_i^2 + \epsilon) \dots \dots (3)$$

where :  $r_i^2 = (x - x_i)^2 + (y - y_i)^2$  and

$$\sum F_i^2 = \sum x_i \cdot F_i = \sum y_i \cdot F_i = 0 \quad \dots \dots \dots (4)$$

Differential equations can be derived from Eq.1 :

$$\partial W(x,y) / \partial x = a_1 + 2 \sum F_i [ r_i^2 / (r_i^2 + \epsilon) + L_\epsilon(r_i^2 + \epsilon) ] (x - x_i) \dots \dots \dots (5)$$

$$\partial W(x,y) / \partial y = a_2 + 2 \sum F_i [ r_i^2 / (r_i^2 + \epsilon) + L_\epsilon(r_i^2 + \epsilon) ] (y - y_i) \dots \dots \dots (6)$$

Two sets of displacement component data, U and V, measured from N points are respectively substituted into Eq. 1, the two simultaneous equations , for U and V , can be obtained respectively . Considering the additional Eq. 4, the simultaneous equations can be solved , thus all the undetermined constants ,  $a_0$  ,  $a_1$  ,  $a_2$  and  $F_i$ , are determined respectively and form the U spline space curved surface and V spline space curved surface. Hence, the displacement components and partial derivatives of any point on the model surface can be calculated by Eq. (1), (5), (6) and the stress and strain can also be determined as follows:

$$\epsilon_x = \partial U / \partial x \quad , \quad \epsilon_y = \partial V / \partial y$$

$$\gamma_{xy} = \partial V / \partial x + \partial U / \partial y$$

$$\sigma_x = E \cdot (\partial U / \partial x + \mu \cdot \partial V / \partial y) / (1 - \mu^2) \quad \dots \dots \dots (8)$$

$$\sigma_y = E \cdot (\partial V / \partial y + \mu \cdot \partial U / \partial x) / (1 - \mu^2)$$

$$\tau_{xy} = 1/2 \cdot E \cdot (\partial V / \partial x + \partial U / \partial y) / (1 + \mu)$$

For mode I fracture, the J-integral is defined as :

$$J = J_w - J_T \quad \dots \dots \dots (9)$$

where  $J_w = \int_{\Gamma} w \cdot dy \quad \dots \dots \dots (10)$

$$J_T = \int_{\Gamma} T \cdot \partial U / \partial x \cdot ds \quad \dots \dots \dots (11)$$

$\Gamma$  is the integral path, w is the density of deformation energy and

$$w = 1/2 \cdot [ \sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy} \gamma_{xy} ] \quad \dots \dots \dots (12)$$

$T_i$  is the tension vector along the outside normal of the integral path, and the components are respectively given by :

$$T_x = \sigma_x \cdot \cos \beta + \tau_{xy} \cdot \sin \beta \quad \dots \dots \dots (13)$$

$$T_y = \tau_{xy} \cdot \cos \beta + \sigma_y \cdot \sin \beta$$

The selected integral path is reasonably divided into a number of segments, m, the  $U(x_i, y_i)$ ,  $V(x_i, y_i)$ ,  $\Delta x$  and  $\Delta y$  at the middle point of the individual segment can be obtained from equations mentioned above. Finding sum for all segments and considering  $\cos \beta \approx dy/ds$ ,  $\sin \beta \approx dx/ds$ , We can get from Eq. (10),(11):

$$J_w = \sum w_i \cdot dy_i = 1/2 \sum [ \sigma_{x_i} \epsilon_{x_i} + \sigma_{y_i} \epsilon_{y_i} + \tau_{xy} \gamma_{xy} ] \cdot \Delta y_i \quad \dots \dots \dots (10')$$

$$J_T = \sum [ (\sigma_{x_i} \cdot \partial U_i / \partial x + \tau_{xy_i} \cdot \partial V_i / \partial x) \cdot \Delta y_i + (\tau_{xy_i} \cdot \partial U_i / \partial x + \sigma_{y_i} \cdot \partial V_i / \partial x) \cdot \Delta x_i \quad \dots \dots (11')$$

THE EXPERIMENTAL VERIFICATION FOR DETERMINING J-INTEGRAL OF THE HEEL OF THE GRAVITY DAM

The structural model of a gravity dam which is made of gypsum material is shown in Fig. 3. An initial crack ( a = 1 cm ) is preformed at

the heel of gravity dam model, the model surface is divided 181 meshes for measuring and calculating ( see Fig.5 ). The heel crack on load of water, silt and gravity is a mixed fracture, but it is close to mode I fracture according to the studied result of the reference (5). By means of the method mentioned above, three integral paths are selected ( see Fig. 6 ) and the J-integrals are determined respectively with five load conditions (  $1 p_0$ ,  $2 p_0$ ,  $3 p_0$ ,  $4 p_0$  and  $5 p_0$  ). Under the same conditions, the J-integrals are computed by eight-node isoparameter element of F.E.M. Both results are shown in Fig. 6, and relative error between laser speckle method and F.E.M is less than 6%. These show that the results of experimental method coincide quite with the numerical analysis method.

#### CONCLUSION

We have proposed a hybrid method whereby laser speckle technique and calculating method can be effectively applied to the determination of J-integral in fracture mechanics.

#### SYMBOLS USED

- D = the displacement of the point on the object surface
- $dx, dy$  = the displacement components along X, Y axes, respectively
- $\lambda$  =  $0.6328 \times 10^{-4}$  cm, the laser wave length used in optical system
- L = distance between the photoplate and the screen
- $\Delta$  = the interval of Young's fringes of the point
- M = the magnification factor of the recording optical arrangement
- $\alpha$  = the angle that the normal of Young's fringe makes with positive direction of the X axis.
- $a_0, a_1, a_2$  and  $F_i$  ( $i = 1, 2, \dots, n$ ) = undetermined constants
- $\epsilon$  = the micro which modulate curvity of the spline curved surface and  $\epsilon = 10^{-5} \sim 10^{-6}$
- $\beta$  = the angle between  $\vec{\pi}$  and X axis
- $\vec{\pi}$  = the displacement vector at the calculation point and U, V are

horizontal and vertical displacement components respectively

E = deformation modul

$\mu$  = Poisson ration

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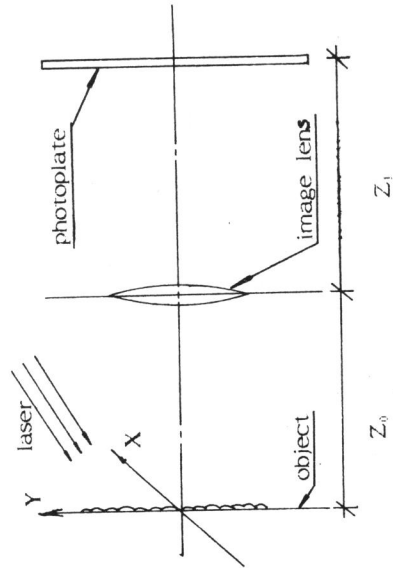


Fig.1 laser speckle image system

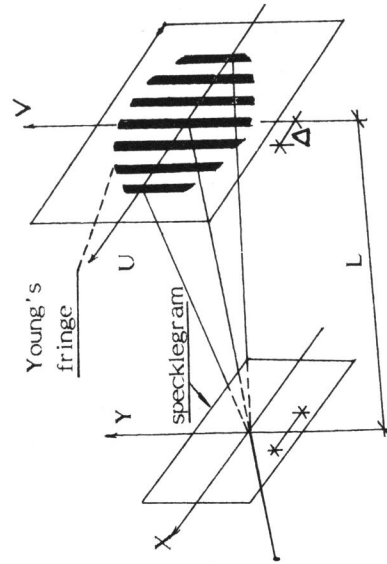


Fig.2 scheme of acquisition data from Young's fringe of a point

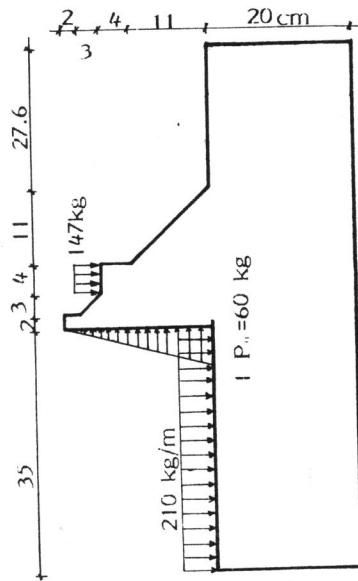


Fig.3 scheme of the gravity dam model made of gypsun material

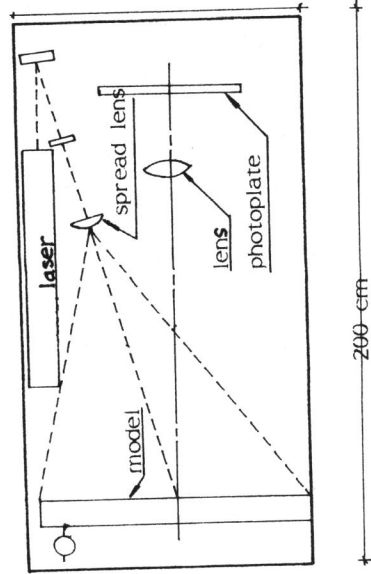


Fig.4 optical arrangement of the structural model test

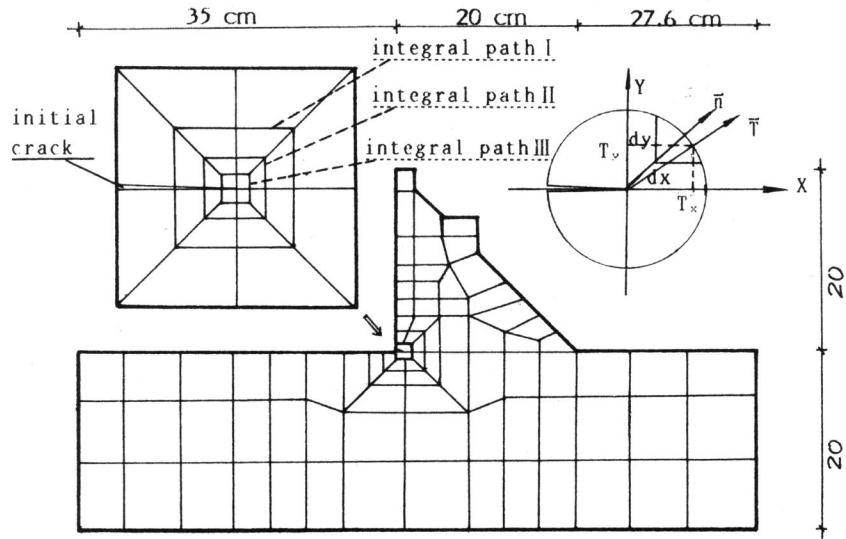


Fig.5 Meshes of the model and the integral paths

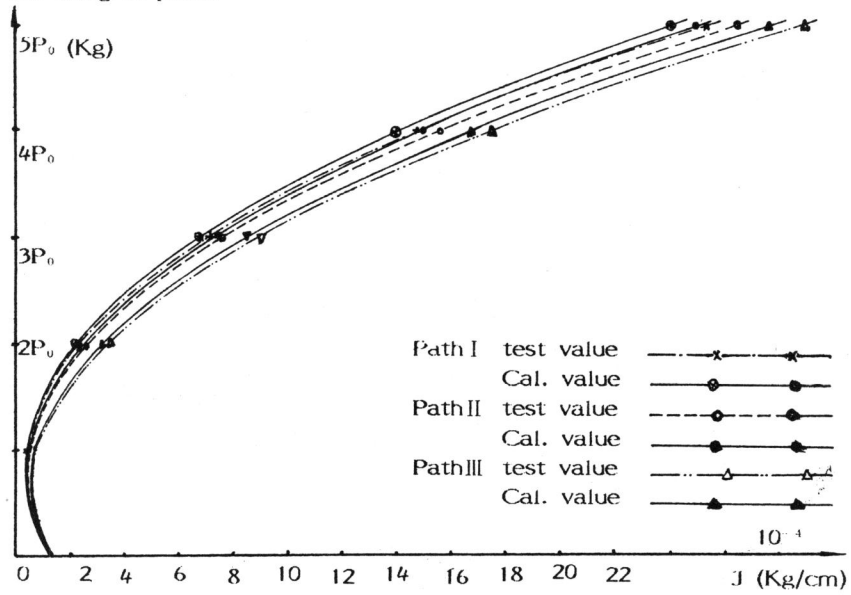


Fig.6 Comparison of calculated and measured J-integral values