ANALYSIS OF THE CRACK PROPAGATION UNDER BIAXIAL CYCLIC LOAD TAKING INTO ACCOUNT THEIR ORIENTATION

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The method of investigation of characteristics of the structural materials cyclic crack-stability under biaxial load of arbitrary direction was elaborated. This method is directed to studying of the angled cracks propagation in whose vertices the I and II pure modes are realized. The following parameters are determined: the crack growth rate; the trajectory and direction of its growth at different biaxial tension stress ratios; the strain energy densities characterizing the material fatigue crack growth rate curve.

INTRODUCTION

The character of load of modern technique products is such that during their usage they are under the conditions of complex stressed state, the noncoincidence of the direction of both the external load and the normal line to the orientation plane of possible defects being the rule rather than exception.

The available theoretical and experimental information on the influence of the stressed state form on the fracture characteristics of bodies having cracks is contradictory. The influence of the crack initial orientation angle on the fracture parameters at complex stressed state is undetermined. These circumstances have stimulated the development of theoretical and experimental studies on elaboration and argumentation of the criteria and parameters of crack mechanics under biaxial load.

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ELASTOPLASTIC ANALYSIS

Analysis of the known elastic solution of the problem of determination of the singular stressed-strained state (SSS) in the angled crack vertex has allowed to present elastoplastic formulation of the investigations. Here the following three problems have been formulated: to determine the elastoplastic singularity amplitude; to compute the dimensionless stress fields and to take into account the boundary conditions. The nonlinearity is introduced in terms of the Ramberg and Osgood model. The stress function is selected as follows

$$F = r^{\lambda+1}\Phi(\lambda,\theta); \quad \Phi(\lambda,\theta) = K\tilde{\Phi}(\lambda,\theta); \quad \lambda = m/(m+1) \ldots\ldots (1)$$

Dimensionless stress fields computation

By Hutchinson (1) the differential compatibility equation of strains governing the stress function in a crack vertex has been obtained. This equation has been initially solved for two special cases of load: a normal separation and a pure shear. The compatibility equation of strains has been solved with a help of the Runge-Kutt iterative procedure of the fourth-order. As a result the dimensionless fields of the stress components $\tilde{\sigma}_r$, $\tilde{\sigma}_\theta$, $\tilde{\sigma}_{r\theta}$ and $\tilde{\sigma}_e$ for the normal separation and a pure shear at the different values of the strain hardening exponent has been obtained.

Further a transformation of the known criteria of the crack propagation direction (the strain energy density in elastic (U) and elastoplastic (W) treatments and the stress intensities (σ_e)) has been performed:

$$\frac{\partial}{\partial\theta}(U, W, \sigma_e) = \frac{\partial^3\tilde{\Phi}}{\partial\theta^3}\left(C_1\frac{\partial^2\tilde{\Phi}}{\partial\theta^2} + C_2\tilde{\Phi}\right) + \frac{\partial\tilde{\Phi}}{\partial\theta}\left(C_3\frac{\partial^2\tilde{\Phi}}{\partial\theta^2} + C_4\tilde{\Phi}\right) \ldots\ldots (2)$$

$$\frac{\partial^3\tilde{\Phi}}{\partial\theta^3} = \frac{\partial\tilde{\Phi}}{\partial\theta} = 0 \qquad \text{or} \qquad \frac{\partial^2\tilde{\Phi}}{\partial\theta^2} = \tilde{\Phi} = 0 \ldots\ldots (3)$$

Analysis of dimensionless θ-distribution of stresses in combination with a minimax-analysis 3 has allowed to formulate the hypotheses and to establish the following consequences of the theory under development.

Hypotheses and consequences

1. The angled crack propagation is possible either as a normal separation (a local min on $\tilde{\sigma}_e(\theta)$-distri-

bution), or as a pure shear (a local max on $\tilde{\sigma}_e(\theta)$ -distribution).

2. At a pure shear the two extrema of $\tilde{\sigma}_e(\theta)$ take place: at $\theta^* = 0°$ and $\theta^* = 70°$. The angle θ^* determines the crack propagation direction. At $\theta^* = 0°$ an unstable equilibrium is observed when the crack can propagate in its plane. The value $\theta^* = 70°$ corresponds to the experimental data, and $\tilde{\sigma}_{r\theta} = 0$, $\tilde{\sigma}_\theta(70°)$ - being max, which corresponds to the normal separation boundary conditions. Thus, at the initial crack orientation and load, corresponding to pure shear, its further growth takes place as a normal separation.

3. Any θ^* -distribution of dimensionless stress functions can be obtained by the setting the normal separation boundary conditions in the point $\theta = \theta^*$, corresponding to a crack growth direction as a function of its initial orientation.

4. Domain of integration of the compatibility equation of strains has to be variable and it has to be displaced by the value of θ^* relatively to $0°$, i.e. $\theta^* = f(r/a, \eta, \alpha, m)$, where $\eta = \sigma_x^\infty / \sigma_y^\infty$.

For the quantitative evaluation of dimensionless stress values the scaling of their θ -distributions, using the conditional theoretical factors of the stress and the strain concentrations has been proposed. To determine the singularity amplitude in the angled crack vertex domain the unified elastoplastic stress factor K has been introduced, which together with a crack initiation angle describes the SSS fields at the mixed fracture modes. Thus, the general method of analysis of elastoplastic SSS in the angled crack vertex under biaxial load has been elaborated. In accordance with this method the calculation of the stress fields for the different crack orientations with variation of the strain hardening exponent m has been carried out.

NUMERICAL ANALYSIS BY MEANS OF THE FINITE ELEMENT METHOD

The theory of elastic and elastoplastic versions of the finite element method (FEM) taking into account a singularity for the mixed types of the crack propagation has been elaborated. Generalization of the Hilton and Sih method (2) on the case of biaxial load of the angled crack with an arbitrary orientation has been given. By the computational values $\tilde{\sigma}_r, \tilde{\sigma}_\theta, \tilde{\sigma}_{r\theta}$ of dimensionless stress functions, the expressions for elastic (V_{ec}^*) and elastoplastic (V_{pc}^*) strain energy of the domain restricted by the circle with a centre in the angled crack vertex, has been obtained. Potential energy of the whole structure was considered as a sum of contributions from the two care regions and the structure part divided on the finite elements.

By the common energy minimization over the unknown parameters the problem has been reduced to equation system allowing at the same time as SSS parameters of structure to calculate elastic and elastoplastic stress intensity factors (SIF). The algorithm elaborated was realized by means of a subconstructions method. This algorithm was used for calculation of the contours of plastic strain domains in eight-petal samples (fig. 1) at the experiments on biaxial tension with cracks of different initial orientation and also for the SIF calculation at mixed fracture modes. By means of this method the eight-petal samples K-taring has been obtained which could be used further at the experimental results interpretation.

METHODS OF EXPERIMENTAL INVESTIGATIONS

The method for an experimental investigation of characteristics of the cyclic crack-stability for the mixed crack propagation modes has been elaborated. We have proposed to interpret the experimental data over the parameter of the strain energy density. On the basis of the introduced concept of an equivalent straight crack, the method for constructing of the computational-experimental trajectory of the crack growth has been elaborated. Consequent step procedure connecting the preceding (a_{i-1}, α_{i-1}) and the next (a_i, α_i) positions of the crack vertex on its propagation trajectory is described by the following equations Shlyannikov and Dolgorukov (3)

$$a_i=[a_{i-1}^2+\Delta a_i^2-2a_{i-1}\Delta a_i\cos(\pi-\theta_{i-1}^*)]^{1/2} \quad ..(5)$$

$$\Delta\alpha_i=\arcsin\frac{\Delta a_i\sin(\pi-\theta_{i-1}^*)}{a_i}; \qquad \alpha_i=\alpha_{i-1}+\Delta\alpha_i \quad ......(6)$$

From the same positions the simple analytical equation allowing to calculate the theoretical trajectory of the crack growth under biaxial arbitrarily directed load has been obtained Shlyannikov and Ivanyshin (4)

$$a_i=a_{i-1}[\cos(\alpha_i-\alpha_{i-1})+\sin(\alpha_i-\alpha_{i-1})\,tg(\pi/2-\theta_i^*+\alpha_i-\alpha_{i-1})]. \quad (7)$$

It should be noted that the equations proposed are invariant relatively to the criteria of the crack growth direction. In above methods of the experimental results processing we have used the Lagrange interpolation polynom to describe the crack growth direction as a function of its initial orientation angle and to calculate the current SIF values (over the known K-taring funtions, obtained by the FEM method taking into account the singularity) by means of the double interpolation over the crack length and over the angle of its inclination. The method of calculation proposed is united into the block diagram (fig. 2) for the automatical processing of empirical data, realized on the mini-computer Shlyannikov and Dolgorukov (3).

Over the given relations of biaxial tension stresses a geometry of plane eigth-petal samples (fig. 1) has been obtained by means of iterative computation for the experiments under both equally-biaxial (η =1) and biaxial tension with η =0.5.

RESULTS OF EXPERIMENTS

On the aluminium alloys having the different properties (whose main machanical characteristics are presented in the table 1) the experimental investigation and analysis of the crack growth taking into account their orientation under uniaxial $\eta=0$ and biaxial $\eta=1$ and $\eta=0.5$ tension have been performed on the electrohydraulical stand at antisymmetrical cycle of load R = =0.05 with a frequency 3.5 1/s on the rectangular (80x x320mm) and eigth-petal samples by the thickness 3-5mm. The diagrams of the crack propagation direction θ^* as a function of the angle of its initial orientation α under the pointed types of SSS (fig. 3) for the every of materials have been obtained.

In fig.4 and fig.5 the experimental trajectories of crack growth at their initial orientation α =0,25,45, 65,90° for some of materials η =0 and η =0.5 are shown. It was established that SSS type results in change of the trajectory curvature. Moreover the range of the trajectory changes is essentially greater at biaxial tension than at uniaxial one, depending on material properties. It was that the properties of materials observed have the same quantitative influence upon the crack growth, as the SSS type.

The fatigue fracture diagrams (FFD) have been obtained at the following relatonships η =0;0.5;1 and α = =0,25,45,65,90° for all the materials, some of them being shown in fig.6 and in fig.7. Interpretation of empirical data was carried out in accordance with the block diagram (fig.2) and it is based on the parameter of strain energy density S. The linear section of FFD was described by the following equation

$$\frac{da}{dN} = \left(\frac{da}{dN}\right)^* \left(\frac{S_{max}}{S^*}\right)^n \qquad (8)$$

in which $(da/dN)^*=10^{-7}$m/cycle, S^* and n - are the ex-

TABLE 1 - Mechanical characteristics of materials.

Marker	Material	σ_B-MPa	$\sigma_{0.2}$-MPa	δ-%	$\sigma_B/\sigma_{0.2}$
o	Al.alloy 1	320	160	20	2.00
+	Al.alloy 2	390	225	14	1.74
△	Al.alloy 3	440	285	20	1.54
◇	Al.alloy 4	430	335	13	1.28
▽	Al.alloy 5	345	300	9	1.15
□	Al.alloy 6	570	510	11	1.12

perimentally determined constants. Dependence of resistance characteristics to the crack growth under biaxial cyclic tension on the SSS type, on the angle of the crack orientation and on the material properties has been established. In order to describe the effects observed dimensionless parameter

$$T=(n_1/n)(S^*/S_1^*) \qquad (9)$$

was proposed, in which the current values of constant n and S^* (for any combinations η and α) were related to their values at η =1. In fig.8 the diagrams of this parameter change are presented. It shoud be noted that the crack-stability under biaxial tension is greater than under uniaxial one. Moreover, the influence of the crack orientation angle and of the materials properties is greater under biaxial tension with η=0.5.

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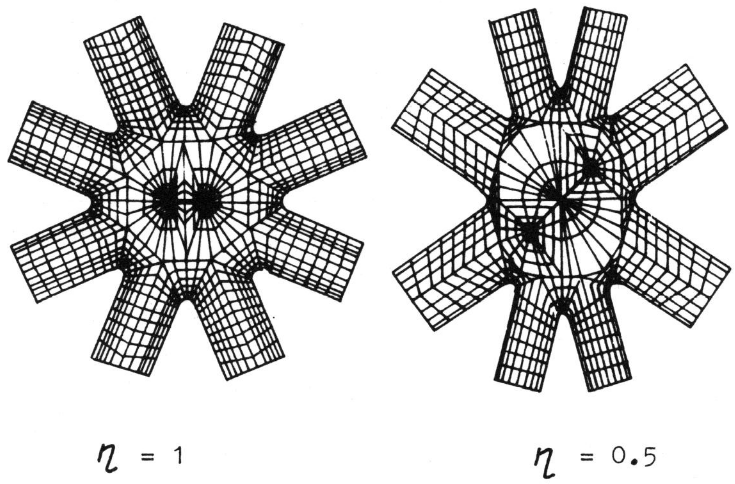

Figure 1 Samples for the testing at biaxial tension and their finite-element models

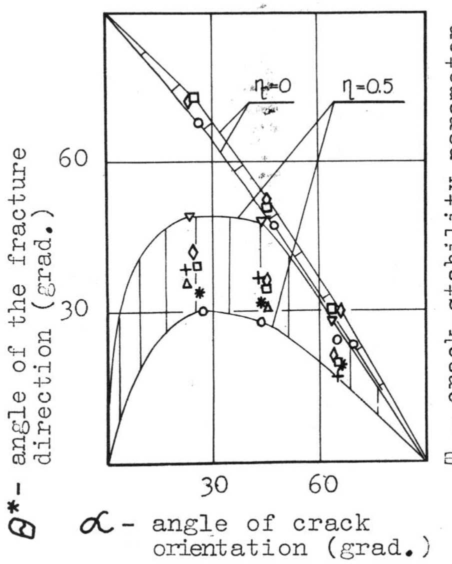

Figure 3 Experimental diagrams of dependence

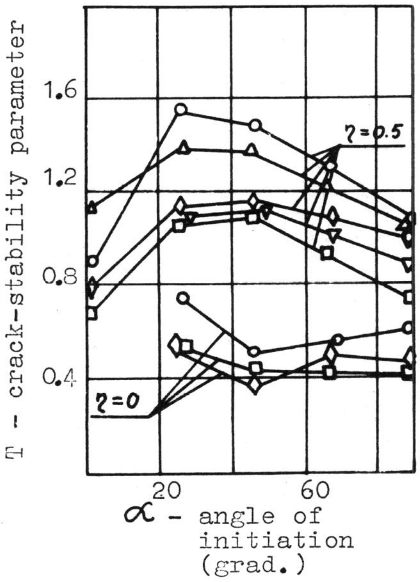

Figure 8 Change of the crack-stability parameters

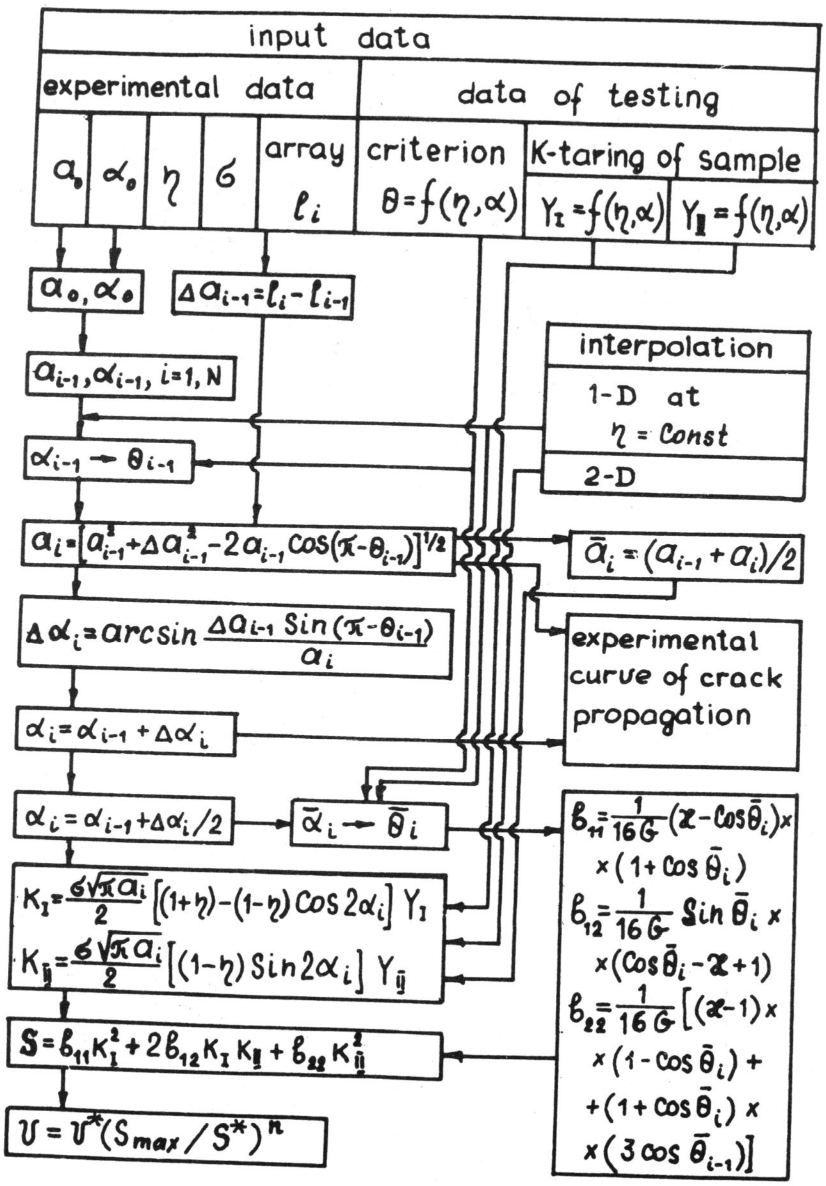

Figure 2 Block diagram of interpretation of the experimental data on the basis of equivalent crack

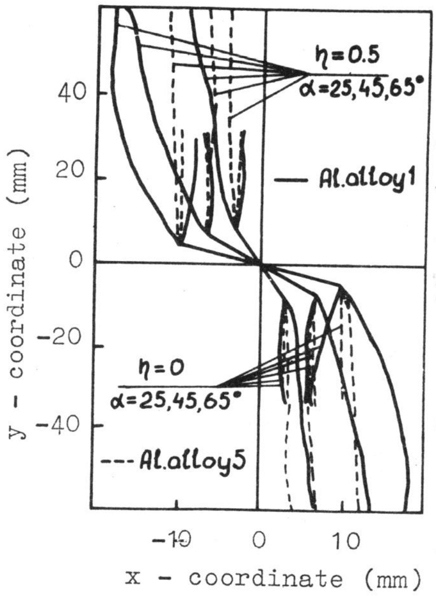

Figure 4 Experimental crack trajectories at η=0, η =0.5

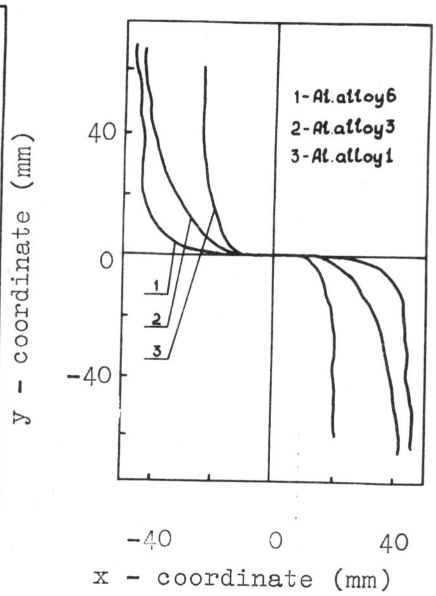

Figure 5 Experimental crack trajectories at η =0.5, α =0°

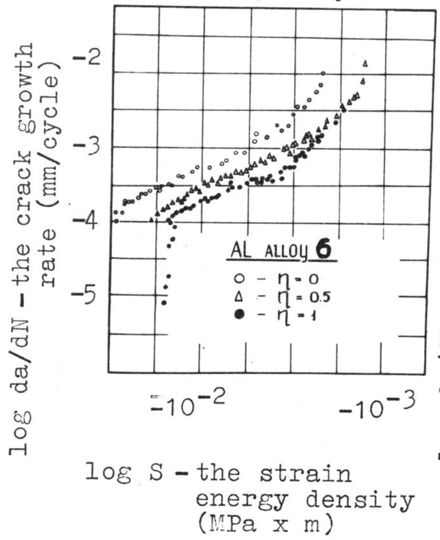

Figure 6 Diagrams of fatigue fracture at α =90°

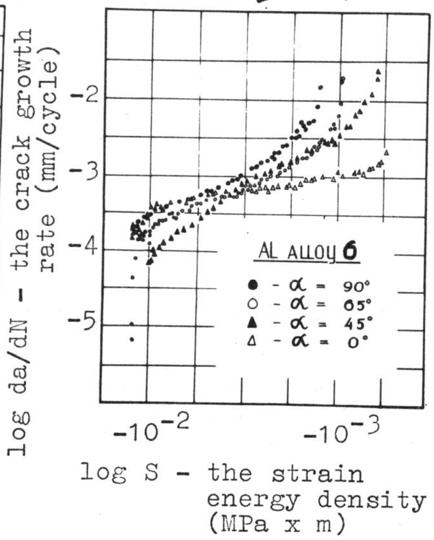

Figure 7 Diagrams of fatigue fracture at η =0.5