

## A TECHNICAL CREEP THEORY FOR CONCRETE

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A technical creep theory for computer-aided analysis of reinforced concrete structures is developed. In the proposed theory no experimental data are required except the characteristics given in the standart regulations for calculation of reinforced concrete structures. This theory is applied in programs for computer-aided design of several structures.

### INTRODUCTION

The up-to-date methods for statical investigation of reinforced concrete structures require a non-linear analysis, taking into account the creep of concrete. The well-known and widely used creep theories are the theory of the elastic-creeping body, developed by Maslov, Arutunjan et al, the theory of the "elastic heritage", developed by Boltzmann, Volter et al and the theory of "ageing" developed by Dischinger, Witney, Ulizki et al. All these theories require many experimental data depending on the class and quality of the examined concrete. This is not convenient for designing

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procedures. For this reason in the proposed work a technical theory is developed, which gives possibility to imitate the three classic theories mentioned above, but does not require experimental data.

THEORETICAL RESULTS

As shown by Rüschi and Jungwirth (1) and by Ulizki et al (2) the creep curves of different types of concretes can be described with difficulty by a single mathematical equation because of the different composition of the material influencing essentially its creep properties.

In the proposed theory the creep characteristics  $C_t$  is obtained from the following two equations:

$$\epsilon_p(t) = C_t \cdot \sigma_c \dots \dots \dots (1)$$

$$\epsilon_p(t) = \epsilon_{c(t)} - \epsilon_s = \frac{\sigma_c}{\nu_t \cdot E_c} - \frac{\sigma_c}{\nu_e \cdot E_c} = \frac{\nu_e - \nu_t}{\nu_e \cdot \nu_t} \cdot \frac{\sigma_c}{E_c}, \dots \dots (2)$$

$$C_t = \frac{\nu_e - \nu_t}{\nu_e \cdot \nu_t} \cdot \frac{1}{E_c} \dots \dots \dots (3)$$

When the process of creep is over, the value of the creep characteristics becomes

$$C_t = \frac{\nu_e - \nu_d}{\nu_e \cdot \nu_d} \cdot \frac{1}{E_c} \dots \dots \dots (4)$$

In equations (2), (3) and (4)  $\nu_e$ ,  $\nu_t$ ,  $\nu_d$  are coefficients of elasticity, respectively for the short-time acting load, for an arbitrary time  $t$  and for the time when the creep process is over. These coefficients are

$$\nu = \frac{\epsilon_e}{\epsilon_c}, \dots \dots \dots (5)$$

which means the ratio between the elastic and the total deformations for each of the three cases mentioned above.

The coefficient of elasticity for short-time acting loads is determined as

$$\nu_e = \nu_{oe} + (1 - \nu_{oe}) \left( \frac{R - \sigma_c}{R} \right)^{\beta_1} \dots \dots \dots (6)$$

In equation (6)

$$\nu_{oe} = 0.45 + \frac{R - 10}{100} \cdot [0.786 + 0.0122(80 - R)] \dots \dots \dots (7)$$

$$\beta_1 = 1 - \frac{R}{80} \dots \dots \dots (8)$$

When the creep process is over, the respective value of the coefficient of elasticity decreases to

$$\nu_d = \nu_{od} + (1 - \nu_{od}) \left( \frac{R - \sigma_c}{R} \right)^{\beta_2} \dots \dots \dots (9)$$

where

$$\nu_{od} = 0.15 + \frac{R - 10}{100} \cdot [0.607 + 0.0041(150 - R)] \dots \dots \dots (10)$$

$$\beta_2 = 1 - \frac{R}{150} \dots \dots \dots (11)$$

An appropriate mathematical law describing well average creep process in the case when the compression stress is applied at an age of 28 days after casting the concrete is

$$C_t = \begin{cases} C_d \cdot \sqrt{1 - \frac{(t_d - t)^2}{t_d^2}} & (0 \leq t \leq t_d) \\ C_d & (t > t_d) \end{cases} \quad \dots (12)$$

In the case when the stress is applied at an arbitrary time  $\tau$  after the age of 28 days, the description of the creep process follows the law

$$C_{(t,\tau)} = \begin{cases} \alpha \cdot C_d \sqrt{1 - \frac{(t_d - t)^2}{(t_d - \tau)^2}} & (0 \leq t \leq t_d) \\ \alpha \cdot C_d & (t > t_d) \end{cases} \quad \dots (13)$$

In equation (13)  $\alpha$  is a characteristic, by the help of which the creep process can be described similarly to the three mentioned classic theories. When  $\alpha = 1$  the description of the process is similar to Boltzman-Volter's theory. When

$$\alpha = 1 - \frac{C_\tau}{C_d}, \quad \dots (14)$$

the creep process is described similarly to Dischinger's theory.  $C_\tau$  is calculated according to equation (12) for  $t = \tau$ . For values of  $\alpha$  between the two mentioned above, the description of the process resembles Maslov-Arutunian's theory. A graphic presentation of the results for  $C_{(t,\tau)}$ , when  $\alpha = 1$  (curves "a") and when  $\alpha$  is calculated according to formula (14) (curves "b") is shown of figure 1.

On the following figures (2,3 and 4) are shown the total creep deformations, calculated by the equation

$$\varepsilon_{cr}(t) = \frac{\sigma_c}{E_c} \cdot \left( \frac{1}{\gamma_e} + \frac{\gamma_e + \gamma_d}{\gamma_e \cdot \gamma_d} \cdot \sqrt{1 - \frac{(t_d - t)^2}{t_d^2}} \right) - \frac{\sigma_c}{E_c}. \quad (15)$$

They are calculated for different classes of concrete, for different values of the applied stress and show their influences over the crepp process. It should be emphasized that the proposed technical theory is a non-linear one. That is clear from the results on figure 3.

#### APPLICATION OF THE THEORY

The proposed technical creep theory was developed especially for the purposes of the computer-aided design of reinforced concrete structures in non-linear stage.

For the time being special programs for the analysis of plane frames, slabs (smooth and ripped), shear walls, and some special structural details are carried out on the basis of the proposed theory in CESSI - Sofia. The programs are already widely applied in practice.

#### SYMBOLS USED

- $\varepsilon_c$  = long-time creep strain
- $\varepsilon_s$  = short-time strain
- $\varepsilon_p$  = total long-time strain
- $\varepsilon_{cr}$  = total creep strain
- $\sigma_c$  = compressive stress (MPa)
- $E_c$  = initial module of elasticity for concrete (MPa)
- $R$  = cubic strength of concrete (MPa)
- $C$  = creep characteristics (1/MPa)

- $\nu$  = coefficient of elasticity  
 $t$  = examined time of creep (days)  
 $\tau$  = time of stress application (days)

REFERENCES

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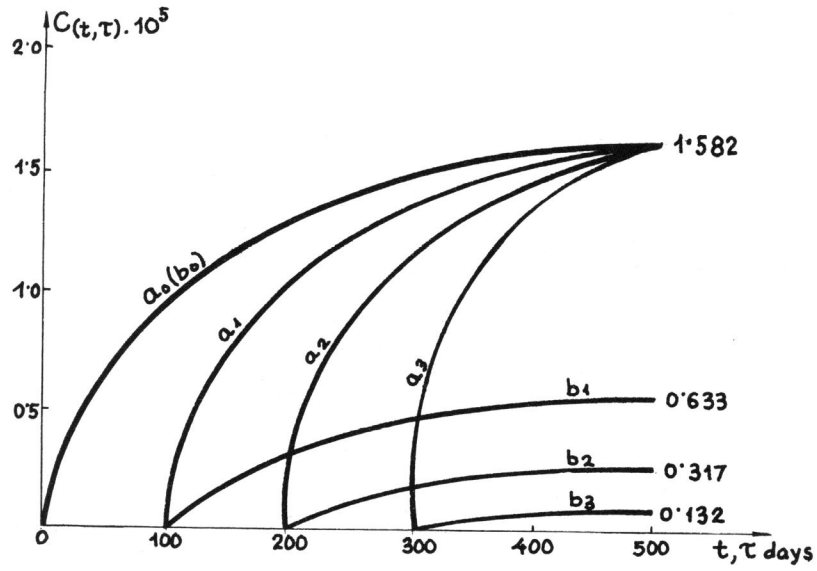


Figure 1 C-curves for  $\alpha = 1$   
 (a) and  $\alpha = 1 - C\tau / C_d$  (b)

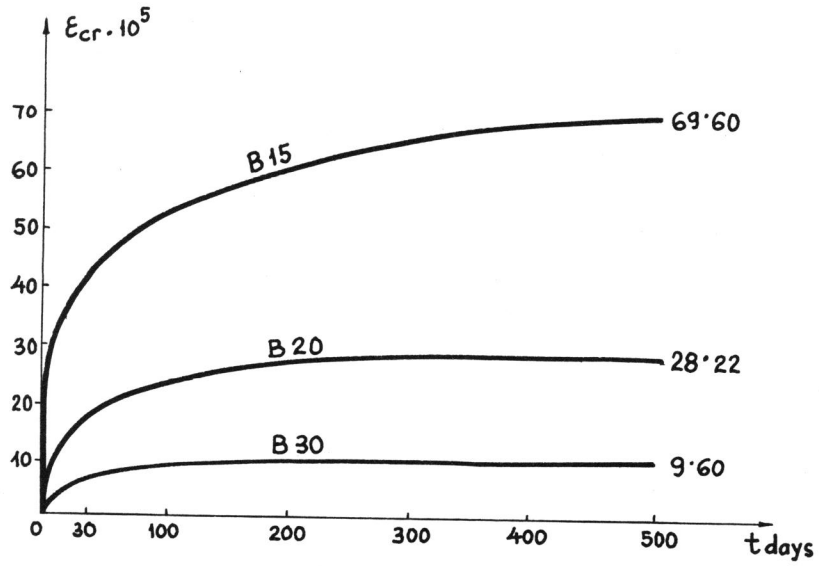


Figure 2 Creep strain for  
 $\sigma_c = 11,25 \text{ MPa}$

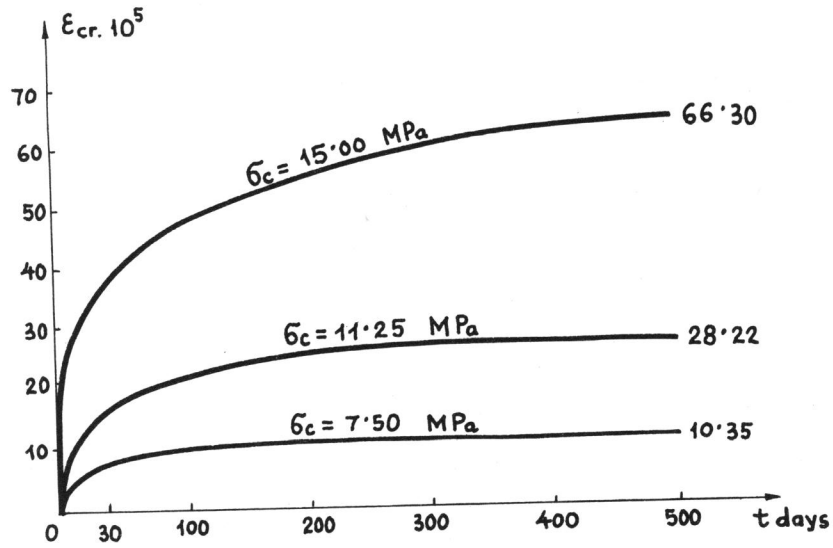


Figure 3 Creep strain for concrete class B20

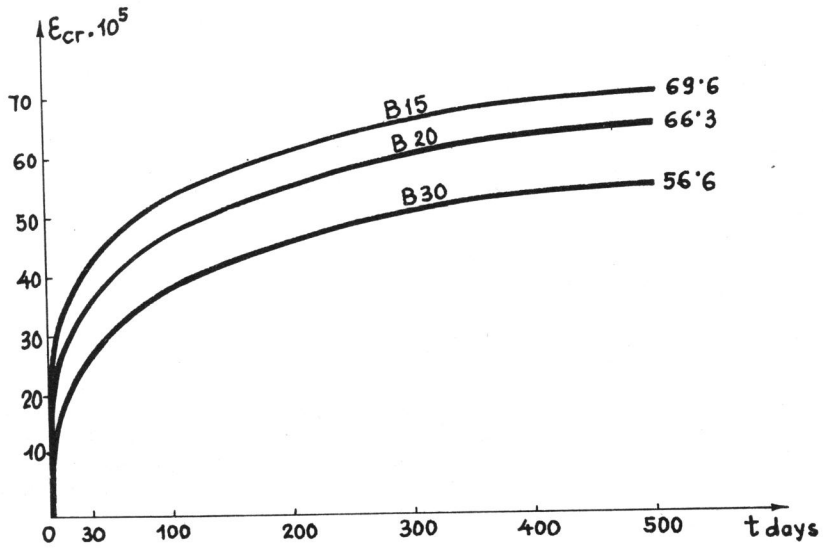


Figure 4 Creep strain for  $\sigma_c = 0,75 R$