NUMERICAL SIMULATION OF DYNAMIC CRACK PROPAGATION PHENOMENA BY MEANS OF THE FINITE ELEMENT METHOD

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A virtual crack extension method for the calculation of energy release rates associated with fracture phenomena during dynamic crack propagation has been implemented in a general purpose finite element program. This paper discusses the application of this method to the calculation of dynamic stress intensity factors for a stationary crack in an impulsively loaded center-cracked panel. The results obtained do compare well with both theoretical results and with numerical data. Analyses are also performed for a crack that propagates with constant and variable speed in a tensile loaded center-cracked panel. The results indicate that the calculated dynamic stress intensity factors depend on the manner in which crack propagation is simulated in the finite element model. It is shown that for dynamic crack propagation node relaxation techniques may yield erroneous results.

INTRODUCTION

For the safety assessment of structures, it is necessary to answer the question whether, for a particular loading system, existing or potential defects will start to grow and/or under what circumstances failure of the structure will occur. The concepts of fracture mechanics provide appropriate tools for describing fracture processes. They are frequently applied nowadays for the prediction of crack initiation, as well as slow stable crack growth of statically loaded structures and for the prediction of fatigue crack growth in cyclically loaded structures.

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In those cases where inertia effects cannot be ignored, application of quasi-static fracture mechanics techniques may lead to erroneous conclusions, which causes the necessity to use dynamic fracture mechanics concepts. The main emphasis of dynamic fracture mechanics is to predict the initiation of stationary cracks in structures, which are subjected to impact loading. It also focuses on the conditions for the continuous growth of fast propagating cracks and on the conditions under which a crack is arrested.
Over the past decade much research work, both numerically and experimentally, has been and is currently still being carried out in the field of dyna- mic fracture mechanics. The problem of predicting the growth rate and the possible crack arrest point is quite complicated. Various research workers treat this problem by means of a so-called dynamic fracture methodology, which requires the combined use of experimental measurements and of detailed finite element analyses. Results obtained by means of this methodology have been reported, amongst others by Brickstad [1] and Kanninen [2]. An essential step in this approach is formed by the numerical simulation of propagating cracks by means of finite element methods (FEM). The FEM programs used to carry out such dynamic analyses should enable the evaluation of dynamic fracture mechanics parameters. This paper concentrates on two aspects of the application of the finite element method to dynamic fracture problems, i.e.:

- the calculation of dynamic energy release rates for cracked bodies which are subjected to arbitrary thermal and mechanical loadings including initial stresses;
- the modelling of crack propagation phenomena in relatively coarse finite element meshes using a gradual node relaxation technique.

First the aforementioned combined numerical and experimental approach is explained. Subsequently the derivation of an extended version of the J-integral originally proposed by Rice [3], is presented which takes into account the effect of inertia and body forces, thermal and mechanical loading and initial strains. In addition it is shown how the expression for the extended J-integral may be transformed into a surface integral in order to enable a more straightforward evaluation of this expression by means of numerical integration techniques commonly used in finite element programs. The extended J-integral has been implemented in the commercially available general purpose finite element program MARC [4]. The paper discusses the application of this extended J-integral to the problem of an impulsively loaded center-cracked panel with a stationary crack. In addition results are presented for a propagating crack in a center-cracked panel under constant loading assuming both constant and varying crack speeds. The present paper finally discusses the observed limitations of node relaxation techniques for the modelling of dynamic crack propagation phenomena.

DYNAMIC FRACTURE METHODOLOGY

In complete similarity with static fracture mechanics concepts, it is assumed that dynamic crack growth processes for linear materials are governed by the following condition:

$$K_I(t) = R_{ID}\ (\dot{a},T,B) \qquad \dot{a} \geq 0 \qquad (1)$$

where $K_I(t)$ is the dynamic stress intensity factor for mode I, $\dot{a}$ is the crack velocity and R_{ID} is the dynamic crack propagation toughness, which is assumed to be a material parameter that in general will depend on crack velocity $\dot{a}$, temperature T and specimen thickness B.

The dynamic stress intensity factor will depend on crack length (a), applied loading (σ), time (t), specimen dimensions (D), temperature (T) and initial stress fields (σ_i) caused by residual stresses or by an initial strain field. The prediction of the crack propagation history and crack arrest event, demands complete knowledge of the R_{ID} vs. $\dot{a}$ relation. Kanninen [2] and Brickstad [1] have adopted a procedure using both experiments and numerical analyses to determine this relation. This procedure, which is referred to as the "Dynamic Fracture Methodology" consists of the following two phases:

a. Generation phase
   In this phase a crack arrest experiment is performed yielding a crack propagation versus time curve. In addition a numerical simulation of the experiment is carried out by using the measured crack propagation curve. This is used as input for the numerical model. This produces calculated dynamic stress intensity factors as a function of time. Combination of the latter relation with the measured crack propagation curve will result in a curve, which may be considered as the dynamic crack propagation toughness versus crack velocity relation.
b. Application phase
   In order to predict the crack growth and possible crack arrest point in a structural component, the inverse problem is solved. Now the actual stress intensity factors are calculated for the structural component, that is subjected to a particular loading history, by means of a dynamic FEM analysis. These calculated values are compared to the fracture toughness curve obtained during the generation phase, eqn. (1), and from this the crack growth is predicted.

As may be clear from the above, the availability of accurate computational techniques to calculate dynamic stress intensity factors will to a great extent determine the successful application of the methodology. In similarity with static problems these factors can be obtained directly from displacement and/or stress fields around the crack tip. However this causes severe problems in practical situations due to the complex nature of the expressions to be used and because of the lack of theoretical solutions which incorporate the effects of reflecting stress waves. In addition such a procedure would require a very fine distribution of finite elements around the crack tip. For these reasons an approach in which the stress intensity factors are derived from quantities that are evaluated away from the crack tip is preferable. One of the quantities commonly employed for this purpose is the elastic energy release rate G. For a structure in which a crack propagates with arbitrary velocity $\dot{a}$ the net change of energy may be denoted by:

$$\dot{Y} = \dot{P} - \dot{W} - \dot{T} = G\,\dot{A} \tag{2}$$

where $\dot{P}$ is the rate of work done by external forces, $\dot{W}$ is the rate of deformation energy, $\dot{T}$ is the rate of kinetic energy and A is the crack surface. The dynamic energy release rate G may therefore be expressed as:

$$G = \frac{dP}{dA} - \frac{dW}{dA} - \frac{dT}{dA} \tag{3}$$

For elastic material behaviour the energy release rate G is identical to J and thus according to Nishioka, et al [5] the following holds:

$$G = J = \frac{K_I^2}{E'} A_I(\dot{a}) \tag{4}$$

Where $E' = E$ for plane stress and $E' = \frac{E}{1-\nu^2}$ for plane st ain.

$$A_I(\dot{a}) = \frac{\beta_1(1-\beta_2^2)}{(1-\nu)\{4\beta_1\beta_2 - (1+\beta_2^2)^2\}} \quad \text{with } \beta_i^2 = 1-\left(\frac{\dot{a}}{C_i}\right)^2 \quad i = 1,2 \tag{5}$$

and C_1 = longitudinal wave speed

C_2 = shear wave speed

For the limiting case $\dot{a} \to 0$, $A_I(\dot{a}) = 1.$ and thus eqn. (4) reflects the well known relation between J and K for static applications. With the aim of calculating dynamic stress intensity factors from J-values, an expression for J that is suitable for a straightforward implemenation in a FEM program is described in the next section.

DERIVATION OF AN EXTENDED J-INTEGRAL

Since the J-integral was proposed by Rice, various researchers have presented extensions of the initially proposed J-integral in order to account for isolated effects like plastic deformation, body forces, thermal loading, inertia forces, large displacements and large strains. In the following section we shall present a modified J-integral, which incorporates most of the aforementioned effects.

Conservation laws

In order to evaluate the forces acting on defects like cavities or cracks in elastic structures both Rice [3] and Eshelby [6] used path independent integrals. It was shown by Knowles and Sternberg [7] that the integrals used by Eshelby and Rice are related to a class of conservation laws in elastic continua, which may be derived from the principle of minimum potential energy and from the invariance of strain energy density with respect to particular coordinate transformations. Along a similar scheme as described by Bakker [8] a conservation law for (non)linear elastic continua can be derived which takes into account the effects of inertia forces, body forces, thermal strains and initial strains.
Consider a structure with a subregion Ω that has a volume V and a boundary Γ with surface S to which an arbitrary coordinate mapping $\underline{x}' = \underline{x} + \delta\underline{x}$ is applied. When the coordinate change $\delta\underline{x}$ is infinitesimal, it can be shown that the following holds:

$$\int_{\Gamma} \delta x_k \left(W\,\delta_{jk} - \sigma_{ij}\frac{\partial u_i}{\partial x_k}\right) n_j \, dS + \qquad (6)$$

$$- \int_{\Omega} \left\{ \frac{\partial \delta x_k}{\partial x_j}\left(W\,\delta_{jk} - \sigma_{ij}\frac{\partial u_i}{\partial x_k}\right) + \delta x_k \left((f_i - \rho\ddot{u}_i)\frac{\partial u_i}{\partial x_k} + \sigma_{ij}\frac{\partial \varepsilon^{o}_{ij}}{\partial x_k} \right) \right\} dV = 0$$

In the derivation of this conservation law use has been made of the following definitions and assumptions:

- The strain energy is defined by:

$$W = \int_{0}^{\varepsilon} \sigma_{ij}\, d\varepsilon^{e}_{ij} \qquad (7)$$

If W is a function of elastic strains only, the stresses follow from:

$$\sigma_{ij} = \frac{\partial W(\varepsilon^{e}_{ij})}{\partial \varepsilon^{e}_{ij}} \qquad (8)$$

- The elastic strains are defined as:

$$\varepsilon^{e}_{ij} = \varepsilon^{tot}_{ij} - \varepsilon^{o}_{ij} \qquad (9)$$

with: $\varepsilon^{tot}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ = total strain components and

ε^{o}_{ij} = initial strain components consisting of eigen strains, thermal strains, etc.

- For a dynamic loaded structure the equation of motion reads:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = \rho \ddot{u}_i \qquad (10)$$

where f_i are the body forces, ρ the mass density and $\ddot{u}_i$ the acceleration components.

- In addition δ_{jk} stands for Kronecker's delta and n_j are the components of the outward normal on Γ.

For a unit virtual translation of Ω in the x_k direction and in the absence of body forces, inertia effects and initial stresses eqn. (6) reduces to

$$\int_{\Gamma} \left(W\, n_k - t_i \frac{\partial u_i}{\partial x_k}\right) dS = 0 \qquad (11)$$

where the traction vector on Γ is defined by $t_i = \sigma_{ij} n_j$.

Equation (11) states that during such a translation of a subregion of the body the change of the strain energy is equal to the work done by the external tractions on the surface of the subregion. It represents the first conservation law derived by Knowles and Sternberg, which is related to the momentum tensor equation used by Eshelby and to the J-integral proposed by Rice.

Application to sharply notched defects

The conservation law according to eqn. (6) will now be applied to a structure containing a sharply notched defect in order to derive a general path-independent integral, which characterizes the fracture process.
In the following we will restrict ourselves to a plane structure only without the loss of generality. Consider a plate of thickness t containing a crack which coincides with the x_1 direction. The x_2 axis is perpendicular to the crack surface. As illustrated in Fig. 1 we consider a subregion Ω_1 with a closed surface $\Gamma_1+\Gamma_{s_1}+\Gamma_p$ where Γ_1 is any arbitrary chosen contour surrounding the crack tip, Γ_{s_1} the fracture surface and Γ_p is a circular contour with radius $r \to 0$ surrounding the process zone that is of negligible size and in which continuum mechanics may not be applied. Note that the normal direction on Γ_p is chosen opposite to the normal on Γ_1.
If a crack extension Δa is considered, which may be treated as a constant virtual coordinate movement $\delta \underline{x} = (\Delta a, 0, 0)$ in Ω_1, application of the conservation law of eqn. (6) to the region Ω_1 with its closed surface consisting of Γ_1, Γ_{s_1} and Γ_p will result in:

$$\int_{\Gamma_1} (W n_1 - t_i \frac{\partial u_i}{\partial x_1})\, dS - \int_{\Gamma_{s_1}} t_i \frac{\partial u_i}{\partial x_1}\, dS - \lim_{r\to 0} \int_{\Gamma_p} (W n_1 - t_i \frac{\partial u_i}{\partial x_1})\, dS + \tag{12}$$

$$- \int_{\Omega_1} ((f_i - \rho \ddot{u}_i) \frac{\partial u_i}{\partial x_1} - \sigma_{ij} \frac{\partial \overset{o}{\varepsilon}_{ij}}{\partial x_1})\, dV = 0$$

The amount of energy that is dissipated in the process zone per unit crack extension, i.e. the rate of energy released at the crack tip, is expressed by the limiting value of the integral along Γ_p in the above relation. We will define this rate of energy as:

$$\hat{J} = \lim_{r\to 0} \int_{\Gamma_p} (W n_1 - t_i \frac{\partial u_i}{\partial x_1})\, dS \tag{13}$$

Based on eqn. (12), $\hat{J}$ may be written as:

$$\hat{J} = \int_{\Gamma_1} (W\, n_1 - t_i \frac{\partial u_i}{\partial x_1})\, dS - \int_{\Gamma_{s_1}} t_i \frac{\partial u_i}{\partial x_1}\, dS + \qquad (14)$$

$$- \int_{\Omega_1} ((f_i - \rho \ddot{u}_i) \frac{\partial u_i}{\partial x_1} - \sigma_{ij} \frac{\partial \overset{\circ}{\varepsilon}_{ij}}{\partial x_1})\, dV$$

For the situation that body forces, inertia forces, initial strains and tractions on the crack surfaces are absent, $\hat{J}$ reduces to the path-independent integral proposed by Rice:

$$\hat{J} = J = \int_{\Gamma_1} (W\, n_1 - t_i \frac{\partial u_i}{\partial x_1})\, dS \qquad (15)$$

The stress/strain fields at the crack tip may be characterized by evaluating an integral away from the crack tip. In the case that the previously mentioned effects are present the property that J can be determined based on the far field solution is no longer valid, because the value of the integral along Γ_1 will depend on the distance of Γ_1 to the crack tip. The expression for the extended J-integral according to eqn. (14) is identical to the one derived by Kishimoto et al [9]. It is worth mentioning that there no assumptions were made for the particular choice of Γ_1. This choice is therefore arbitrarily. Evaluation of $\hat{J}$ however requires full knowledge of the mechanical state within the Ω_1 region. The only restriction imposed so far is that a strain energy density function exists which relates stresses to strains.

Numerical evaluation of $\hat{J}$

The solution of realistic fracture problems in most cases only can be carried out using numerical techniques like e.g. the finite element method. The numerical evaluation of $\hat{J}$ in 2D situations requires the calculation of both a line and a surface integral. This usually causes problems, because of the discontinuities of stresses at element boundaries. These discontinuities make it necessary to apply some smoothing of the stress distribution.

In the following an alternative formulation of $\hat{J}$ in which only surface integrals are involved has been derived. Consider a region

Ω_2 surrounded by the contours Γ_1, Γ_2 and Γ_S (see Fig. 2). By using the conservation law of eqn. (6) and by choosing δx_k such that:

$$\delta x_2 = \delta x_3 = 0 \text{ both in } \Omega_2 \text{ and on its boundary}$$
$$\delta x_1 = \Delta a \text{ on } \Gamma_1$$
$$\delta x_1 = 0 \quad \text{on } \Gamma_2,$$

the following expression can be derived:

$$\Delta a \int_{\Gamma_1} (W \delta_{1j} - \sigma_{ij} \frac{\partial u_i}{\partial x_1}) n_j \, dS = - \int_{\Gamma_{S_2}} \delta x_1 \, t_i \frac{\partial u_i}{\partial x_1} dS + \tag{16}$$

$$- \int_{\Omega_2} \{ \frac{\partial \delta x_1}{\partial x_j} (W \delta_{1j} - \sigma_{ij} \frac{\partial u_i}{\partial x_1}) + \delta x_1 ((f_i - \rho \ddot{u}_i) \frac{\partial u_i}{\partial x_1} - \sigma_{ij} \frac{\partial \overset{\circ}{\varepsilon}_{ij}}{\partial x_1}) \} dV$$

Note that δx_1 will vary in Ω_2 between Δa and 0.

Since $\frac{\partial \delta x_1}{\partial x_j} = 0$ in Ω_1, substitution of eqn. (16) into (14) yields:

$$\hat{J} = - \int_{\Omega} \frac{1}{\Delta a} \frac{\partial \delta x_1}{\partial x_j} (W \delta_{1j} - \sigma_{ij} \frac{\partial u_i}{\partial x_1}) dV + \tag{17}$$

$$- \int_{\Omega} \frac{\delta x_1}{\Delta a} ((f_i - \rho \ddot{u}_i) \frac{\partial u_i}{\partial x_1} - \sigma_{ij} \frac{\partial \overset{\circ}{\varepsilon}_{ij}}{\partial x_1}) dV - \int_{\Gamma_S} \frac{\delta x_1}{\Delta a} t_i \frac{\partial u_i}{\partial x_1} dS$$

where $\Omega = \Omega_1 + \Omega_2$ and $\Gamma_S = \Gamma_{S_1} + \Gamma_{S_2}$.

Within a finite element program this expression for $\hat{J}$ can be readily evaluated by means of the commonly used numerical integration methods.
In case of the absence of body forces, free surface tractions, inertia effects and initial strains the above expression reduces to:

$$\hat{J} = - \frac{1}{\Delta a} \int_{\Omega} \frac{\partial \delta x_1}{\partial x_j} (W \delta_{1j} - \sigma_{ij} \frac{\partial u_i}{\partial x_1}) dV \tag{18}$$

An alternative technique to evaluate J values within a FEM context was independently developed by Parks [10] and by Hellen [11]. With this technique, which is known as the stiffness derivative method or the virtual crack extension (VCE) method, the change in potential energy caused by an infinitesimal crack extension is calculated. For elastic materials the energy release rate obtained via this technique equals the path independent J-integral of Rice. The expression that forms the basis of the VCE method reads:

$$J = -\frac{1}{\Delta a} \int_{\Omega} (W\, \delta|J| + \delta W\, |J|)\, dV \qquad (19)$$

where $|J|$ is the determinant of the Jacobian of the element mapping.
This technique is available as a standard feature in the MARC program. For the evaluation of eqn. (19) a numerical differentiation is performed in order to obtain approximations for $\delta|J|$ and δW. Although this is not a serious drawback in practical situations, the distribution of the virtual coordinate movement δx_i must be selected such that not very large nor too small crack extensions are considered, because of the risk of obtaining rounding off errors or wrong approximations of the derivatives involved. In contrast to this, numerical evaluation of expression (18) will not require such numerical differentiations. It was shown by Bakker [8] that $\hat{J}$ according to eqn. (18) and J of eqn. (19) are identical.
Although a different starting point was used, deLorenzi [12] has derived an identical expression for eqn. (17) for the case that inertia forces and initial effects are absent.

Physical interpretation of $\hat{J}$

The J-integral which was proposed by Rice as defined in eqn. (15) can be interpreted as the amount of energy which flows through the contour Γ_1 per unit crack advance. When during crack propagation no work is done by tractions within the contour Γ_1 or on the crack surface and if no energy is dissipated other than the energy that is associated with crack growth in the process zone, J equals the rate of energy that is released at the crack tip. This energy rate is absorbed in the process-zone when advancing the crack forward. In case the previously mentioned conditions are not fulfilled, the energy rate J that flows through Γ_1 will depend on the choice of Γ_1 and does thus not equal the energy released at the crack tip. In contrast $\hat{J}$ according to eqn. (14) defines the net difference between the rate of energy that flows through Γ_1 and the energy that is consumed within the region enclosed by Γ_1. $\hat{J}$ therefore expresses the energy flow to the crack tip per unit crack advance and it is independent of the choice of Γ_1.

EXAMPLES OF DYNAMIC FRACTURE APPLICATIONS

As a first step towards the prediction of dynamic crack propagation and crack arrest phenomena by means of the extended J-integral, several dynamic fracture problems have been analyzed. In order to demonstrate and to verify the applicability and reliability of the $\hat{J}$-integral concept to dynamically responding structures, dynamic stress intensity factors for an impulsively loaded cracked panel with a stationary crack have been calculated. For the purpose of investigating the numerical modelling of crack propagation, the problem of a fast running crack in a uniform loaded center-cracked plate has been analyzed.

Dynamically loaded center-cracked rectangular plate

This first problem consists of a center-cracked plate which is initially at rest and which is subjected to a uniform tensile load which is suddenly applied and then maintained. This problem was originally analyzed by Chen [13] who used a finite difference method. Details on the dimensions, material properties and loading conditions are given in Fig. 3. The dynamic stress analysis has been carried out by means of the MARC finite element program. Because of symmetry only one quarter of the plate was modelled with 90 eight noded isoparametric plane strain elements.
The implicit Newmark-beta method with a constant time step of 0.15 μsec. perform the direct time integration. A consistent mass matrix has been used in this analysis. The calculated $\hat{J}$-integral values have been converted into dynamic stress intensity factors. Fig. 4 shows stress intensity factors as a function of time normalized with respect to the static stress intensity factor for an infinite plate. The solution obtained by Chen is denoted too in Fig. 4 and as may be concluded from this a good agreement between both solutions is obtained. In this figure characteristic time intervals are indicated at which particular (reflected) waves arrive at the crack tip. At L_1 the longitudinal wave, initiated at the boundary, will arrive at the crack tip. At R_1 the Rayleigh wave generated by the previously mentioned longitudinal wave has travelled from one crack tip to the other. At P_1 the scattered longitudinal wave has travelled from the crack tip to the nearest boundary and back to the same tip. At S_1 the scattered transverse wave has travelled from the crack tip to the nearest boundary and back to the same tip. Similar indications are given for arrival times R_2, P_2 and S_2 of Rayleigh-, longitudinal- and transverse

waves that are generated at L_2 when the longitudinal wave has crossed the complete length of the strip and has reflected back to the tip. The results of this test problem clearly demonstrate the capabilities of the employed finite element approach to simulate stress wave propagation and to accurately calculate dynamic stress intensity factors. The results obtained agree with FEM results reported by other researchers e.g. Mall et al [14] and Brickstad [1] who derived the stress intensity factors from the crack opening displacement. It should be pointed out that the extended J-integral enables the accurate calculation of dynamic stress intensity factors with standard isoparametric elements. A separate analysis in which so-called quarter point elements were employed to represent the $1/\sqrt{r}$ singularity of stress and strain distributions around the crack tip has indicated that the difference in the resulting stress intensity factors versus time was less than 1%.

Fast running crack in a center-cracked square plate

The second test problem consists of a tensile loaded center-cracked plate in which a crack propagates at constant speed. This problem has been chosen, because the results can be compared with those of other authors. The plate is loaded by a uniform tensile stress in the direction perpendicular to the crack. This uniform load remains constant in time. The crack is initially symmetrical with a half-crack length a of 0.2 W, where W is the half-width of the plate. This crack is allowed to propagate symmetrically at speeds of 0.2, 0.4 and 0.6 times the shear wave speed of the material. Details on the material properties, dimensions and loading conditions are given in Fig. 5.
The main purpose of this analysis is to investigate the ability to simulate crack growth in relative coarse meshes by applying incremental crack propagation that results in partly cracked elements. This ability forms an essential requirement for performing efficient generation- and/or application phase crack propagation analyses.
Crack growth is modelled by means of the so-called node relaxation technique, which consists of the following steps (see Fig. 6)

- at the moment the crack tip position passes an element border the boundary conditions at point 1 and 2 are removed. In addition external forces are applied with the reaction forces at these points computed at the instant the crack-tip reaches point 1;
- these external forces are then assumed to decay in a given number of increments until the crack-tip location reaches the next element.

In the present test problem a quarter of the plate has been modelled by means of eight-noded elements. Both the forces at the corner node (point 1) and at the mid-side node (point 2) are released simultaneously in the following manner:

$$F_i = F_{oi} \{1 - \delta(t)/d\}^{\alpha} \qquad i = 1, 2 \tag{20}$$

where d is the element length, $\delta(t)$ the amount of crack propagation along the element, F_{oi} the reaction force acting at node i at the time instant the crack reaches node 1 and F_i the force acting at node i at time t. In the literature various suggestions have been made for the choice of the parameter α. Most commonly a linear decay function is suggested i.e. $\alpha = 1$. Rydholm [15] has applied a value of .5 in order to generate a constant energy release rate for quasi-static crack propagation along the current element. Malluck and King [16] have suggested a decay function based on $\alpha = 1.5$. In the present example decay functions have been used with α values of .5 and 1.0 . Fig. 7 shows the normalized dynamic stress intensity factors versus the relative crack length. The K_I values have been derived from the calculated $\hat{J}$ values using eqn. (3). The velocity dependent correction factor A_I of the latter equation is depicted in Fig. 8. The solid horizontal lines in Fig. 7 represent the theoretical values for cracks that propagate with constant velocity in an infinite, tensile loaded plate. These values were derived by Broberg [17]. Since they are applicable to cracks that do have initial lengths equal to zero, deviations are observed up to a relative crack length of a/W = .3. Subsequent to this initial effect a quasi-static state is reached during which nearly constant values are obtained up to the point where reflecting waves reach the crack tip. Since this is not the case for the problem with the highest velocity, it must be concluded that the oscillating behaviour of the K_I values is caused by another effect. In general the results obtained are, when averaged, similar to the theoretical solution. For a realistic crack propagation analysis it will be necessary to consider a varying crack propagation velocity. For this reason the same problem was analysed assuming a crack velocity that increases linearly with time from 0 to 1600 m/s and then decreases linearly until crack arrest occurs. In order to determine the influence of the element size used, two different meshes were considered. From Fig. 9 it may be concluded that even for the fine mesh, oscillations occur. Application of a decay function that is based on $\alpha = .5$, for a constant velocity case in which no reflections are present, produced results as shown in Fig. 10. From this figure it may be concluded that the calculated K_I values depend on the decay function. It should be stressed that the values corresponding with the points where the crack passes an element border are strongly influenced by the chosen decay function. The latter values are commonly employed by research workers when simulating dynamic crack propagation. Mesh refinement results in even larger oscillations as is shown in Fig. 11.

CONCLUSIONS

The extended J-integral $\hat{J}$, that is presented in this paper, enables the accurate calculation of dynamic energy release rates.

The conversion of the expression for $\hat{J}$ into surface integrals allows for a straightforward implementation into FEM programs. The numerical simulation of crack propagation in dynamically loaded structures results in calculated fracture parameters that are influenced by the particular choice of the decay function in the node relaxation technique. Additional research will be required for the numerical simulation of dynamic crack propagation phenomena. Until the accurate calculation of dynamic fracture parameters for fast propagating cracks is possible, results obtained with the commonly used node relaxation techniques have to be interpreted with care.

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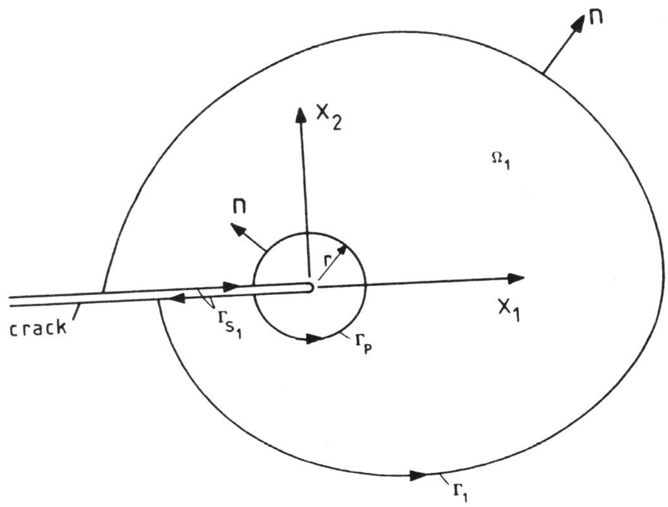

Figure 1 Closed contour used in the definition of the $\hat{J}$-integral

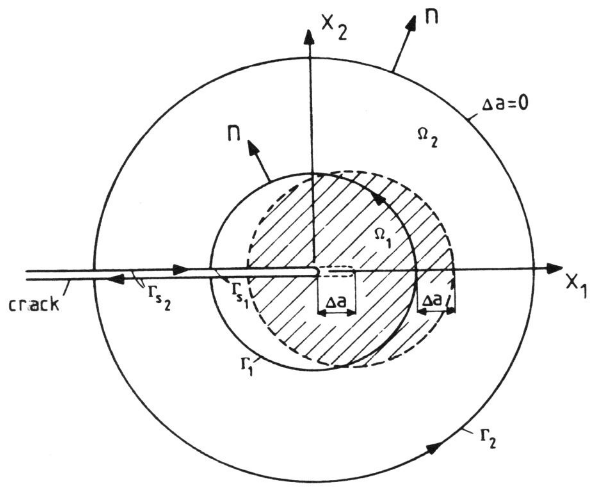

Figure 2 Definitions used to transform contour integral along Γ_1 into surface integral over Ω_2 for numerical evaluation of $\hat{J}$

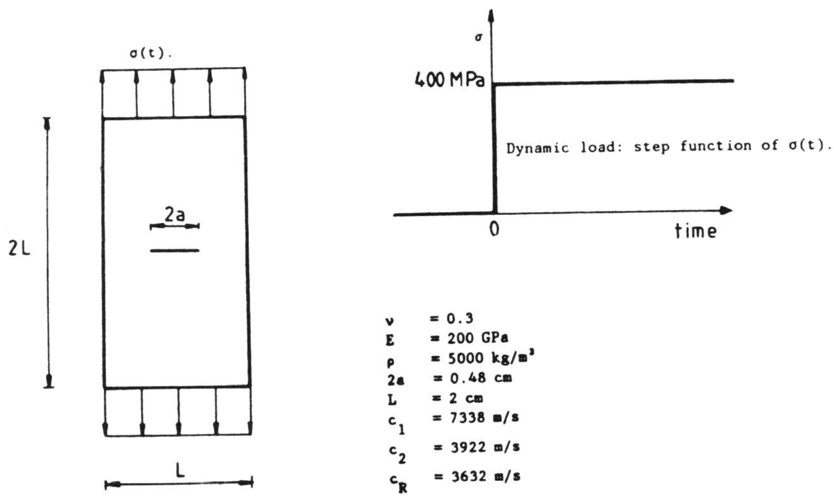

Figure 3 Dynamically loaded center-cracked rectangular plate

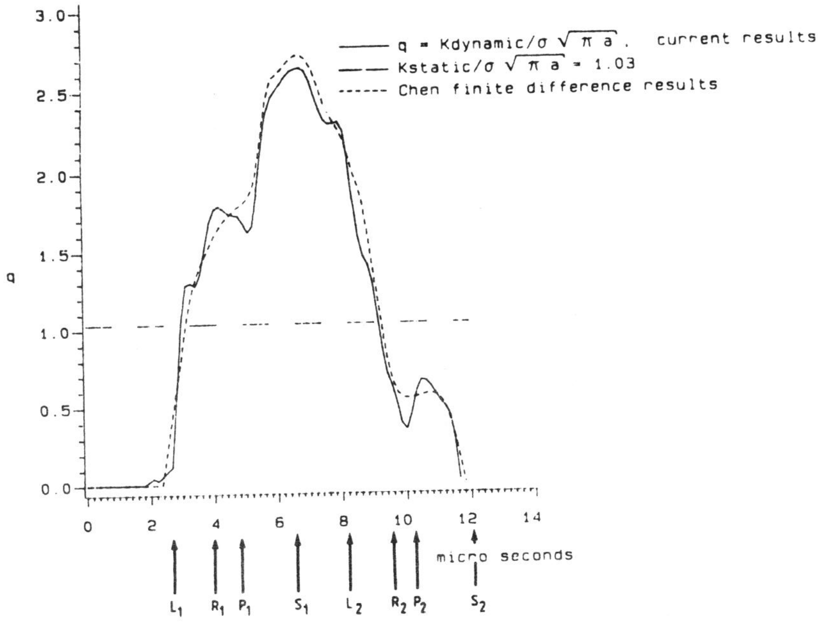

Figure 4 Comparison of calculated dynamic stress intensity factors for a dynamically loaded center-cracked rectangular plate

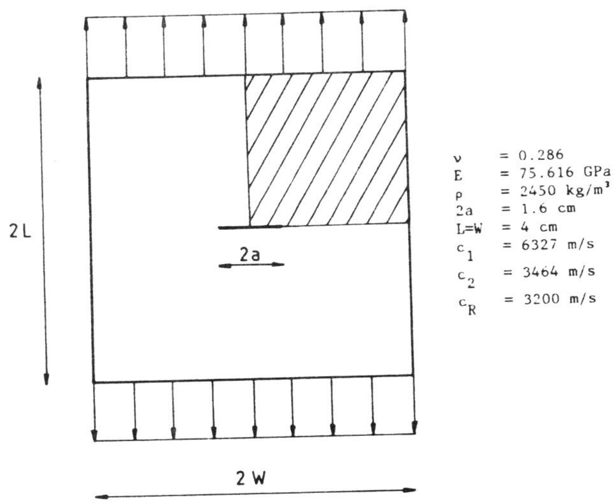

Figure 5 Center-cracked plate model used to study the effects of node relaxation

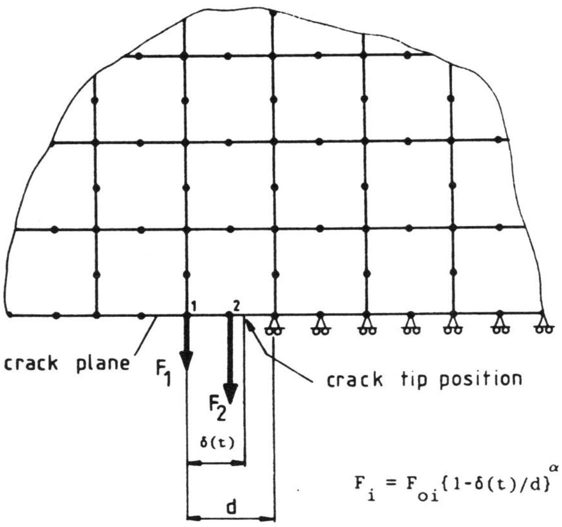

Figure 6 Node relaxation technique

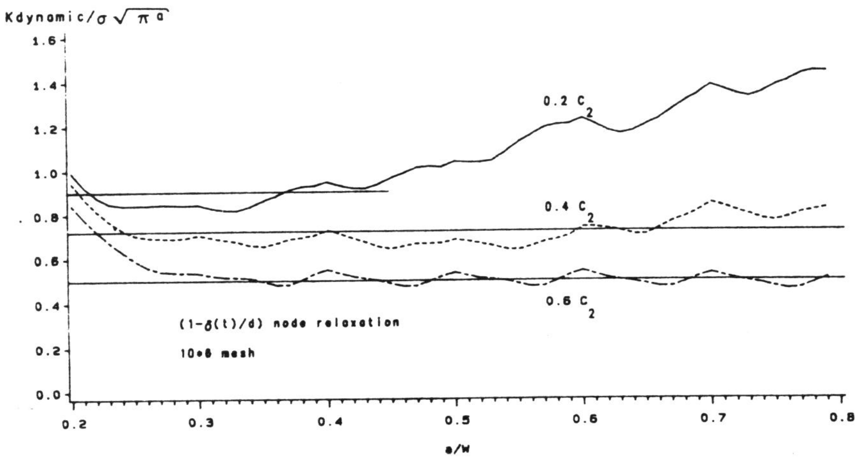

Figure 7 Dynamic stress intensity factors for crack propagating at constant speeds of 0.2, 0.4 and 0.6 times the shear wave speed

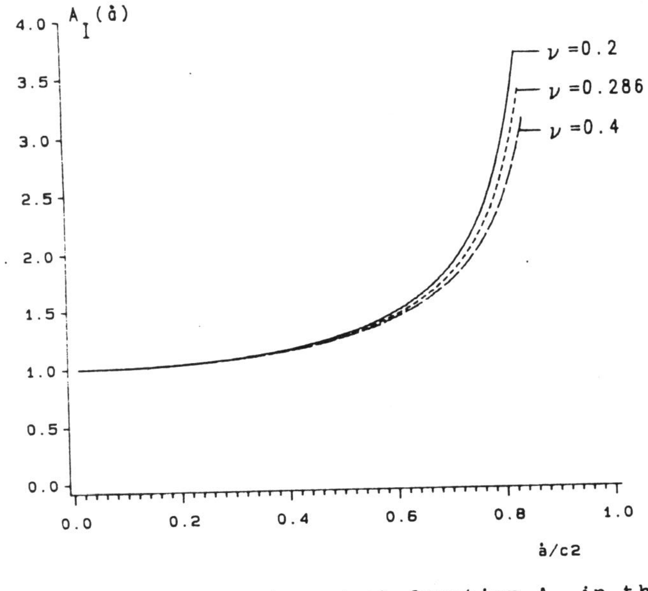

Figure 8 The velocity dependent function A_I in the relation between J and K_I

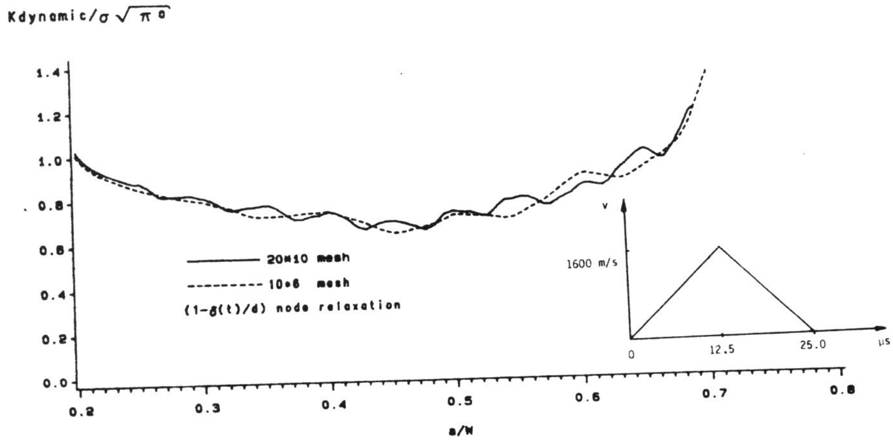

Figure 9 Dynamic stress intensity factors for a crack propagating at varying speed

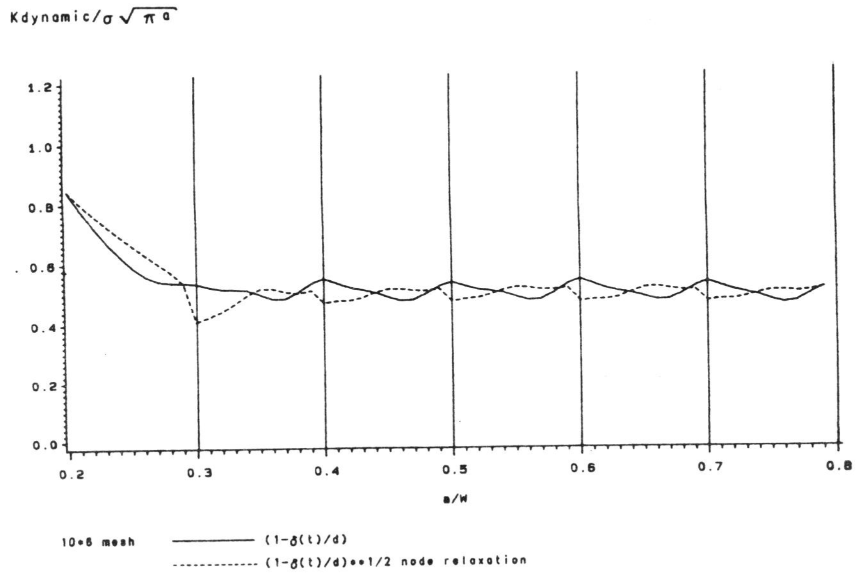

Figure 10 Dynamic stress intensity factors for cracks propagating at constant speed of 0.6 times shear wave speed for various decay functions

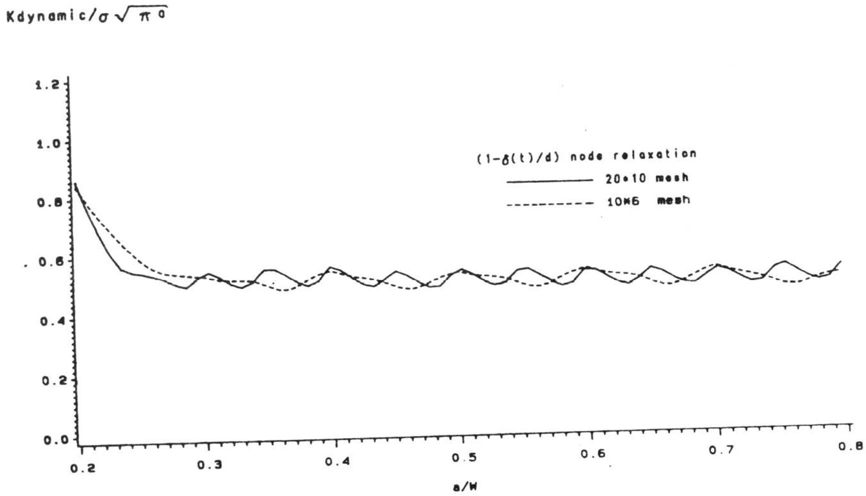

Figure 11 Dynamic stress intensity factors for crack propagating at constant speed of 0.6 times shear wave speed for two different element sizes