PREDICTION OF THE INFLUENCE OF TEMPERATURE ON FATIGUE OF POLYMERS

J.A.M. Remmerswaal*

A generalized model for the prediction of the effect of stress, strain rate and temperature on the fatigue properties of polymers is presented. With the aid of a thermal activation model is it possible to construct a master S/n-curve, which can be used to predict the combined effect of the service conditions on fatigue properties. The validity and the applicability of the model has been tested using specimens of ABS and PC. Extensive fatigue tests were carried out under stringent temperature control and various frequencies. The effects of strain rate and stress concentrations are also reported.

INTRODUCTION

Because of the increasing use of polymers in load-bearing applications, the need for thorough characterization of the material properties in intermittent loading situations has been increased. It is important to be able to judge the reliability of a component whilst still in the design stage. At least the most critical areas should be analysed to meet the required fatigue resistance.

However not only is our current knowledge inadequate for analysing all stress situations, but the material properties of the polymers are inherently difficult to predict.

They are effected by processing (thermal degredation, water content), design (orientation, welds) and operation. Due to the visco-elastic nature the fatigue properties of polymers are strongly dependent on time and temperature. Thus the actual service conditions have to be considered.

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*Department of Industrial Design Engineering,
Delft University of Technology

In common engineering practice Miner's rule for fatigue prediction is widely used. In this law the fatigue damage is calculated as the sum of the individual fatigue damage from each cycle, usually expressed with:

$$D = \sum_i \frac{n_i(\sigma)}{N_i(\sigma)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For this, the number of possible cyclic loadings N_i have to be determined from a suitable S/n-curve. This S/n-curve must be determined using the service conditions under consideration, for each polymer.
Up to now very little suitable fatigue data has been reported. This is one cause of the poor accuracy of the life time predictions derived from existing S/n-curves, which were carried out under slightly different conditions. The accuracy could be improved if the effect of strain rate and temperature changes were better understood.

To improve the accuracy of fatigue calculations a study has been performed which took these factors into consideration when predicting lifetime under cyclic load. Practical applicability was considered paramount. Therefore the influence of notch sensitivity has been investigated which means the results can be generalized.

THE THEORY; A UNIFIED APPROACH

In polymer science there exists a large number of models which describe yield stress, ultimate strength and fracture as a process dominated by chain slip and chain scission. This class of models is based primarily on the assumption that chain scission results when the mechanical load exceeds a certain level. Mechanical stress and increase of temperature lowers the effective strength, which is also time-dependent. Zhurkov et al (1) proposed this behaviour to obey:

$$\sigma_c = \frac{U_o - RT \ln \omega_o \eta_o \tau_o}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Many experiments have been performed (1) to confirm that this model is correct. It is accepted that the model agrees well, at least for highly oriented chains. Evidently in unoriented polymers the behaviour deviates from this ideal due to inhomogenities and entanglement effects. Numerous investigations on the effects of stress and time have been performed using more or less extended versions of this model, usually written as an Arrhenius equation:

$$\ln t/t_o = (U_o - v\sigma)/RT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Among others Ratner et al (2) studied the elementary strength of a polymer, Mindel et al (3) the relation between strainrate and strength and Weaver (4) employed this model for compressive fatigue experiments.

Wide spread agreement is apparantly obtained using this sort of relation.

Many authors have proposed that the parameters in this class of models have a more general significance. There is for instance a correspondence between T_o and the weakest polymer chain bond and v is often referred to as the activation volume. However it has been shown by Kausch (5) that the values of these parameters depend on the mechanism of fracture and so are not true material constants. The fundamentals of this are still a subject of discussion and agreement seems far off.

It is expected that the activation energy theory may apply to homogeneous fatigue, at least for the same failure mechanism; for the observation is that in the time-strength region (sloping part of the S/n-curve) the logarithm of the number of reversals increases linearly with decreasing stress amplitude and varies with temperature. Failure at short lifetime is accomponied by extensive plastic deformation, so that, like monotonic fracture, it is controlled by flow processes.

And in the endurance limit region failure may be related to creep or interrupted creep.

Provided that we restrict ourselves to one failure mechanism we hypothise that, without any particular physical significance to the constitutive constants, the time to failure is expressed by the constitutive equation:

$$\log t = C_1 + \frac{C_2}{T} - \frac{C_3\sigma}{T} \quad \dots\dots\dots\dots\dots\dots(3)$$

If this is true for periodic varing loads with amplitude σ then the number of reversals must obey:

$$\log n = C_1 + \log \omega + \frac{C_2}{T} - \frac{C_3\sigma}{T} \quad \dots\dots\dots\dots(4)$$

From this we can derive the constants C_1, C_2 and C_3:

$$C_3 = - T \frac{\partial \log t}{\partial \sigma} \quad \dots\dots\dots\dots\dots\dots(5)$$

$$C_2 = \frac{\partial \log t}{\partial\, 1/T} + C_3\sigma = \left\{\frac{\partial \log t}{\partial\, 1/T}\right\}_{\sigma=0} \quad \dots\dots\dots(6)$$

$$C_1 = C_3 \cdot \frac{\partial \sigma}{\partial T} + \log t = \left\{C_3 \frac{\partial \sigma}{\partial T}\right\}_{t=1} \quad \dots\dots\dots(7)$$

By finding these first order derivatives from measurements or appropiate graphs (transformations of S/n/T-curves), one can calculate C_1, C_2 and C_3.
As already mentioned, the validity is restricted to one failure-mechanism which is assumed to correspond to one distinct region of the S/n-curve. It must be expected that several factors connected with fatigue may disturb the behaviour. Well known are creep, the initiation and propagation of cracks and thermal softening. But one should also be concerned about deviation due to residual stress, flaws and inhomogenities, thermal history and physical aging. Some of these may be eliminated by suitable experimental conditions and others by careful specimen preparation. Obviously the model only predicts fatigue behaviour within the region between two transition zones.
An extra comment has to be made on the discrimination between homogeneous and heterogeneous fatigue, as two distinct phases of a fatigue process. It has been shown (5) that both crack initiation and slow crack growth can both be described by most kinetic theories of failure. This implies that discrimination between the two phases is not necessary: they can be described by the same constitutive equation.

EXPERIMENTAL

Equipment

The testing equipment consists of a hydraulic fatigue machine and a electric-mechanical device both suitable for torsion experiments and of high precision. Extra care has been taken to perform the tests under iso-thermal conditions. With the hydraulic machine is this achieved using an active temperature control system and forced air heating. A comparable system is used with the electric machine. In this case the temperature is maintained at the desired level using a thermo-couple attached at a critical location at the specimen. This made isothermal fatigue tests possible with a maximum error of 1 K at frequencies up to 12 Hz. The experiments can be done in either constant stress amplitude or in constant displacement amplitude modes, with automated counting, failure detection and protection.
Also constant monitoring of the important parameters is provided and the test is stopped if the limits of error are exceeded.

Specimen

The types of specimen were chosen according to the objectives of the investigation. One is a tubular specimen with a slender waist section and a longitudinal hole, see fig 1. The other specimen used, fig 2, is a coupling which consists of two short tubes connected by four small bridges. The first was neccesary to confirm the theory and to collect basic fatigue data. The second specimen was chosen to test the extrapolation procedure. It is expected that stress concentration and failure definition will hamper translation of fatigue behaviour derived with neat specimen to real products.

To mimimize errors arising from the method of production, all specimens were injection moulded.

Naturally care was taken to obey the pre-conditioning and moulding instructions of the material suppliers.

The materials investigated were ABS cycolac T, a general purpose ABS with moderate fatigue properties, and a standard Polycarbonate polymer.

Test conditions

As mentioned previously several secondary factors affect the fatigue life.

We may recall for instance residual stress, thermal history and physical aging. In our tests their influence is excluded, or at least mitigated by thermally treating the specimen before the fatigue test. The specimen is heated to slightly above T_g and then gradually cooled to roomtemperature. After an elapse time t_e of 1000 sec the specimen is placed in the fatigue machine. Then the specimen was allowed 10 minutes to reach thermal equilibrium at the test temperature, before the fatigue was started.

All the experiments were performed in torsion. This suppresses the generation of crazes and free volume. An additional advantage is that the hydrostatic stress is zero: some polymers show yield behaviour dependent on hydrostatic stress. And necking and creep will not occur in fully reversed cyclic loading situations.

It is known that a strong relation exists between strain rate and yield stress, and this relation should have a strong effect on fatigue behaviour. Hence all the experiments have been performed under constant mean strain rate conditions. This requirement is obeyed with the constraint:

$$\left\{\frac{dE}{dt}\right\}_{\text{mean}} = A_1 \frac{\sigma\omega}{E} \rightarrow \sigma\omega \simeq \text{constant} \quad \ldots\ldots\ldots\ldots(8)$$

Corresponding to the objectives of our investigation our attention has been restricted mainly to the time-strength region.

RESULTS FROM THE FATIGUE EXPERIMENTS

If the fatigue lifetime corresponds with the expected behaviour described in equation (3) then the number of reversals endured must be a function of the frequency. This is shown in fig 3. When $t=n/\omega$ than the point n_1 will shift to n_2 if the frequency is lowered because $t= n_1/\omega_1 = n_2/\omega_2$, hence $n_2/n_1 = \omega_2/\omega_1$. So the dotted line represents the S/n-curve in case of $\sigma.\omega$ = constant.

Correspondingly a parallel curve may be derived originating from τ_1, n_2 and representing our S/n-curve for a different constant value of $\sigma\omega = K_2$.

The experiments seem to agree with this aspect of the constitutive law as may be seen from fig 4. A better test of this condition is derived if the results are plotted as a graph of the endurance against frequency, on a logarithm base, see fig 5.
With $\log t = \log n - \log \omega$, $d \log \omega / d \log t = 1$.
So independency from frequency corresponds to a vertical line with infinite slope and linear dependency gives a slope of +1. The graphs of fig 5 reveal a strong tendency towards the expected dependency although full accordance is not reached.
Additional support is possible from evaluation of the influence of the wave-form on endurance. To conform to the constitutive law, the fatigue damage must be proportional to logarithm of time during which the load acts. Therefore tests have been performed with triangle-, sinusoïdal- and square wave-forms of the same frequency. The results displayed in fig. 6 show how the results correspond to this expectation. There is good agreement with the theory that the endurance is governed by the time under load to a reasonable approximation, the lifetime is proportional to the normalised area under the wave-form.

In accordance with the main objective of the experiments, the effect of temperature on fatigue endurance over a technically relevant temperature range has been investigated. Typical results are shown in figures 7 and 8. Increase of temperature results in greater mobility of the polymer molecular chains.
This is assumed to explain the increase of molecular flow and the decrease of yield stress. The observed shift to shorter lifetimes is in accordance with theory. Within the time-strength region the S/n-curves are approximately straight and equidistant for equal temperature intervals.
The same general behaviour was established when stress concentrations are involved as in the coupling specimen. Figures 9 and 10 show clearly that although there is a significant difference in strength between PC and ABS, the overall appearance of the curves is similar.

From these S/n-curves the fatigue behaviour can be described using the thermal activation model. The constants C_1, C_2 and C_3 (eq. 5, 6, 7) may be obtained by measurement or calculation. The values determined from the S/n-data presented here are tabulated in tabel 1.
The similarity between the figures for the same material is obvious. Notice for instance that the coordinates of the loci to which all the lines diverge, C_1, C_2/C_3, are irrespective of type of specimen.
It should be noted that for this calculation the nominal stresses are used. This point is of particular importance when specimens contain discontinuities. Commonly the actual stress concentration for fatigue is unknown and difficult to define because it needs information about the notch sensitivity of the material involved. This is only rarely available.

Specimen Material	Fatigue loading, $\sigma\omega$=constant	Temp. [°C]	C_1	C_2	C_3	C_2/C_3
Tube ABS cyc.T	torsion	0-60	-2.0	2276	33.5	68
Coupling ABS cyc. T	torsion	20-60	-2.0	1831	27	68
Tube PC	torsion	0-60	-1.4	2355	29,9	79
Coupling PC	torsion	30-60	-1.4	1918	24.6	78

Table 1 CALCULATED MODEL CONSTANTS OF THE ACTIVATED ENERGY MODEL

Of course one cannot omit the influence of stress concentration but instead of a calculation from first principles, an alternative procedure is suggested. This method is based on the idea that the apparent stress concentration K_f for fatigue is a function of (amongst other factors) stress level, stress concentration factor K_T, strain rate and temperature. The linear elastic K_T reduces to K_f due to visco-elasticity of the polymer. Hence K_f must depend on strain rate and temperature. It is well-known that the inelastic component of deformation of polymers causes fatigue. It is assumed that under a certain threshold the damage will be sufficiently small to result in infinite life. From our experiments K_f has been derived over the entire time-strength region. At high nominal stresses and corresponding short lifetimes K_f approaches 1, see fig. 11, wheras at low stresses K_f increases to approximately K_T.
We suppose that this variation can be modelled, to a first order approximation, by the dotted lines depicted in the figure.
The K_f associated with a certain life expectation will then be found by interpolation. But again it is stipulated that this procedure only applies to the time-strength region of fatigue.

There are many ways to test the model. A first check is obtained by inspection of figures 7, 8, 9 and 10. The lines represent the estimated average lifetime for several temperatures. Agreement with experiment is good.
Additional confirmation comes from the waveform and frequency dependence, which obey the model. So the description enables us to transform a particular experimental S/n-curve to another S/n-curve valid for another temperature and frequency.

A combined demonstration to prove the abilities of the model is shown in figure 12. Here a comparison is made between a calculated S/n-curve for the coupling and the measured S/n-curve for that product. The procedure consists of several steps and starts with the transformation from the basic S/n-curve to the temperature and frequency conditions involved. After that, correction to true stresses is neccessary, which requires the stress-strain relations, for the conditions mentioned, to be known.
Subsequently the derived stresses are divided by the K_f involved

and then turned from real stresses into apparant stresses, usually expressed as nominal stress. Comparison between the measured and predicted S/n-curve shows promising agreement.

CONCLUSIONS

The reported results show that the usual presentation of resistance against cyclic loads is not satisfactory for ABS and PC. S/n-curves for those materials are strongly dependent on several experimental conditions. In the absence of this information the S/n-curve cannot be applied to real situations.
In agreement with numerous publications on the strong dependence of the properties of polymers on temperature, a strong temperature influence on fatigue resistance has been shown. This accentuates the need to consider the temperature in each application.

With the aid of a thermal activated energy model it is possible to predict the influence of both temperature and frequency. However an uncoupled approach is used so hysterysis heating is not considered. Hence the method is applicable in situations without significant selfheating. Prediction capability is only gained in the time strength region and between the state transition temperatures of the polymer.

Considering the preceeding results it would be of value to present the fatigue resistance of a particular material in a unified way, the so-called mastercurve. We suggest an S/n-curve at 1 Hz sine-wave and 23°C would be appropiate.

An alternative presentation of fatigue resistance may be the values of the constants C_1, C_2 and C_3 in the activated energy model presented here. This should suffice for calculating fatigue performance for design purposes.

Practical application of the theory involves the transformation of the general S/n-curve into a S/n-curve valid for the product.
A procedure is suggested for this which takes account of stress concentration and visco-elasticity. The promising results obtained with this method proves its utility.

The described model can be extended to cover more of the S/n-curve. Available information supports strongly the idea that the low cycle region may also be adequately described with the proposed activated model. Owing to the different failure mechanism different constants would be expected.
Although the conclusions strictly only apply to the investigated materials ABS and PC, a wider application must be envisaged.
Visco-elasticity seems the factor responsible for the reported temperature and frequency dependency. Hence it is probable that many other ductile polymers will obey this model. Additional research is needed to establish this suggestion.

SYMBOLS USED

C_1 = constant (s)

C_2 = constant (s.K)

C_3 = constant (s.K/MPa)

D = damage due to fatigue

E = modulus of elasticity (N/m^2)

n = number of reversals

N = number of reversals to break

η_o = number of polymer units involved

R = universal gas constant (Nm/mol.K)

σ = characteristic stress (N/m^2)

t = time to break (s)

t_o = reference or start time (s)

T = temperature (K), (°C)

τ = shear stress (N/m^2), (MPa)

τ_o = average time of a polymer bond to break (s)

U_o = bond energy of a polymer (nm/mol)

v = activated volume of a polymer (m^3)

ω = frequency of cyclic loading (Hz)

ω_o = vibration of a polymer chain element (rad/s)

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(2) Ratner, S., Stinskas, A. and Shpakovskaya, E., "Analysis and test methods, Long-term strength of plastics", Plasticheskie massy 9, 1961.

(3) Mindel, N. and Brown, J. Materials sc., No. 9, 1974, pp. 1661-1669.

(4) Weaver, J., Beatty, C., Polymer Eng. and Sc., No. 18, 1978, pp. 1117-1126.

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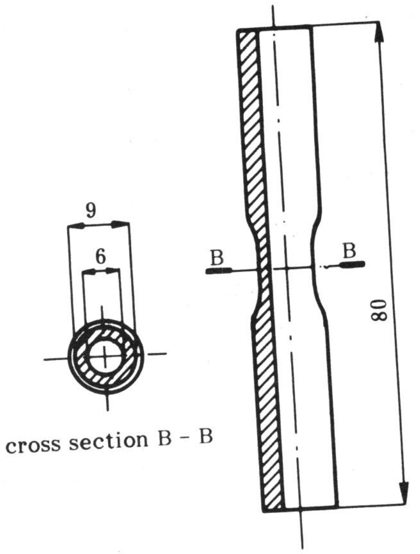


Figure 1 Tubular specimen

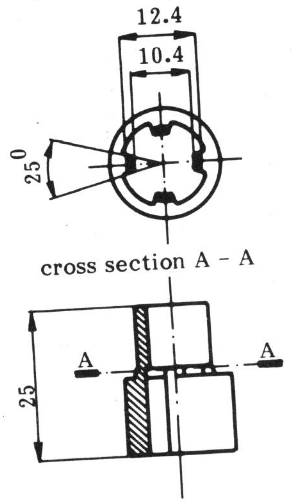
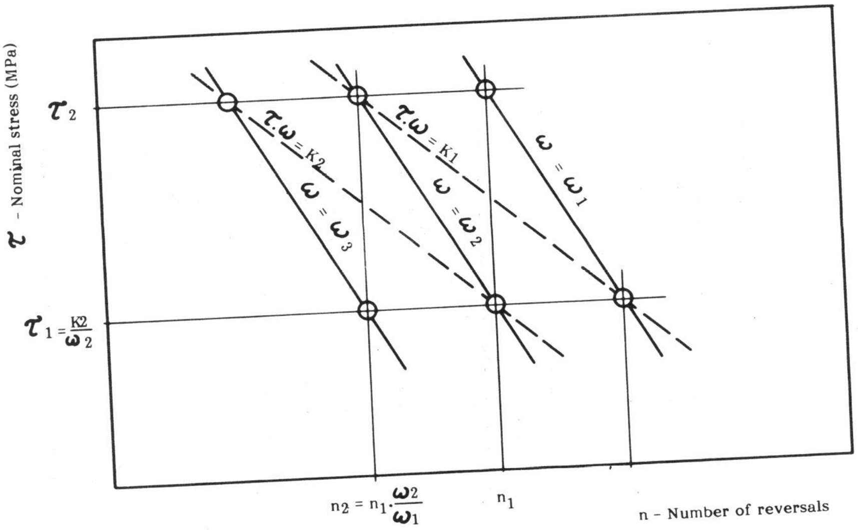


Figure 2 Plastic product used for fatigue experiments



gure 3 Relation between the number of reversals to break and the frequency if $t = n/\omega$.

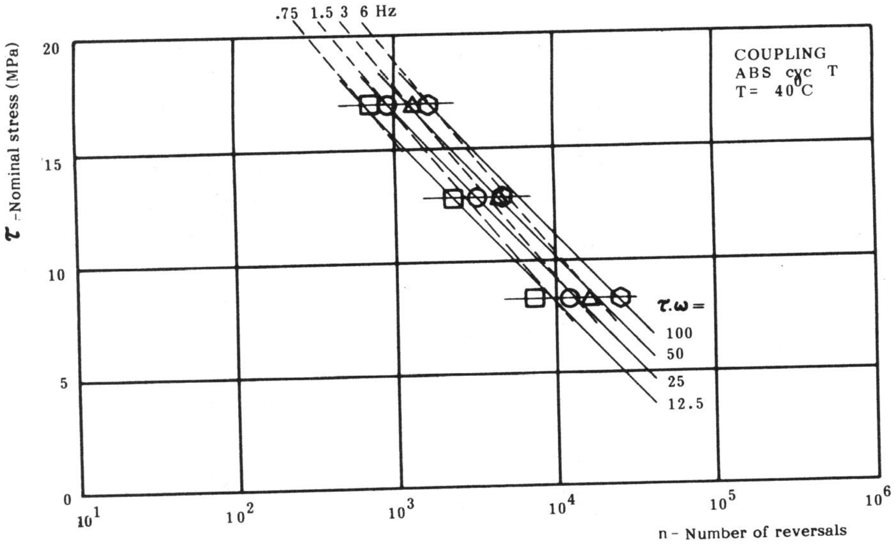


Figure 4 S/n-curves of ABS cyc. T at 40°C performed for several strainrates (reversed torsion)

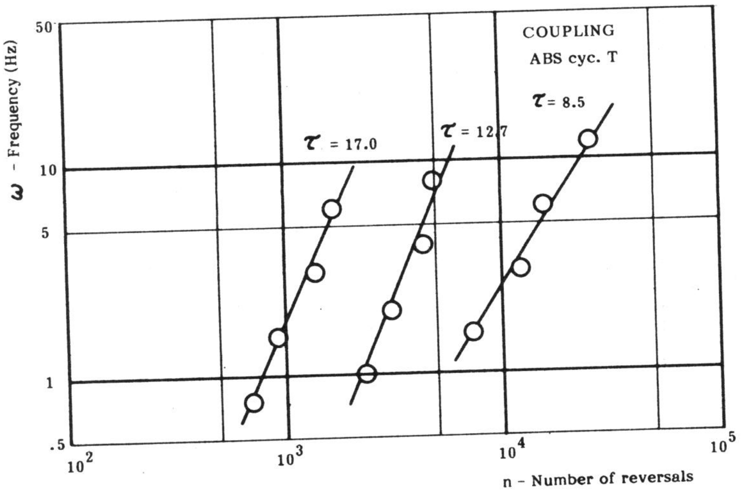


Figure 5 The experimental data of fig. 4 presented in a ω/n-plot to show the frequency sensitivity (reversed torsion)

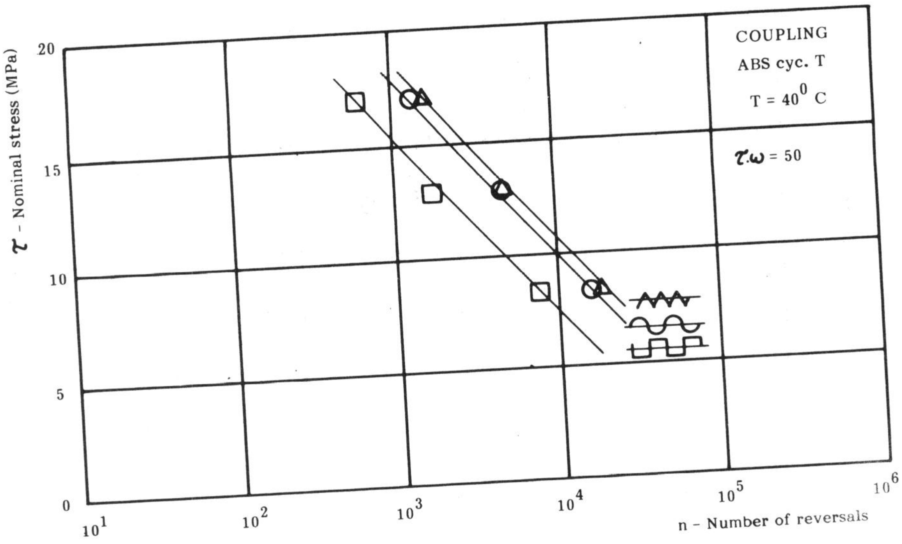
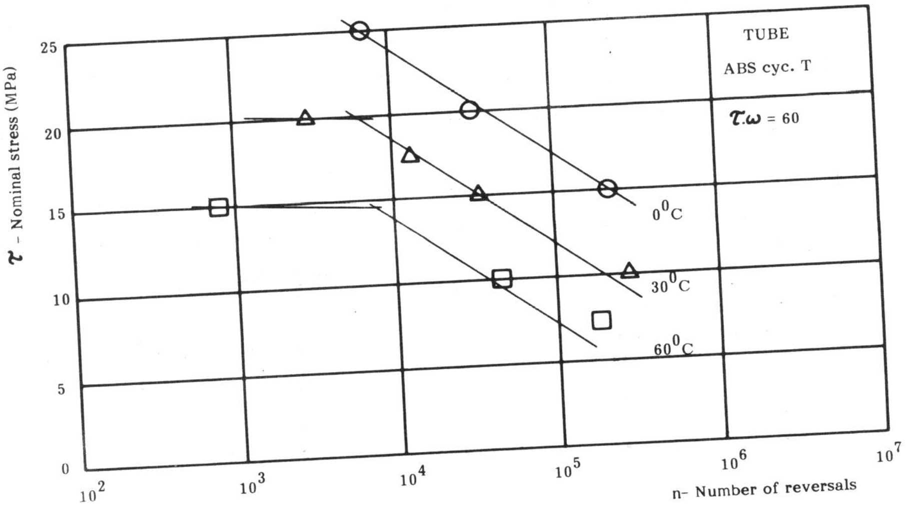


Figure 6 The influence of waveform on lifetime for the ABS coupling at 40°C (reversed torsion)



gure 7 Effect of temperature on endurance for reversed torsional sinewave on ABS tubes

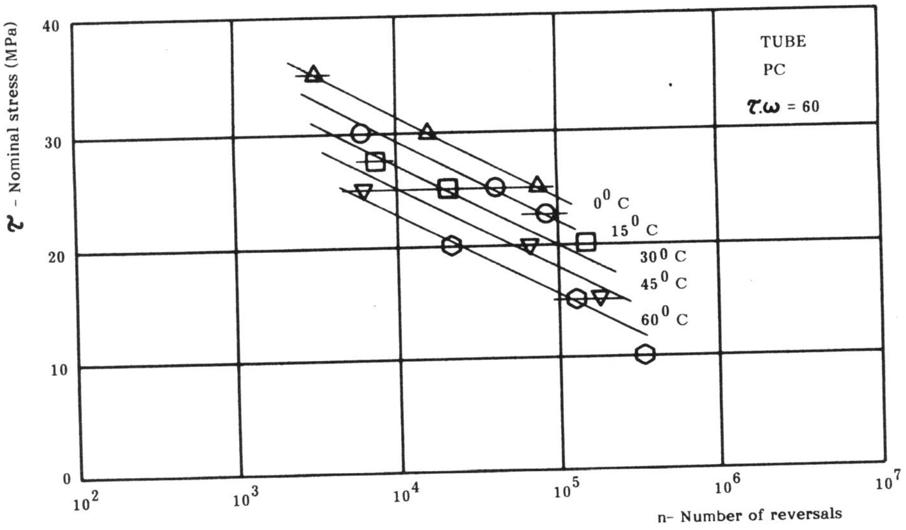


Figure 8 S/n-curves showing the temperature dependency of Polycarbonate loaded with reversed torsion

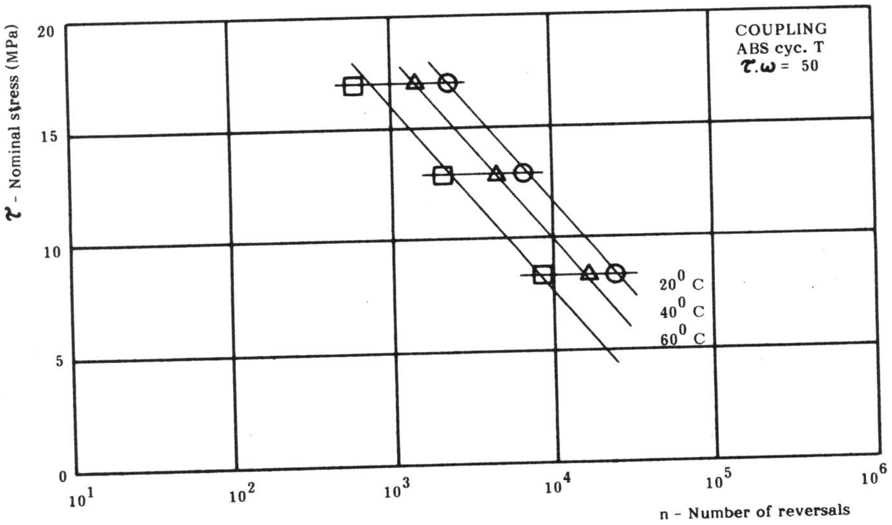


Figure 9 S/n-curves of ABS couplings at several temperatures (reversed torsion)

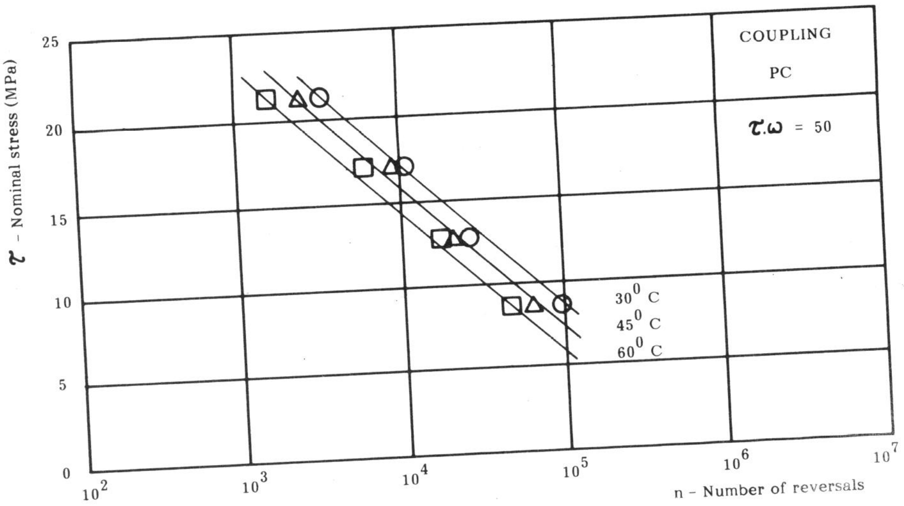
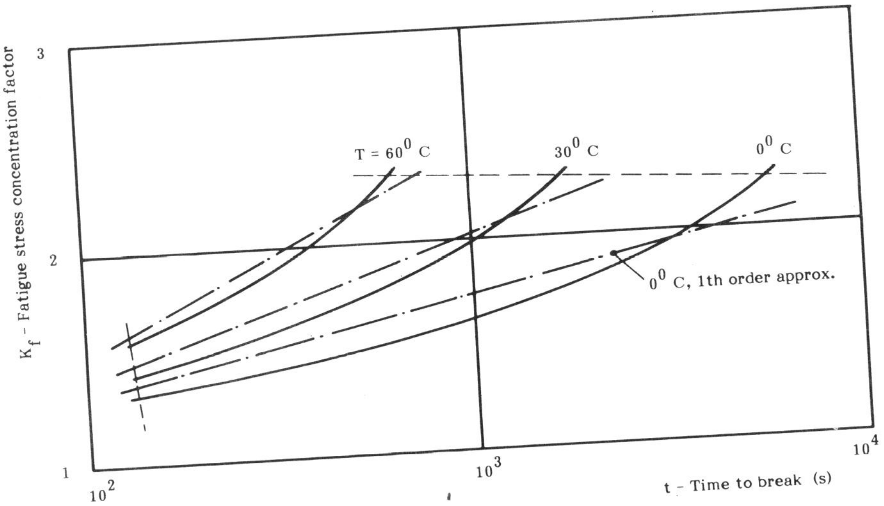


Figure 10 Effect of temperature on lifetime for the Polycarbonate coupling (reversed torsion)



igure 11 The from fig. 7 and 9 derived stress concentration factor K_f valid for the ABS cyc.T. coupling at 0, 30 and 60°C.

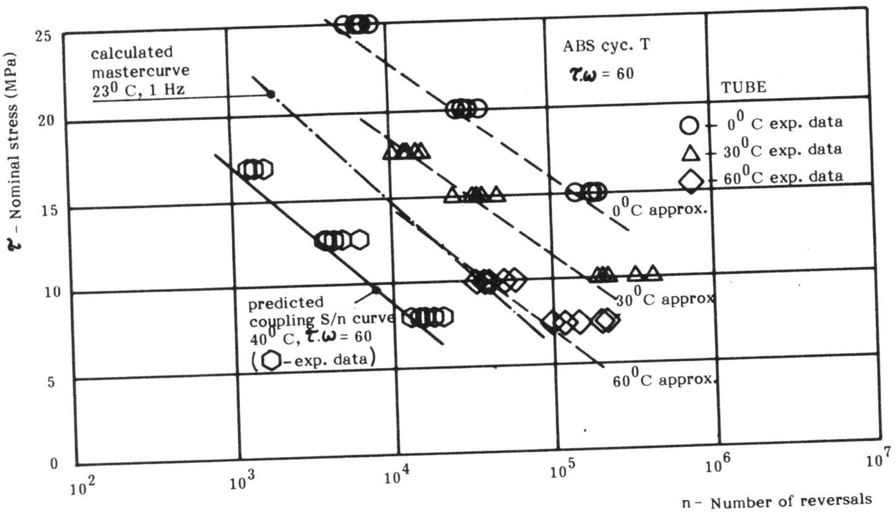


Figure 12 S/n-diagram showing the agreement of the model (dotted) and the predicted S/n-curve for the coupling, with experimental data