EXPERIMENTAL AND THEORETICAL DETERMINATION OF LEAKAGE AREAS DUE TO SUBCRITICAL CRACKS

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Leakage areas are evaluated with respect to
- minimum values for the design of leak detection systems and
- maximum values for controlling jet and reaction forces.

Results of experiments and finite element calculations are compared with analytical predictions. The analytical approach is able to supply both the minimum values and, assuming an empirical factor of 3, the maximum values, too.

INTRODUCTION

Leakage areas are evaluated with respect to
- minimum values for the design of leak detection systems and
- maximum values for controlling jet and reaction forces.

For subcritical leaks leak before break behaviour has to be shown by means of fracture mechanics. The leak detection system has to assure that leakage areas can safely be detected with respect to leakage rate and detection time. Therefore the evaluation of minimal realistic leakage areas due to leaking cracks is a design criterion for leak detection systems.

For the design of nuclear power plants leaks and loads due to jet and reaction forces have to be considered. Subcritical cracks are taken into account in longitudinal and circumferential welds. For these flaws

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leakage areas are determined by fracture mechanics analysis and limited to a maximum area of 0,1 times nominal pipe cross section.

ANALYTICAL APPROACH

The analytical model is based on solutions of crack opening areas in plates containing through wall flaws.

General Considerations

Analogous to the assumptions in plates relations between crack area, through wall crack length and loading ding are established for through wall cracks in cylindrical structures (pipe lines, pressure vessels etc.).

In general, the method is founded on well known relationships between the displacement of the crack surface in the vicinity of the crack tip and the stress intensity factor for mode I loading.

General Expression of Leakage Areas

The displacements are expressed by

$$V(r,\varphi) = \frac{K_I}{2G} \sqrt{\frac{r}{2\pi}} \cdot \sin\frac{\varphi}{2} \left(k+1-2\cos^2\frac{\varphi}{2}\right) \qquad (1)$$

where

$$K_I = \sigma \cdot \sqrt{\pi \cdot c} \qquad G = \frac{E}{2(1+\nu)} \qquad (2)$$

and σ = applied stress

c = half through wall crack length

ν = Poisson's ratio

E = Young's modulus

k = $3-4\nu$ for plane strain

k = $(3-\nu)/(1+\nu)$ for plane stress

Assuming plane stress and

$\varphi = 180°$, $r = c-x$ with $0 \leqq x \leqq c$

a parabolic shape of the crack opening will be obtained

$$V(x) = \frac{4}{\sqrt{2\pi}} \cdot \frac{K_I}{E} \sqrt{c-x} \qquad (3)$$

The resulting area is the integral of the crack opening displacements

$$F(2c) = 4 \int_0^c V(x)\, dx \qquad (4)$$

which is, using eqn. (3)

$$F(2c) = \frac{32}{3\sqrt{2}} \cdot \frac{\sigma}{E} \cdot c^2 \qquad (5)$$

$$= \frac{8}{3\sqrt{2}} \cdot \frac{\sigma}{E} \cdot (2c)^2 \qquad (5a)$$

Putting the maximum crack opening displacement originating from eqn. (3) in the centre of the crack ($x = 0$)

$$V(0) = \frac{4}{\sqrt{2\pi}} \cdot \frac{K_I}{E} \sqrt{c} \qquad (6)$$

$$= \sqrt{2} \cdot \frac{\sigma}{E} \cdot (2c) \qquad (6a)$$

and inserting it in eqn. (5), the area is calculated by

$$F(2c) = \frac{4}{3} \cdot V(0) \cdot (2c) \qquad (7)$$

It is obvious that the opening area increases with c^2

$$F(2c) \sim c^2 .$$

Effect of Plastification at the Crack Tip

In the vicinity of the crack tip yielding normally takes place in metallic structures. The extension of the plastic zones depends essentially on the material behaviour under applied load.

Following Irwin (1) the crack length is corrected by the radius of the plastic zone rp

$$c_{eff} = c + rp \qquad (8)$$

where c = half crack length

$$rp = \frac{1}{2\pi} \cdot \left(\frac{K_I}{\sigma_F}\right)^2 \qquad (9)$$

σ_F = flow stress.

This correction results in a modification of the stress intensity factor (SIF), eqn. (2)

$$K_I\ (c) \longrightarrow {}^{pl}K_I\ (c + rp) \tag{10}$$

Then the plastic SIF is calculated by

$${}^{pl}K_I = \sigma \sqrt{\pi\ (c + rp)} \tag{11}$$

$$= \sigma \sqrt{\pi\ (c + \frac{1}{2\pi} \cdot (\frac{{}^{pl}K_I}{\sigma_F})^2} \tag{11a}$$

and

$${}^{pl}K_I^{\,2} = \frac{\sigma^2 \cdot \pi \cdot c}{1 - \frac{1}{2} (\frac{\sigma}{\sigma_F})^2} = \frac{K_I^2}{1 - \frac{1}{2} (\frac{\sigma}{\sigma_F})^2} \tag{12}$$

and a relative radius of the plastic zone is obtained

$$\frac{rp}{c} = \frac{\frac{1}{2} (\frac{\sigma}{\sigma_F})^2}{1 - \frac{1}{2} (\frac{\sigma}{\sigma_F})^2} \tag{13}$$

The relative extension of the plastic zone depends only on the ratio of $\frac{\sigma}{\sigma_F}$.

Calculation of Opening Areas for Cracks in Plates

The analytical model is based on

- integration of crack displacements,
- parabolic shape of opening area and
- correction of crack length by size of plastic zone.

Generally

$${}^{pl}F\ (2c) = 4 \int_0^c V\ (x)\ dx \tag{4a}$$

results in a parabolic area of

$${}^{pl}F\ (2c) = 4 \cdot V(0) \cdot \sqrt{1 + \frac{rp}{c}} \int_0^c \sqrt{(1 + \frac{rp}{c}) - \frac{x}{c}}\ dx \tag{14}$$

The integration leads to

$${}^{pl}F\ (2c) = 4 \cdot V(0) \cdot \sqrt{1 + \frac{rp}{c}} \cdot c \cdot \frac{2}{3} \left[(1 + \frac{rp}{c})^{3/2} - (\frac{rp}{c})^{3/2} \right] \tag{14a}$$

V(0) and $\frac{rp}{c}$ are expressed according to eqn. (6a) and equ. (13), respectively.
After inserting (6a) and (13) into (14a) it follows:

$${}^{pl}F\ (2c) = \frac{4\sqrt{2}}{3} \frac{\sigma}{E} (2c)^2 \frac{1 - X^3}{(1 - X^2)^2}$$

where $X = \frac{1}{\sqrt{2}} \frac{\sigma}{\sigma_F}$

Leakage Area in Cylinder

It depends on crack length and loading conditions and in cylindrical structures additionally on crack orientation (longitudinal and circumferential) and on the curvature of the cylinder.

The bulging factor $\alpha(\lambda)$ due to the curvature of the cylinder results in an increased leakage area of cracks in cylinders with respect to those in plates:

$$\alpha(\lambda) = \frac{F(2c)_{cylinder}}{F(2c)_{plate}} \tag{15}$$

where

$$\lambda = \sqrt[4]{12(1-\nu^2)} \cdot \frac{c}{\sqrt{R \cdot t}} \tag{16}$$

with R = average radius of cylinder
t = wall thickness
ν = Poisson's ratio

Bulging factor for axial cracks. An approximation of the crack opening displacements given by Erdogan and Ratwani (2) is derived

$$\alpha(\lambda) = 1 + 0{,}1\lambda + 0{,}16\ \lambda^2 \tag{17}$$

Bulging factor for circumferential cracks. Similar to axial cracks the cylinder curvature is taken into account by

$$\alpha(\lambda) = \sqrt{1 + 0{,}117\ \lambda^2} \tag{18}$$

Verification of bulging factors. In Figure 1 solutions of Tada (3), Wüthrich (4) and above derived formulae are compared.

EXPERIMENTS AND FINITE ELEMENT CALCULATION

Tests on pipes were performed by MPA (5) and KWU (6,7). Linear elastic finite element calculations for determination of leakage areas in cylinders with through wall cracks are described in Kastner et al. (8).

Longitudinal Cracks

As leakage areas have to be calculated, only experiments on pipes with through wall cracks are considered. The experimental data are obtained from unstable crack lengths.

A compilation of the data is given in Table 1 refering to experiments in (5).

Applying the theoretical procedure on these data leads to a ratio of experimental and calculated leakage areas versus normalized geometry according to Figure 2.

The experimental results are compared with the cross sectional area of the pipes in Figure 3.

Circumferential Cracks

Only through wall cracks based on finite element calculations (8) are considered, Table 1 summarizes all important data. The results are presented analogous to longitudinal cracks in Figures 2 and 3.

Conclusion

From Figure 2 it is evident that minimum leakage areas can be calculated according to eqn. (1) - (18). The experimental values are at the most 2.5 times higher than the analytical values. Therefore an empirical factor of 3 will be used for prediction of maximum leakage areas.

The ratio of leakage area F to pipe cross section area A is in the order of 10 % ($F = 0{,}1 \cdot A$), see Fig. 3.

APPLICATION OF CALCULATIONS OF LEAKAGE AREA

Two fields of evaluation of leakage areas are of interest:

- the prediction of minimum leakage areas caused by leaking cracks, in order to determine the capabilities of leakage detection systems and to quantify the margin of safety between the detectable crack length and the critical crack length;
- the calculation of maximum leakage areas resulting from maximum stable crack lengths in order to determine the maximum jet and reaction forces.

The analytical procedures are applied to the analysis of the leakage areas of a hypothetical longitudinal crack in reactor piping systems.

The curves in Figure 4 show the calculated critical through wall crack length and the leakage areas of a hypothetical longitudinal crack in the main coolant line of a PWR.

Minimum Leakage Area

To ensure the leak before break behaviour the requirements of the guidelines of the German Reactor Safety Commission (9) have to be fullfilled.
The argumentation for leak before break is explained in more detail in Figure 5 and (6). The main ideas of this argumentation are that a postulated maximum crack corresponding to the results of non-destructive examinations during fabrication has to be significantly smaller than the minimum critical crack size, even if the maximum calculated crack growth due to the specified loadings of the whole service life of the component is added. Even in the case of a multiple life time of the component it has to be shown that the postulated crack only reaches a through wall crack length (after breaking through the ligament of the wall) which results in a stable leak with a small leakage area. This stable through wall crack length grows very slowly and can reliably be detected by leakage monitoring systems before it reaches the critical through wall crack length.

Main coolant line. The leak before break behaviour of the main coolant line is demonstrated using fracture mechanics (7). A critical through wall crack length of 680 mm and a corresponding leakage area of 2600 mm² are calculated, see Figure 4.
In comparison to this minimum critical crack length the size of a hypothetical crack resulting from non-destructive testing (25 mm long and 2,8 mm deep) is very small (the calculated crack growth included). This crack needs a multiple life time of specified loadings to grow through the wall thickness and to produce a stable leak with a length of 100 mm, the area of which is 15 mm², see Figure 6.
This stable leak can reliably be detected by the non-destructive inservice examination and the leakage monitoring system (3 to 10 mm²) and grows slowly (about 5 mm) during the specified operational time.
The detected leakage area can be transformed to detectable crack length and then be compared with the critical crack length.
The safety margins resulting from this evaluation are large.

Maximum Leakage Area

As stated in the German RSK-Guideline (9) postulated leaks have to be taken into account.
For the main coolant lines a leakage area of 0.1 A have to be taken for the evaluation of the effects of

pressure waves, jet and reaction forces on pipings, components and buildings. For the main steam line and the main feed water line leakage areas are needed for calculation of jet and reaction forces.

The leakage area due to a subcritical crack is determined using fracture mechanics or is limited to 0.1.A.

The maximum leakage area is calculated for the main coolant line (assuming maximum flow stress σ_F = 1,2 . 365 N/mm² = 438 N/mm² and an empirical factor of 3) and results in a ratio of F/A = 0.03, which is well below the upper limit of F/A = 0.1, see Figure 6.

CONCLUSIONS

Leakage areas are evaluated with respect to
- minimum values for the design of leak detection systems and
- maximum values for controlling jet and reaction forces.

It is shown that for the relevant steels used in reactor technology the analytical approach is able to supply both the minimum values and, assuming an empirical factor of 3, the maximum values, too.

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LIST OF SYMBOLS

A	= pipe cross section area (mm^2)
c	= half through wall crack length (mm)
c_{eff}	= effective crack length (mm)
E	= Young's modulus (Nmm^{-2})
F	= crack opening area (mm^2)
^{pl}F	= plastic corrected crack opening area (mm^2)
G	= shear modulus (Nmm^{-2})
K_I	= stress intensity factor for mode I loading ($Nmm^{-3/2}$)
$^{pl}K_I$	= plastic stress intensity factor ($Nmm^{-3/2}$)
k	= $(3-4\nu)$ for plane strain or $(3-\nu)/(1+\nu)$ for plane stress
R	= mean radius of cylinder (mm)
r	= distance from crack tip (mm)
r_p	= radius of plastic zone at the crack tip (mm)
t	= wall thickness (mm)
V	= crack opening displacement (mm)
X	= stress ratio σ/σ_F
x	= distance from crack tip (mm)
α	= bulging factor
λ	= shell correction factor
ν	= Poisson's ratio
σ	= applied nominal stress (Nmm^{-2})
σ_F	= flow stress (Nmm^{-2})
φ	= angle of r (polar coordinates) (°)

TABLE 1 - Throughwall flaw experiments and calculations

Experiment No.		Experiments acc. to MPA (5)				FEM-Calc. acc. to KWU (8)					Applic. MCL/PWR	
		BVZ010	BVZ011	BVZ012	BVS010	17	18	19	22	23	"min"	"max"
Material:		20MnMoNi55			22NiMoCr37	20MnMoNi55					20MnMoNi55	
Dimensions:												
D_A	mm	797,5	798	798	792	844	844	844	844	844	864	864
t	mm	47,6	47,5	47,5	47,6	42	42	42	42	42	52	52
Loads:												
p	bar	238	148	114	175	155	155	155	155	155	160	160
ϑ	°C	20	20	20	155						300	300
$M_{Bending}$	10^8 Nmm								1,92	3,84		
Material properties:												
E-Modulus	kN/mm²	204	204	204	190	185	185	185	185	185	185	185
$R_{p0,2}$	N/mm²	520	515	515	420	277	316	475	302	332	363	436
R_m	N/mm²	633	632	632	544						513	616
σ_F	N/mm²										365	438
Stress:												
σ_{hoop}	N/mm²	176	110	107	176						117	117
σ_{axial}	N/mm²					66,5	66,5	66,5	66,5	66,5		
$\sigma_{ax.bend.}$	N/mm²								9,0	18,0		
Crack length:		longitudinal cracks				circumferential cracks					longitudinal	
$2c_{exper.}$	mm	730	1105	1285	1480							
$2c_{calc.}$	mm					1008	1078	1260	1008	1008	680	830
Leakage area:												
$F_{exp.}$	mm²	7748	13800	60000	42100	2040	2450	6564	2116	2264		
$F_{exp.}/A$	10^{-3}	20,0	35,6	155	110	4,5	5,4	14,5	4,7	5,0		
$F_{calc.}$	mm²	5220	13940	23900	19400	1900	2270	3430	2160	2430	2600	15200
$F_{calc.}/A$	10^{-3}	13,5	35,9	61,6	50,9	4,2	5,0	7,7	4,8	5,4	5,7	34

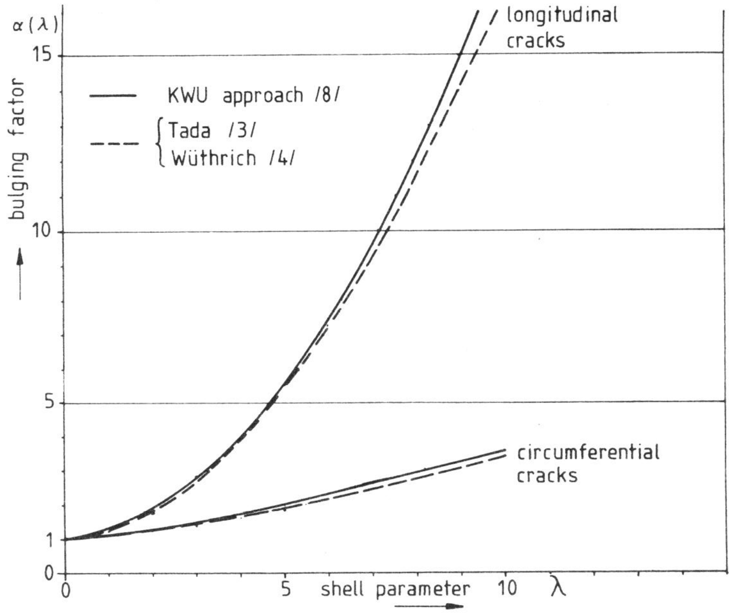

Figure 1 Comparison of bulging factors $\alpha(\lambda)$

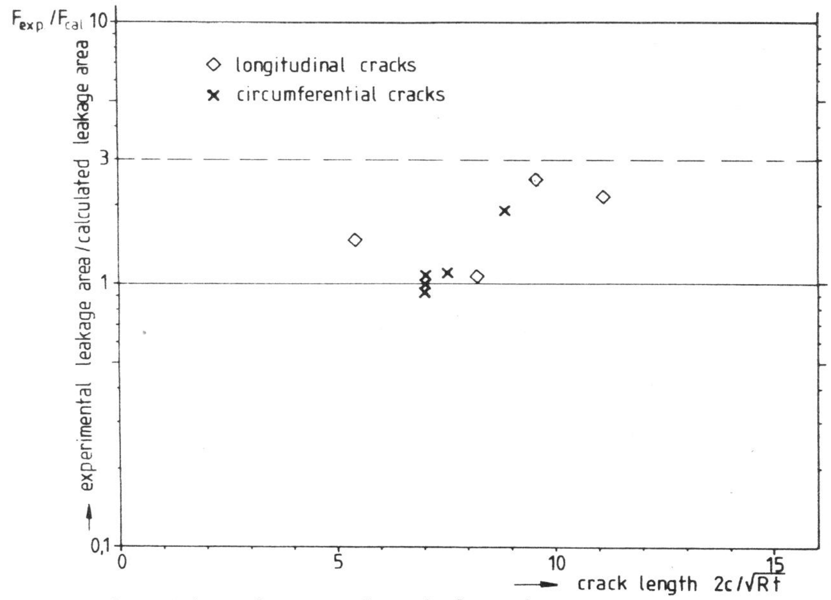

Figure 2 Ratio of experimental and analytical leakage areas versus $2c/\sqrt{Rt}$

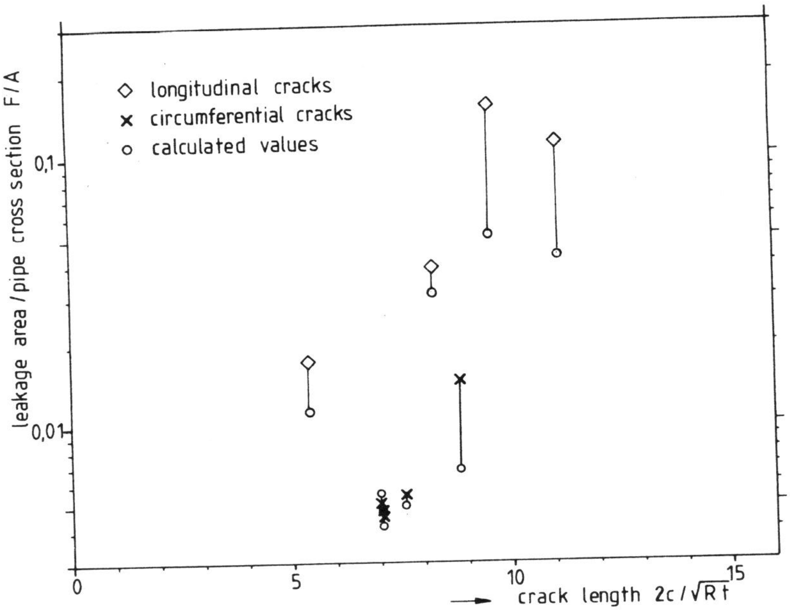

Figure 3 Leakage areas related to pipe cross section versus $2c/\sqrt{Rt}$

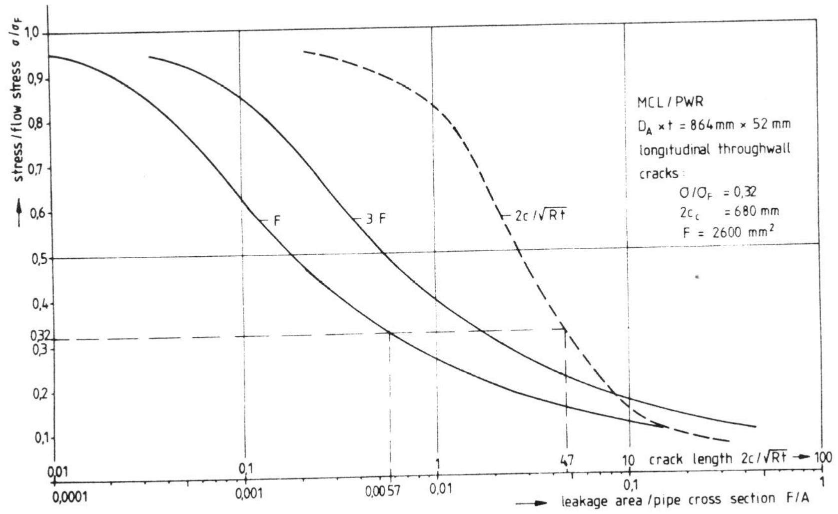

Figure 4 Normalized critical throughwall crack length ($2c_c/\sqrt{Rt}$)and leakage area (F/A) in dependence on normalized hoop stress (σ / σ_F)

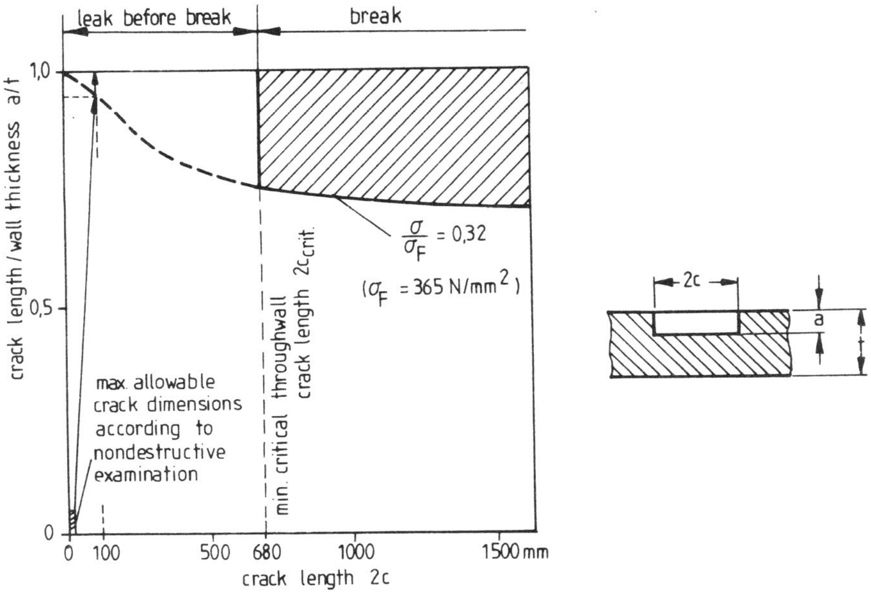

Figure 5 Critical crack depth (a_c/t) and critical crack length ($2c_c$) for main coolant line

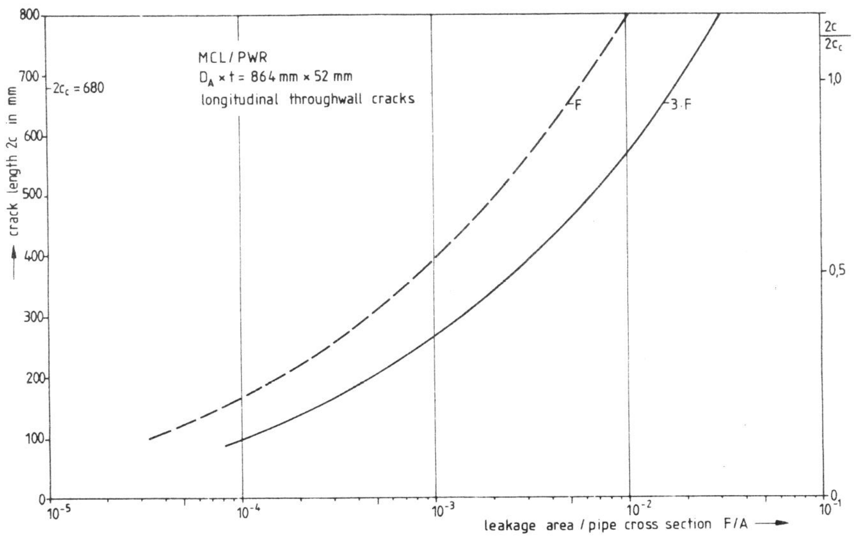

Figure 6 Normalized leakage area (F/A) versus through-wall crack length