APPLICATION OF STATISTICAL CLEAVAGE CRITERIA TO FRACTURE CONTROL OF PRESSURE VESSELS

A. PELLISSIER-TANON*, J.C. DEVAUX, B. HOUSSIN

Results of experiments featuring the most important characteristics of situations encountered in pressure vessel integrity analysis, provide a very good qualification of the criterion :

$$\frac{1}{B}\int_L \left(\frac{K_I}{K_{IC}^R}\right)^4 dL \leq 1$$

in which the integration is performed on the applied value of K_I and on the toughness K_{IC}^R all along the crack front of length L, and for which the curve expressing the reference toughness K_{IC}^R as a function of temperature is a lower bound to a sufficient number of valid K_{IC} measurements with specimens of thickness B.

INTRODUCTION

Evaluation of margins with respect to fast fracture in the brittle-ductile transition for ferritic steels is currently achieved by linear elastic fracture mechanics analyses in which the computed value of stress intensity factor K_I is compared to a reference toughness K_{IC}^R , expressed as a function of temperature. The K_{IC}^R temperature curve is defined in principle in order to be a lower bound to a sufficient set of experimental measurements.

A first problem arose when accumulation of experimental results, and particularly results of tests on a long cracked cylinder under thermal shock loadings [1] set forth initiation of cleavage instable propagation at values of K_I lower than the lower bounds of previous specimen testing programs. This cast doubt on the validity of the lower bound reference curve approach.

* FRAMATOME, Tour FIAT, cédex 16, 92084 PARIS LA DEFENSE

At the same time, the requirement to increase computation accuracy in nuclear safety assessment [2] led to consider the behaviour of surface cracks under high stress gradients, presenting large variations of K_I along the crack contour. Application of an analysis criterion, in which the value of K_I at each point on the crack front was compared to the K_{IC} given by the reference curve for the temperature at that point, led to a much more pessimistic appraisal of the resistance to fracture than previous assessments had done.

Solution of these two problems appeared clearly to be related to an objective account of the finding made in some large experimental programs, involving specimens of different thicknesses, that the lower bounds of toughness obtained at a given temperature decreased with specimen thickness [3] [4].

These results clearly pointed towards a weakest link statistics probabilistic approach of cleavage fracture. Recent works from different countries have firmly established the physical grounds for such an approach and its interest for solving engineering application problems [5] [6] [7] [8]

This paper presents in a first part the mechanical basis of a simple engineering probabilistic cleavage initiation criterion, which brings a practical answer to the two problems expressed above in small scale yielding conditions. A qualification of this criterion is looked for in a second part, by applying it to the interpretation of two different sets of experiments featuring these two problems.

PROBABILISTIC ASSESSMENT OF RESISTANCE TO CLEAVAGE FRACTURE IN SMALL SCALE YIELDING CONDITIONS

Basic results on probabilistic approach to cleavage fracture

Probabilistic approach to cleavage fracture is discussed in [5] [8] [9]. The most basic elements are :

- cleavage fracture initiation exhibits a weakest link statistical behaviour, which expresses the probability of no fracture $1-P$ of an aggregate as the product of the probabilities of no fracture $\Pi(1-p_i)$ of the elements constituting the aggregate.

- the mean volume Vo or mean characteristic length $\sqrt[3]{V_o}$ of these elements are of the order of size below which continuum mechanics analysis no longer has a meaning. They are related to the size of the crystallographic microstructural elements.

- the probability of fracture of an element is a function $h(\bar{\sigma}/\sigma_u)$ of the mean principal stress $\bar{\sigma}$ through this element, σ_u being a material characteristic, which has been found to be nearly independent of temperature.

- onset of plastic deformation through the element is necessary to initiate cleavage fracture.

Thus, the probability of fracture P obeys the relation :

$$\mathrm{Ln}\,(1-P) = \sum_{V_o \in V_p} \mathrm{Ln}\left[1 - h\left(\frac{\bar{\sigma}}{\sigma_u}\right)\right] \qquad (1)$$

in which Vp represents the volume of the plastic zone.

Putting :

$$\mathrm{Ln}\left[1 - h\left(\frac{\bar{\sigma}}{\sigma_u}\right)\right] = H\left(\frac{\bar{\sigma}}{\sigma_u}\right)$$

gives :

$$\begin{cases} P = 1 - \mathrm{Exp} \displaystyle\int_{V_p} H\left(\frac{\bar{\sigma}}{\sigma_u}\right) \frac{dV}{V_o} \\ \text{For } r_y > \sqrt[3]{V_o} \end{cases} \qquad (2)$$

$$\begin{cases} P = 0 \\ \text{For } r_y < \sqrt[3]{V_o} \end{cases} \qquad (2)'$$

in which ry is the extension of the part of the plastic zone in which the principal stresses are the largest ahead of the crack tip.

MUDRY, PINEAU and DEVAUX [5] [6] [7] have established and checked a particular expression of (2), recalling the WEIBULL cleavage statistics :

$$\begin{cases} P = 1 - \mathrm{Exp}^{-\left(\frac{\sigma_w}{\sigma_u}\right)^m} \\ \sigma_w = \left[\int_{V_p} \sigma^m \frac{dV}{V_0} \right]^{\frac{1}{m}} \end{cases} \tag{3}$$

Relations for small scale yielding conditions

The plastic zone is contained in a limited volume extending along the crack front, according to figure 1. At each point M of the crack front, marked by its curvilinear coordinate l the cross section of the plastic zone in a plane normal to the crack front is called Sp. Equation (2) can be expressed as :

$$\mathrm{Ln}\,(1-P) = \int_{\ell} \int_{S_p} \frac{1}{V_0} H\left(\frac{\sigma}{\sigma_u}\right) dS\, d\ell \tag{4}$$

In small scale yielding, the geometrical distribution of stresses in Sp follows the pattern a of geometrical affinity, with center at the crack tip, and which is such that :

- for two loading cases $K_I^{(1)} - \sigma_y^{1}$ (T1) and $K_I^{(2)} - \sigma_y^{(2)}$ (T2) the same value of stress tensor σ_{ij}, is found at same polar coördinate θ and at radial coordinates ρ_1 and ρ_2 such that :

$$\frac{\rho_1}{\rho_2} = \left(\frac{K_I^{(1)}}{\sigma_y^{(1)}}\right)^2 \Big/ \left(\frac{K_I^{(2)}}{\sigma_y^{(2)}}\right)^2$$

- and the plastic zone cross section elements dS1 and dS2 which are to be associated with the maximum principal stress σ are such that :

$$\frac{dS_1}{dS_2} = \left(\frac{K_I^{(1)}}{\sigma_y^{(1)}}\right)^4 \Big/ \left(\frac{K_I^{(2)}}{\sigma_y^{(2)}}\right)^4$$

σ_y is the yield stress at the temperature T at the crack tip ; it is assumed that the strain hardening exponent remains constant in the temperature interval T1-T2. .

These properties imply that :

$$\int_{S_p} \frac{1}{V_0} H\left(\frac{\sigma}{\sigma_u}\right) dS = -\alpha \left(\frac{K_I}{\sigma_y}\right)^4 \tag{5}$$

with α being a material characteristic independent of temperature.

And :

$$\mathrm{Ln}\,(1-P) = -\int_{\ell} \alpha \left(\frac{K_I}{\sigma_y}\right)^4 d\ell \tag{6}$$

Correlation to material testing results

If a large enough number of valid K_{IC} measurements (which imply small scale yielding) have been made on specimens of the same thickness B, the tests can be fitted to relation (6), in order to obtain the value of α.

MUDRY [7] gives direct expressions of α for 15MND6 (A508cl3) forgings, resulting from the elastoplastic computation of the specimens.

If the number of specimens is not large enough, an approximate value of α can be established by correlating the median value of K_{IC} to P = 0,5 or the lower bound value to P = 1/N (N being the number of specimens tested).

Criterion for structural analysis

When a lower bound reference toughness curve is available, which is considered to be safe when tests are performed with specimens of width equal or smaller than the value B, it is possible to express the cleavage fracture criterion in a simple engineering manner, which is the following :

The aim is to ensure that the probability of fracture P in the analyzed structure be not larger that the probability P^R to obtain an experimental value of K_{IC} below the value K_{IC}^R on the reference curve, when making measurements with specimens of width B. P^R is assumed to be independent of temperature, and equation (6) becomes, for the reference toughness :

$$Ln\left(1-P^R\right) = B\alpha\left(\frac{K_{IC}^R(T)}{\sigma_y(T)}\right)^4 \qquad (7)$$

Replacing $\dfrac{\alpha}{[\sigma_y(T)]^4}$ by its value from (7) in équation (6), for the analyzed case gives :

$$Ln\,(1-P) = \frac{Ln\,(1-P^R)}{B}\int_\ell\left(\frac{K_I}{K_{IC}^R}\right)^4 d\ell \qquad (8)$$

And the condition, stated above, to have P not larger than P^R reduces to :

$$\frac{1}{B}\int_\ell\left(\frac{K_I}{K_{IC}^R}\right)^4 d\ell \leq 1 \qquad (9)$$

EXPERIMENTS WITH LARGE VARIATIONS OF K_I ALONG THE CRACK FRONT

The experimental program comprises bend specimens with fatigue precracked semi-elliptical cracks and prismatic bars taken in the same 15 MND 6 forging, according to figure 2, which have been broken at several temperatures in the transition region.

The values of K_I at edge K_{Ib} and deepest point K_{If} computed at fracture for the bend specimens are compared with the K_I at fracture on the prismatic bars, called K_{Ip}, in figure 3. Some averaging must be clearly taken into account to express a fracture criterion, and the experimental averaged ratios K_{If}/K_{Ip} and K_{Ib}/K_{Ip}* for the mean values and for the lower bounds are given in table 1*.

The values computed for K_I along the front of the semielliptical cracks of the bend specimens can accurately be expressed according to a parabolic expression of the elliptical angle ϕ.

$$K_I(\phi) = K_{If} + (K_{Ib} - K_{If})(2\phi/\pi)^2$$

Hence, for the same probability of fracture, equation (6) leads to the following relation between K_{Ip}, K_{If} and K_{Ib} :

$$K_{Ip}^4 B = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \left[K_{If} + (K_{Ib} - K_{If})\left(\frac{2\phi}{\pi}\right)^2\right]^4 (a^2 \sin^2\phi + c^2 \cos^2\phi)^{\frac{1}{2}} d\phi$$

For the range of shape and size of the semielliptical cracks in the bend plates, $K_{Ib}/K_{If} = 1.77$, and the interpretation of the above equation leads to :

$$K_{Ib}/K_{Ip} = 1.15 \quad ; \quad K_{If}/K_{Ip} = 0.65$$

The very good agreement with the experimental values shown in table 1 supports the validity of the probabilistic cleavage criterion.*(Table 1 is given page 11).

BEHAVIOUR OF A ONE METER LONG CYLINDER UNDER LIQUID NITROGEN THERMAL SHOCK

Results of thermal shock experiment

Experimental results on the behaviour of a 1m long 16 MND5 (A508CL3) cylinder during liquid nitrogen thermal shock is presented in [1] together with a preliminary interpretation with two-dimensional analysis. The test cylinder is described on figure 4. An initial 17 mm deep straight front longitudinal crack was produced by charging hydrogen in an electron beam fusion bead. Three successive initiation and arrest events took place during cooling. The crack front shapes after each arrest are shown on figure 5. Temperatures were measured along the four radial plugs shown at figure 4.

Stress intensity factors have been computed by loading the cylinder with the mean of the temperature distributions given by the plugs at each initiation and arrest, under the following assumptions :

1) Infinite straight crack in infinite cylinder, weight function method for maximum depth at all initiation and arrest events.

2) Straight crack in finite cylinder, finite element method, for maximum depth (88 mm) at second initiation.

3) Curved crack front matching crack contour at second initiation, finite element method, at second initiation.

K_I distribution obtained for cases 2 and 3 are presented at figure 6. K_I distribution for case 3 shows a nearly constant value through most of the crack front in center part, the peak near the edge being an artefact related to the finite element meshing. K_I distribution for case 2 is nearly parabolic with a maximum at center and a very small value at edge. A probabilistically equivalent average $\overline{K_I}$ to case 2 distribution can be defined by relation :

$$\overline{K_I}^4 \ell = \int_\ell K_I^4 \, d\ell \qquad (10)$$

The ratios between the finite cylinder finite element computations and the infinite cylinder straight front weight functions results obtained in the analysis of the second initiation event are applied to the first and third initiation event analysis. The results are presented on table 2 :

TIME AT INITIATION EVENT (S)		169	209	356
Crack front shape		straight	curved	curved
Crack depth (mm)	At deepest point : at edge :	17 17	88 44	150 80
K_I (MPa $\sqrt{m}$)	Infinite cylinder analysis	50.6	64.8	98
	Finite cylinder analysis	47.6 (a)	60.9 (b)	92.1 (b)
Temperature (°C)	At deepest point at edge point average (C)	- 40 - 40 - 40	4 - 50 - 17,4	8,8 - 45 - 6

(a) : Probabilistic average according to (10)
(b) : Mean value at plateau in center part
(c) : $\overline{T} = \frac{1}{\ell} \int_\ell T \, d\ell$

Material characterisation results

Several material characterisation programs have been performed on material cut in the cylinder forging. Two programs are of interest here. A first one comprises six valid K_{IC} tests on 75 mm thick (3 TCT) compact tension specimens in which the starter crack was created by the same electron beam fusion and hydrogen charging method as for the first test cylinder initiation. The second one comprises 6 valid K_{IC} tests on 100 mm thick (4TCT) compact specimens, cut in the cylinder itself, with fatigue crack tip at a location corresponding to the depth 70 mm within the cylinder. Results are presented at table 3.

		Program 1	Program 2	
CT specimen thickness (mm)		75	100	
Test temperature (°C)		- 60	- 10	+ 10
Number of specimens		6	2	4
K_{IC} (MPa $\sqrt{m}$)	maximum	92	110	144
	minimum	55	83	86
	average	66	96,5	114

TABLE 3

Correlations between the cylinder and CT specimens results by the probabilistic cleavage criterion

Thermal shock experiment results on the cracked cylinder of length l can be correlated to the series of test results on CT specimens of width B according to the weakest link statistical model, by considering the cracked cylinder as an aggregate of a number N = L/B of CT specimens put side by side.

Let us now consider a hypothetical cylinder constructed by putting side by side along the crack front material aggregates having the toughness properties of the n CT specimen tested, with therefore a length n B. The weakest link statistical model predicts that if this cylinder is loaded uniformly along the crack front at the same temperature as the CT specimens it must break at a value of K_I equal to the lowest toughness, K_{IC} min, measured on the CT specimens. If an actual cylinder of same material and of same length was tested in the same conditions it can be considered that the K_I measured would have a good chance of being close to K_{IC} min.

According to the weakest link statistical model the actual cylinder, as an aggregate of N specimens, would have for K_I = Kmin, a fracture probability P^c different from the one of the aggregate of the nCT specimens considered above, which we call P^s.

The probability of fracture P^s at K_{IC} min is 1/n.

The mean probability at fracture π of each CT specimen at K_{IC} min is such that :

$$1 - \pi = (1 - P^s)^{\frac{1}{n}}$$

The probability of fracture P^c of the cylinder is such that :

$$1 - P^c = (1 - \pi)^N = (1 - P^s)^{\frac{N}{n}}$$

When N is larger than n, as it is the case here, P^c is larger than P^s and the test cylinder has the greatest chance to undergo cleavage fracture initiation at a value of K_I lower than K_{IC} min.

It is physically sound to consider that the most likely value of K_I at fracture for the cylinder would correspond to the same overall cleavage initiation probability as the one of the CT specimen test serie at K_{IC} min.

Application of equation (6) to the cylinder and to the CT test series gives :

- for CT specimen serie : $Ln(1 - P^s) = nB\alpha\left(\frac{K_{Ic}\,min}{\sigma_y}\right)^4$
- for cylinder : $Ln(1 - P^c) = NB\alpha\left(\frac{K_I}{\sigma_y}\right)^4$

For a same crack tip temperature, σ_y is the same, and the above relations give, for $p^c = p^s$

$$K_I = \left(\frac{n}{N}\right)^{\frac{1}{4}} K_{Ic}\,min \qquad (10)$$

In order to apply equation (10) to predict the thermal shock cylinder behaviour, the experimental results on CT specimens, given at table 2, must be adjusted to the average temperature defined for the initiation events in the cylinder. Results of the first serie of tests with cracked electron beam fused beads are used to predict the first initiation event ; results of the second serie of tests with fatigue cracks are used to predict the second and third initiation events. The temperature adjustment for K_{IC} min is made by fitting around each of the lower bound value of the two series of tests a lower bound curve segment similar in slope and curvature to a lower bound curve which had been established previously for a larger set of test results including measurements on another forging with same RTNDT as the thermal shock forging [1] [3].

Table 4 presents all the data relevant to application of equation (10) to the three initiation events and compares the K_I obtained to the values deduced from the experiments (taken at table 2).

AVERAGED TEMPERATURE AT INITIATION EVENT ON CYLINDER		- 40	- 17	- 6
CT Specimen data	Number of specimens in série : n	6	6	6
	Specimen width B (mm)	75	100	100
	Adjusted value of K_{IC} min (MPa $\sqrt{m}$)	62	71	77
Cylinder prediction	Equivalent number of CT specimens N	13,3	10	10
	K_I predicted by equation (10) (MPa $\sqrt{m}$)	51	62,5	68
Cylinder results	K_I measured (MPa $\sqrt{m}$)	48	61	92

TABLE 4

The prediction is very close to the measured value for the first two initiation events which mark the lower bound among the three initiation events. The measured value for the third one, which is above the lower bound marked by the first two is logically larger than the predicted value.

Analysis of the three initiation events could have been performed according to equation (9) if the value K_{IC}^R had been known all along the crack front. To obtain K_{IC}^R (T), it should have been necessary to establish mathematical expressions for the variation of temperature with respect to position along the crack front at the initiation time in the cracked cylinder and also for the lower bound toughness curve K_{IC}^R (T).

Actually, the ratio g :

$$g = \left(\frac{K_I \text{ predicted}}{K_I \text{ measured}} \right)^4$$

in which K_I predicted and K_I measured are the values at the bottoms of table 4 has the same physical meaning as the left member of equation (9), and is a very close approximation to its value, which was easier to determine according to the experimental uncertainties

CONCLUSION

Application of the engineering criterion for cleavage fracture initiation in small scale yielding of equation (9) to the interpretation of two sets of experiments, one with large variation of K_I along the crack front, the other with very large cracks, have led to very good agreements with the experimental findings. In addition to the basis already provided by the general studies on probabilistic approach to cleavage fracture, these results provide a direct qualification of the proposed criterion.

REFERENCES

[1] A. PELLISSIER-TANON, and Al. Paper G/F/1/8, 7th SMIRT, Chicago, august, 1983

[2] A. PELLISSIER-TANON, Advances in Fracture Research (ICF6), Vol.1, 691-706, PERGAMON, 1984

[3] B. HOUSSIN, G. SLAMA, Europeen Nuclear Conf., Hamburg, may, 1979

[4] T. IWADATE and al., ASTM-STP803, vol.2, 531-561, 1983

[5] A. PINEAU, Advances in Fracture Research, Vol.2, 553-580, PERGAMON, 1981

[6] F.M. BEREMIN, Id., Vol.2, 825-832

[7] F. MUDRY, Doctor Thesis, Universite de Technologie Compiègne, 1982

[8] K. WALLIN, Eng.Fr.Mech, 19, 6, 1085-1093, 1984

[9] J.D. LANDES, D.E. Mc CABE, Westinghouse Scientific Paper 82-107-METAL-P2, 1982

Experimental Results						Prediction
Temperature (°C)	- 196	- 100	- 60	Mean among tempe-ratures	General mean value	Probabi-listic Criterion
$\frac{K_{Ib}\ mean}{K_{Ip}\ mean}$	1,19	1,06	1,30	1,19	1,22	1,15
$\frac{K_{Ib}\ min}{K_{Ip}\ min}$	1,11	1,43	1,16	1,24		
$\frac{K_{If}\ mean}{K_{Ip}\ mean}$	0,66	0,59	0,72	0,66	0,67	0,65
$\frac{K_{If}\ min}{K_{Ip}\ min}$	0,64	0,79	0,65	0,69		

Table 1

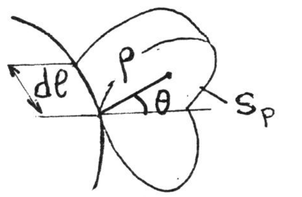


Figure 1

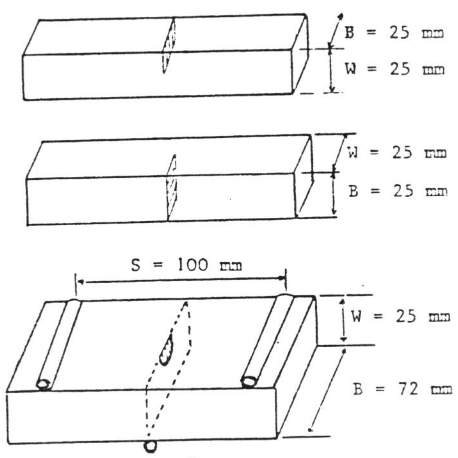


Figure 2

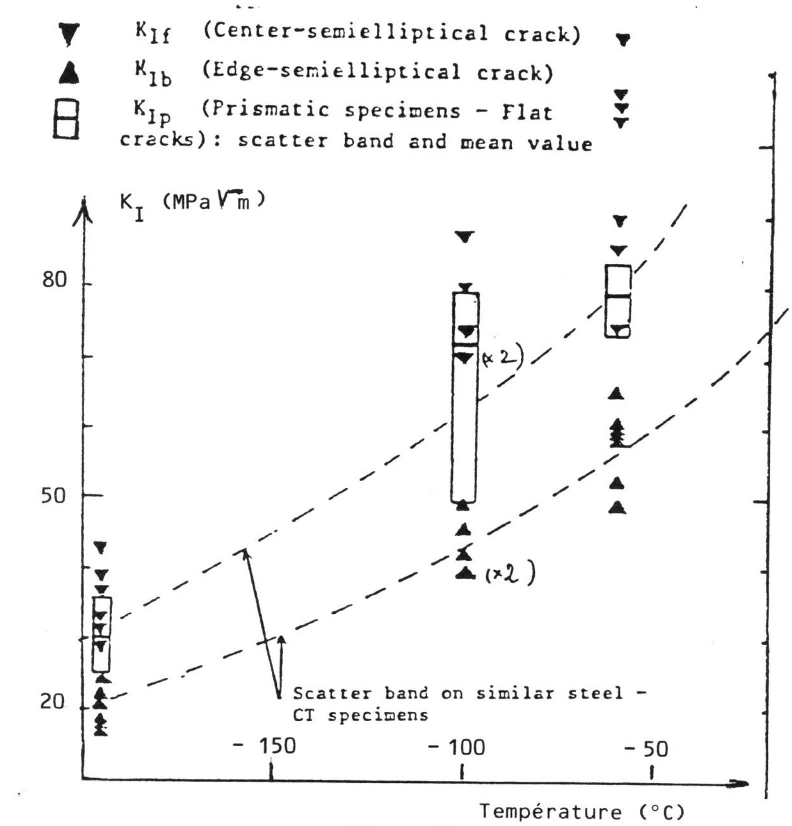
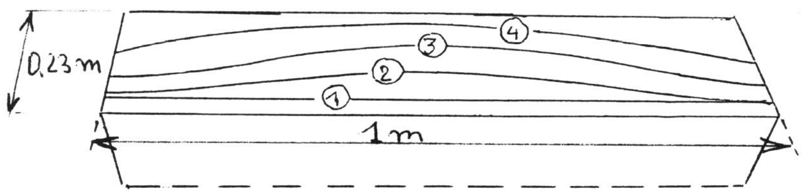


Figure 3



CRACK CONTOURS IN CYLINDER

1 - 1st Initiation at 169 sec. in thermal shock
2 - 1st Arrest, 2nd Initiation at 209 sec
3 - 2nd Arrest, 3rd Initiation at 356 sec
4 - 3rd Arrest

Figure 4

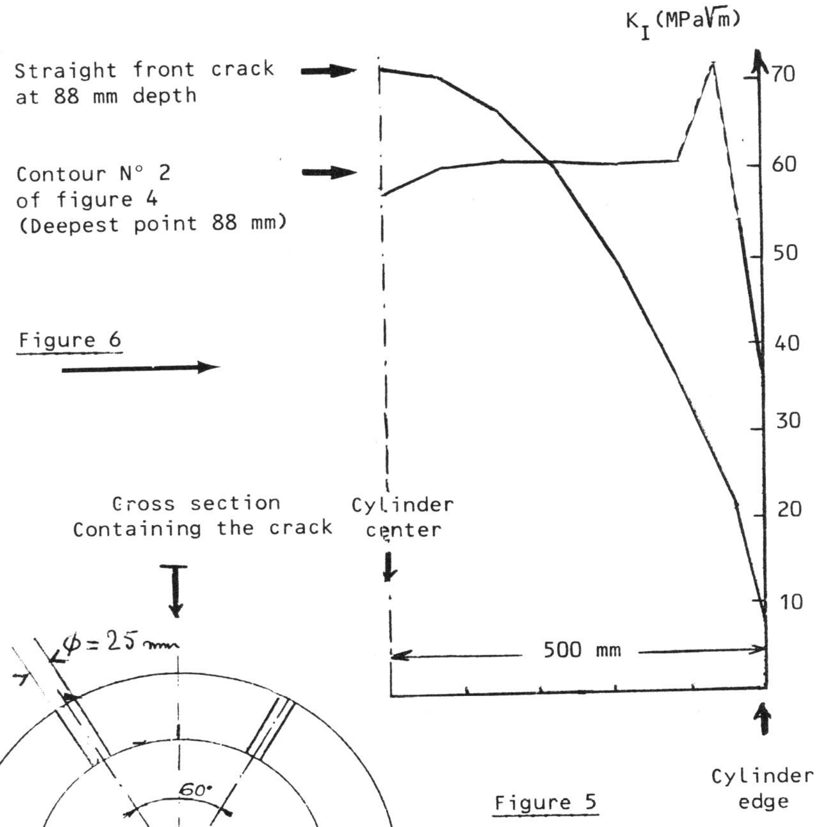


Figure 6

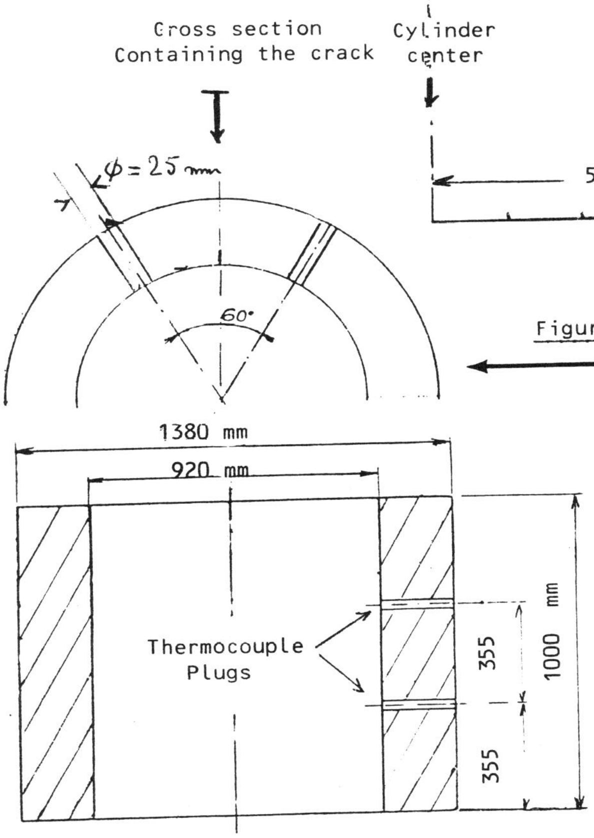


Figure 5