LIFE PROBABILITIES OF ENGINEERING STRUCTURES AND ANALYSIS OF TEST DATA

John C. Radon* and Miklos Prodan**

Specific examples are presented based upon the authors' individual and joint assignments with various companies and institutions. The examples are the following two ones: Failure probability of a buried thickwalled vessel [1] and statistical analysis of combined static and fatigue crack growth data [2, 3]. A number of scientific and technical reports examined during the preparation of this paper lead to interpretations not yet published in the open literature.

INTRODUCTION

Probabilistic failure analysis, as well as deterministic design calculations, receive particular attention since failures occurring with very low probabilities can only be explained by a statistical approach. Simple engineering approaches were used and a number of idealizations were made in our examples with a limited amount of input data. Therefore, the results should be regarded as associated with uncertainties. However, despite developing better methods and models, it should be emphasized that there will always be a scarcity of sufficient input data, because by their nature the structures considered are very safe.

* Imperial College, London SW7 2BX, UK
** Motor-Columbus Consulting Engineers Inc., CH-5401 Baden/ Switzerland

EXAMPLE "THICK WALLED CONTAINER"

A probabilistic failure analysis had to be performed for a container carrying highly toxious contents [1]. In Fig. 1 the geometry and the most important dimensions are shown, Fig. 2 shows the nomenclature and the assumptions made for the probabilistic analysis. The material assumed was cast steel GS 50.

PROBABILITY OF FAILURE BY BUCKLING

The deterministic calculations are the basis of the following probabilistic analysis. The allowable maximum pressure $p_{buckl,z}$ was determined to be 400 bar. The actual external pressure p is 300 bar. The trivial criteria of non failure $p \leqslant p_{buckl,z}$ is satisfied.

For the probabilistic analysis the symbols of Fig. 2 were used and the following assumptions were made:

R_0 = 2000 bar (5 x ($p_{buckl,z}$ = 400 bar))

S_0 = 220 bar (< p = 300 bar)

σ_R = 220 bar

σ_S = 50 bar

$\left.\begin{matrix}\sigma_R \\ \sigma_S\end{matrix}\right\}$ standard deviations of R and S (or $p_{buckl,z}$ and p)

The failure quotient p_f is calculated with the additional assumption of standard distribution and tabulated values from reference books.

$$p_f = \Phi\left(-\frac{R_0 - S_0}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right) = \Phi(-7.9) = 0.15 \cdot 10^{-14} \tag{1}$$

Eq. 1 is based on the general eqs. (2) and (3) where p_f is expressed by the integral

$$p_f = F_v(1) = P(R \leqslant S) = \int_0^\infty F_r(x) \cdot f_s(x)dx \tag{2}$$

$$F_r(x) = P(0 < R \leqslant x) = \int_0^x f_r(\zeta)d\zeta \tag{3}$$

The probability of failure F_1 (t) in the time interval (0, t) is:

$$F_1 (t) = 1 - \left[\int_0^{\infty} \sum_{r=0}^{\infty} p_r (t) \left\{ F_s (x) \right\}^r \cdot f_r (x) \, d x \right] \tag{4}$$

With the help of the reduced equations derived from eq. (4) F_1 (t) is calculated. The method is extensively explained in [1]. However, [1] contains two computational approximations shown in eq. (5) and (6).

$$F_1 (t) \simeq 1 - (1 - p_f)^t \tag{5}$$

$$F_1 (t = 1000 \text{ years}) \simeq 1.5 \cdot 10^{-12}$$

$$F_1 (t) \simeq 1 - \exp (-p_f \cdot t) \tag{6}$$

$$F_1 (t = 1000 \text{ years}) \simeq 1.5 \cdot 10^{-12}$$

The quoted results are explained and commented in Ref. [1]. Due to the extent of reasoning necessary only the additional comment to [1] should be made that a probabilistic reliability analysis is difficult for two reasons:

- there is a variety of technical equipment to consider
- each component is exposed to a variety of influences and mechanism which can lead to failure.

Failure data and statistics from the conventional pressure vessel industry were used for comparison with these theoretical results of the probabilistic analysis.

ANALYSIS OF TEST DATA

In analysing test data, and predicting materials behaviour from the results, it may be necessary to use statistical methods in a different way. Although the loading conditions may be known exactly, a number of separate mechanisms may combine to cause eventual failure. Rhodes et al. [2] used a statistical approach to consider the effects of fatigue and static crack growth occurring together in high ΔK cyclic crack growth. In this case the crack may grow either by micro-void coalescence or by crack blunting. The "choice" depends on the distance between the crack tip and the next inclusion in the material, which varies stochastically both along the crack front and on a cycle by cycle basis. In another case, statistical methods may be applied to near-threshold and short crack fatigue behaviour. When the plastic zone size is of the order of the microstructure size, fatigue crack growth cannot be considered as a continuum process. If the crack front is long, and the number of cycles large (e. g. in a conventional long-crack threshold test) the observed macroscopic crack growth rate may still be a function of a continuum mechanics parameter such as ΔK, but individual crack advance increments may not. In the case of physically small cracks, more complicated analyses may be required [3].

DISCUSSION

Aspects of probabilistic failure analysis, deterministic stress analysis and fracture mechanics analysis of components have been discussed. An attempt shall be made to connect these three analytical methods.

Even though only partial aspects have been considered, a general solution for the analysis of complex components seems possible.

It is obvious that these are not only partial problems but overall and interdisciplinary tasks.

A common problem is that a component is sometimes analyzed under the assumption of no flaws, but in the course of the usage or after manufacturing, flaws can arise.

At first sight, the large amount of time and money spent for the "pure" stress analysis seems in the case of appearance of cracks suddenly useless. Nevertheless, the stress analysis is not without value, but it is not the only evaluation basis. It must be combined with fracture mechanics.

In practice it is generally felt that repair is better than the existence of some small flaws.

Another connection would be between the fracture mechanics and the probabilistic analysis. The question in this case would be e. g.: how to proceed in the case of a flaw in the thick walled vessel? The solution in this case is the so-called probabilistic fracture mechanics, developed from reliability analysis and deterministic fracture mechanics.

SYMBOLS USED

S_o	- Mean value of the load, e. g. external pressure
R_o	- Mean value of the resistance, e. g. buckling load
S_q	- "Maximum" load acc. to the deterministic approach
R_p	- "Minimum" resistance acc. to the deterministic approach
p, q	- Fractions (see shaded areas in Fig. 2)
$\nu_o=R_o/S_o$	- Safety factor acc. to the mean values
$\nu=R_p/S_q$	- Safety factor acc. to min R/max S
p_f	- Failure probability
$f_s(x)$	- Probability density function of the load
$f_r(x)$	- Probability density function of the resistance

REFERENCES

[1] Prodan, M., "Failure and Non-Failure Probability of a Buried Thick-Walled Vessel". Proc. 8th SMIRT Conf., Paper M1J 1/5, Brussels, 1985

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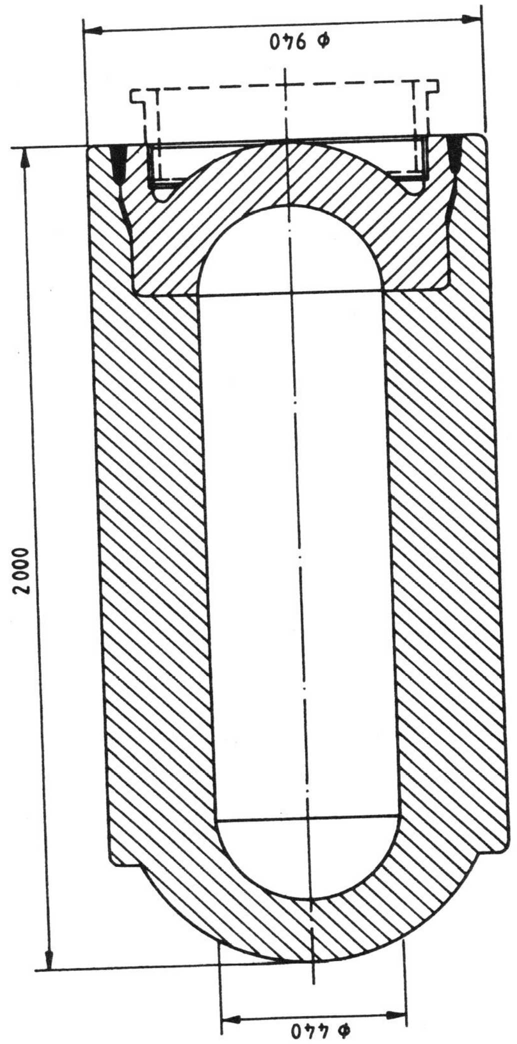

Fig. 1 Geometry of the Thick-Walled Vessel

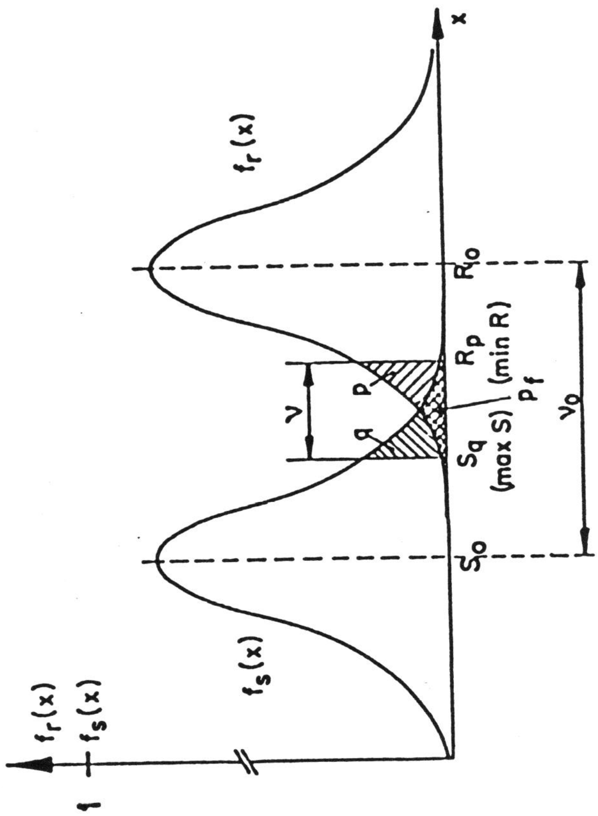

Fig. 2 Probability Density Functions of the Load and the Structural Resistance with Related Notations