THE EFFECT OF THE RESIDUAL STRAIN ON THE CRACK INITIATION

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The main object of this paper is to investigate the effect of the residual strain at a crack tip on crack initiation and crack growth. We present an analytical model of plastic zones at the original crack tip and examine their effect on the stress intensity factor at the extended crack tip. Questions how and when an increasing load leads to unstable crack initiation instead of further growth of the plastic zone are answered. Because of the residual strain, the energy required for the crack initiation increases dramatically with increasing ductility, which leads to the transition from brittle (unstable) crack growth to ductile (stable) one.

INTRODUCTION

Although extensive efforts have been devoted on the characterization of the crack growth phenomena, there seem to exist numerous things to be clarified. The main object of this paper is to quantify the effect of the residual strain at the crack tip, which seems to be necessary to grasp the condition for crack initiation and crack growth. Questions of the adequacy to use the J-integral for the ductile crack growth have been raised, whereas it was first considered to be the nonlinear counterpart of the linear energy release rate. Rice (1) pointed out that a Griffith-type energy balance for crack growth leads to paradoxical results for elastic-plastic materials, since such solids provide no energy surplus for the material separation in the continuous crack advance. The limitation of the J-integral of the deformation theory is discussed for the ductile crack growth where the unloading and the residual strain play dominant roles; Hutchinson and Paris (2). On the basis of those and related discussions, different models

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of crack growth and various versions of modified J-integral and modified energy release rate have been proposed (3-6). The emphasis has been cast on the necessity of constructing a model of the crack tip separation process, and based on that model to find a suitable approach to the crack growth.

How and when does an increasing load lead to unstable crack initiation instead of further growth of the plastic zone, and how is the unstable crack initiation suppressed as the ductility of the material increases? To answer those questions, we propose a model of the unstable crack initiation from the original crack tip at which a plastic zone has grown. The attention is paid to the effect of the residual strain in the plastic zone on the crack extension.

PLASTIC ZONE AT THE CRACK TIP

Since the plastic zone at the crack tip in plane strain under tensile stress is of the shape shown in Fig. 1a, the Dugdale model is modified as is shown in Fig. 1b, where two planes of plastic flow inclined to the plane of the crack are considered; see e.g. Vitek (7). Along the planes of plastic flow the yield condition is assumed. Numerical results provide various features of the plastic zone at the crack tip – size, orientation, CTOD, and so on. No information on the crack growth, however, is drawn from this model.

To study the possibility of the unstable crack growth from the original crack tip, we consider a model shown in Fig. 1c. The extended crack OQ is considered where the residual strain (the displacement gap) is distributed along the plastic zone O'P,O'P' at the original crack tip O'. The distribution of the residual strain is obtained by solving the problem of Fig. 1b. Since we restrict our attention on the unstable crack growth where the strain rate is high enough, the plastic deformation at the tip of the extended crack is neglected. Deformation in mode I under plane strain is considered.

We introduce a loading parameter for this model, "the applied K-value", which is defined as the mode I stress intensity factor at the crack tip when the material is assumed to be elastic. It is known that the stability of the crack growth highly depends on the geometry of the specimen and the way of loadings. In this paper we consider the crack growth under the constant applied K-value. Results may be easily applied for cases where the loading is the function of the crack length.

The problem consists of two parts. First we consider the plastic zone growth prior to the crack extension; see Fig. 1b. The problem is symmetric with respect to the x-axis. The length of the plastic zone, which makes an angle θ with the x-axis, is denoted by ℓ_p. The boundary conditions are given by

$$\sigma_y = \tau_{xy} = 0 \, , \quad \text{on OQ,} \tag{1}$$

$$u_\theta^+ = u_\theta^- \, , \quad \tau_{r\theta} = \tau_Y \, , \quad \text{on OP and OP'}, \tag{2}$$

where superscripts + and − indicate the value of the quantity on the upper and lower surfaces of the plastic zone and τ_Y denotes the yielding shear stress. The condition at the end of the plastic zone,

$$\text{stress is bounded at P and P'}, \tag{3}$$

must be satisfied. With the applied K-value, we find the solution which satisfies conditions (1-3). To solve this problem we use the Green's function technique with the solution of a dislocation near a semi-infinite crack. Distributed dislocations along plastic zones are introduced. The condition (1) is automatically satisfied and the condition (2) leads to the singular integral equation for the dislocation density. It is solved numerically with the condition (3).

Next, with the obtained distribution of the residual strain, we calculate the stress intensity factor at the tip of the extended crack, which is different from the applied K-value because of the effect of the residual strain. Mathematical formulations are shown in the following section. Those who are not interested in the mathematical details can skip the next section without loss of continuity. Note that the mathematical formulation is easily modified for a finite crack; see Horii, Hasegawa, and Nishino (8) for details.

The interaction of plastic zones and cracks is also a fundamental factor in the micromechanism of the brittle-ductile transition under compression. Its analytical model is proposed by Nemat-Nasser and Horii (9), and Horii and Nemat-Nasser (10). The model includes cracks and plastic zones emanating from an initial defect. Various features of the brittle-ductile transition are explained in terms of analytical results.

MATHEMATICAL FORMULATION

For the mathematical formulation, Muskhelishvili's complex stress functions Φ and Ψ are employed (11). In terms of these potentials the stresses and displacements are given by

$$\sigma_x + \sigma_y = 2(\Phi' + \overline{\Phi'}) ,$$
$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2(\bar{z}\Phi'' + \Psi') , \tag{4}$$
$$2\mu(u_x + iu_y) = \kappa\Phi - z\overline{\Phi'} - \overline{\Psi} ,$$

where μ is the shear modulus; $\kappa = 3-4\nu$ for plane strain, ν being Poisson's ratio; $z = x + iy$ with $i = \sqrt{-1}$; overbar denotes the complex conjugate; and prime stands for differentiation with respect to the argument.

To solve the problem we consider a single dislocation at z_0 near a semi-infinite crack. We introduce stress functions $\Phi_D = \Phi_0 + \Phi_R$ and $\Psi_D = \Psi_0 + \Psi_R$ where Φ_0 and Ψ_0 are stress functions for a single dislocation in an infinite plane and Φ_R and Ψ_R are the complementary potentials to satisfy the stress free condition (1) along the crack surface. Φ_R and Ψ_R are obtained by the method of Muskhelishvili (11). They are given by

$$\Phi_0' = \frac{\alpha}{z - z_0} , \quad \Psi_0' = \frac{\overline{\alpha}}{z - z_0} + \frac{\alpha\overline{z_0}}{(z - z_0)^2} ,$$
$$\Phi_R' = -\alpha[F(z,z_0) + F(z,\overline{z_0})] - \overline{\alpha}(z_0 - \overline{z_0})G(z,\overline{z_0}) ,$$
$$\Psi_R' = \bar{\Phi}_R' - \Phi_R' - z\Phi_R'' , \tag{5}$$

with

$$F(z,z_0) = \frac{1}{2}[1 - \sqrt{\frac{z_0}{z}}] \frac{1}{z - z_0}, \quad G(z,z_0) = \frac{\partial}{\partial z_0} F(z,z_0), \tag{6}$$

where $\alpha = \mu([u_r]+i[u_\theta])e^{i\theta}/\pi i(\kappa+1)$, $[u] = u^+ - u^-$, and $\bar{\Phi} = \overline{\Phi(\bar{z})}$.

The stress functions Φ_A and Ψ_A for the applied load are given by,

$$\Phi'_A = \frac{1}{2}\frac{K_{IA}}{\sqrt{2\pi z}}, \quad \Psi'_A = \frac{1}{4}\frac{K_{IA}}{\sqrt{2\pi z}}. \tag{7}$$

K_{IA}, called "<u>the applied K-value</u>" is the loading parameter for the semi-infinite crack.

Stress functions Φ_D, Ψ_D and Φ_A, Ψ_A automatically satisfy the stress free condition (1) on the crack surface. We introduce distributed dislocations along the plastic zones OP and OP'; see Fig. 1b. From the first equation of (2) and the symmetry of the problem, the dislocation density is given by

$$\alpha(\xi) = -i\beta(\xi)e^{i\theta}, \quad \text{at } z_0 = \xi e^{i\theta}, \text{ and}$$
$$\beta(\xi) = i\beta(\xi)e^{-i\theta}, \quad \text{at } z_0 = \xi e^{-i\theta}, \tag{8}$$

where $\beta(\xi)$, which is the derivative of the shear displacement gap across the plastic zone with respect to the distance ξ, is a real function to be determined. The yield condition along the plastic zone – the second equation of (2), leads to the singular integral equation for the dislocation density $\beta(\xi)$,

$$-2\int_0^{\ell_p}\frac{\beta(\xi)}{\xi-\eta}d\xi + \int_0^{\ell_p}\beta(\xi)K(\xi,\eta;\theta)d\xi + \frac{K_{IA}}{\sqrt{2\pi\eta}}\frac{1}{2}\sin\theta\cos\frac{\theta}{2} = \tau_Y, \tag{9}$$

where

$$K(\xi,\eta;\theta) = \mathrm{Re}\{e^{4i\theta}[\frac{2}{\xi-\eta e^{2i\theta}} + \frac{2i\eta\sin 2\theta}{(\xi-\eta e^{2i\theta})^2}]\}$$
$$+ 4\eta\sin^2\theta\mathrm{Re}\{e^{2i\theta}[F'(z,z_0) + F'(z,\overline{z_0})$$
$$- \xi\{e^{-i\theta}G'(z,\overline{z_0}) + e^{i\theta}G'(z,z_0)\}]\}, \tag{10}$$

with Re{ } for the real part of the argument; $F' = \frac{\partial}{\partial z}F$.

The singular integral equation (9) for the dislocation density $\beta(\xi)$ is solved numerically by the method of Gerasoulis and Srivastav (12) with the condition (3) resulting in

$$\frac{K_{IA}}{\tau_Y\sqrt{\pi \ell_p}} = f(\theta) \ . \qquad (11)$$

From the obtained distribution of the dislocation density, the crack opening displacements δ and the dissipated plastic work W_p are obtained as

$$\delta = 2\sin\theta \frac{\pi(\kappa+1)}{\mu} \int_0^{\ell_p} \beta(\xi)\, d\xi \ ,$$

$$W_p = 2\tau_Y \frac{\pi(\kappa+1)}{\mu} \int_0^{\ell_p} \int_\eta^{\ell_p} \beta(\xi)\, d\xi\, d\eta \ , \qquad (12)$$

which are calculated in the forms

$$\frac{\delta}{\tau_Y \ell_p} \frac{\mu}{\pi(\kappa+1)} = d(\theta) \ , \qquad \frac{W_p}{\tau_Y^2 \ell_p^2} \frac{\mu}{\pi(\kappa+1)} = w(\theta) \ . \qquad (13)$$

The obtained distribution of the dislocation density is used as the residual strain when we consider the extended crack; see Fig. 1c. We calculate the stress intensity factor ΔK_I at the tip of the extended crack due to the residual strain along the plastic zones. From Eqns. (4), (5), (6), and (8), ΔK_I is given by

$$\Delta K_I = -4\sqrt{2\pi}\sin\theta \int_0^{\ell_p} \beta(\xi) \mathrm{Re}\left\{ \frac{1}{\sqrt{z_0}} + \frac{\xi e^{i\theta}}{2 z_0 \sqrt{z_0}} \right\} d\xi \ , \qquad (14)$$

which is calculated in the form

$$\frac{\Delta K_I}{\tau_Y\sqrt{\pi \ell_p}} = g(\ell_t/\ell_p;\theta) \ . \qquad (15)$$

In the following section, numerical results are shown and the growth of plastic zone followed by the unstable crack growth is discussed.

RESULTS AND DISCUSSIONS

In the previous section it is shown that by solving the singular integral equation numerically, we obtain

$$\frac{K_{IA}}{\tau_Y\sqrt{\pi \ell_p}} = f(\theta) \ , \qquad \frac{\delta}{\tau_Y \ell_p} \frac{\mu}{\pi(\kappa+1)} = d(\theta) \ ,$$

$$\frac{W_p}{\tau_Y^2 \ell_p^2} \frac{\mu}{\pi(\kappa+1)} = w(\theta) \ , \qquad \frac{\Delta K_I}{\tau_Y \sqrt{\pi \ell_p}} = g(\ell_t/\ell_p;\theta) \ , \tag{16}$$

where δ and W_p are the crack opening displacement and the dissipated plastic work prior to the crack extension, respectively; ΔK_I is the stress intensity factor at the extended crack tip due to the residual strain. The total stress intensity factor at the extended crack tip is given by the summation, $K_I = K_{IA} + \Delta K_I$. The quantities $f(\theta)$, $d(\theta)$, $w(\theta)$, and $g(\ell_t/\ell_p;\theta)$ are the nondimensional values calculated for given θ and ℓ_t/ℓ_p.

The orientation of the plastic zone is set to be 76.1° such that the dissipated plastic work W_p is maximized for a constant applied K-value, K_{IA}. (The orientation which maximizes the length of the plastic zone is slightly less than that which does the dissipated plastic work.) With this orientation, we have [see Eqns. (16)],

$$\frac{K_{IA}}{\tau_Y \sqrt{\pi \ell_p}} = 2.35 \ , \qquad \frac{\delta E \sigma_Y}{K_{IA}^2 (1-\nu^2)} = 0.565 \ ,$$

$$\frac{W_p E \sigma_Y^2}{K_{IA}^4 (1-\nu^2)} = 0.0225 \ , \tag{17}$$

where $\sigma_Y = 2\tau_Y$ and E denotes the Young's modulus. Corresponding to this orientation and the associated distribution of the residual strain, we calculate the stress intensity factor ΔK_I at the extended crack tip for different values of ℓ_t/ℓ_p; see Eqns. (16) and Fig. 2.

So far the introduced material parameter is only the yielding shear stress τ_Y. To discuss the crack extension, we require the fracture toughness G_c or K_c which are related each other through the relation between the energy release rate G and the stress intensity factor K_I,

$$G = \frac{1-\nu^2}{E} K_I^2 \; . \tag{18}$$

Then the material property is represented by two parameters τ_Y and K_c. The characteristic length of the material r_p is defined by

$$r_p = \frac{K_c^2}{8\pi\tau_Y^2} \; , \tag{19}$$

which is the Irwin's plastic zone correction with $K_I = K_c$. Each material has its own characteristic length. For example, the value of r_p is larger than 135mm for low strength Carbon steel and about 0.1mm for 4340 steel. Quantities whose dimensions include the length are non-dimensionalized using r_p. With this characteristic length it follows from Eqns. (17) that

$$\frac{K_{IA}}{K_c} = 0.83 \sqrt{\frac{\ell_p}{r_p}} \; . \tag{20}$$

From Eqns. (16), (19), and (20), we have the stress intensity factor at the extended crack tip as a function of ℓ_t/r_p and ℓ_p/r_p,

$$\frac{K_I}{K_c} = \frac{K_{IA}}{K_c} + \frac{\Delta K_I}{K_c} = \sqrt{\frac{\ell_p}{r_p}} \; [\; 0.83 + g(\ell_t/\ell_p)/2\sqrt{2} \;] \; , \tag{21}$$

which is shown in Fig. 3 where lines for constant values of K_I/K_c are plotted. Above the critical line for $K_I/K_c = 1$, the stress intensity factor K_I at the crack tip is larger than the fracture toughness K_c. Once the crack is extended beyond the critical line, the crack runs away by itself. This instability is possible if the applied K-value is greater than the fracture toughness as is seen in Fig. 3. However, the shaded area below the critical line where K_I is less than K_c obstructs the crack growth. To jump this obstacle some energy must be supplied since the released energy due to the crack extension is less than that required for the material separation. Once energy required to jump the obstacle is supplied, the unstable crack growth is materialized.

To calculate the energy required to initiate the crack extension, we plot the energy release rate (see

Eqn. (18)) as a function of the length of the crack extension for different values of the applied K-value in Fig. 4. It is seen that the energy release rate reaches the straight line, $G = G_c$, when K_{IA} is greater than K_c. The energy required for the crack extension is calculated as the area surrounded by the corresponding curve, the straight line for $G = G_c$, and the vertical axis; the shaded area for $K_{IA} = K_c$. As the applied K-value increases, the associated area, that is the required energy, decreases. Calculated energy per unit thickness required for the crack extension, E_i, is shown in Fig. 5 as a function of the applied K-value.

As is seen from Fig. 5, the maximum energy required for the crack extension at $K_{IA}/K_c = 1$ is given by $E_i = 0.25G_c r_p$. It is equal to the energy for the material separation of length a quarter of r_p. It increases drastically as the ductility of the material increases. For example, E_i is 0.26 J/m for 4340 steels and 7300 J/m for Carbon steels. In Fig. 6 the characteristic length r_p and the maximum energy required for the crack initiation E_i are plotted as a function of τ_Y and K_c together with data for typical metals; to calculate E_i, $E = 20\times10^{10}$ Nm^{-2} and $\nu = 0.3$ are used.

For brittle materials such as Maraging steels, the energy required for the crack extension is so small that it is provided from the external system by dynamic effects and others. Hence the crack extension is supposed to occur at the minimum value of the applied K-value which is the same as the fracture toughness. Therefore it is concluded that the Griffith criterion is valid for brittle materials even with the plastic zone at the crack tip prior to its extension.

As the ductility of the material increases, the required energy for the crack initiation increases drastically and it may not be possible to jump the obstacle at $K_{IA} = K_c$. Then two choices are possible: One is that the required energy for the crack extension decreases with the increasing K_{IA} as shown in Fig. 5, and at a certain stage the unstable crack extension is materialized. The other is that with the increasing K_{IA}

a stable crack growth occurs where the applied K-value must be increased to advance the crack and large plastic zones follow the crack tip as it advances. The prediction requires a criterion of the stable (ductile) crack growth which must be based on the micro-events ahead of the crack tip such as the void growth and their coalescence.

Although the mechanism of the stable crack growth is out of focus in this paper, the residual strain is considered to play an important role in the stable crack growth. As the crack continues the stable growth, the plastic zone follows the advancing crack tip accumulating residual strains behind the crack tip. The resistance to fracture continues to rise due to the accumulated residual stain. With increasing crack length the applied K-value increases, and the required energy for the unstable crack growth decreases similarly to the case shown in Figs. 4 and 5.

It is seen from our results that the brittle crack initiation necessarily accompanies an instability because it is accomplished by jumping the obstacle due to the residual strain. The residual strain also plays an important role in fatigue crack growth, for example, in the retardation effect of overloads. This study shows a way to estimate the increase in the fracture toughness due to the residual strain.

In this paper the applied K-value is introduced as a loading parameter and is fixed constant for the crack extension. In the actual situation, the applied K-value changes as the plastic deformation proceeds and as the crack extends in a very complex manner. The stability of the crack growth depends on the way of the loading, the stiffness of the loading machine and other factors. Our results are easily modified for those cases if their relations are given.

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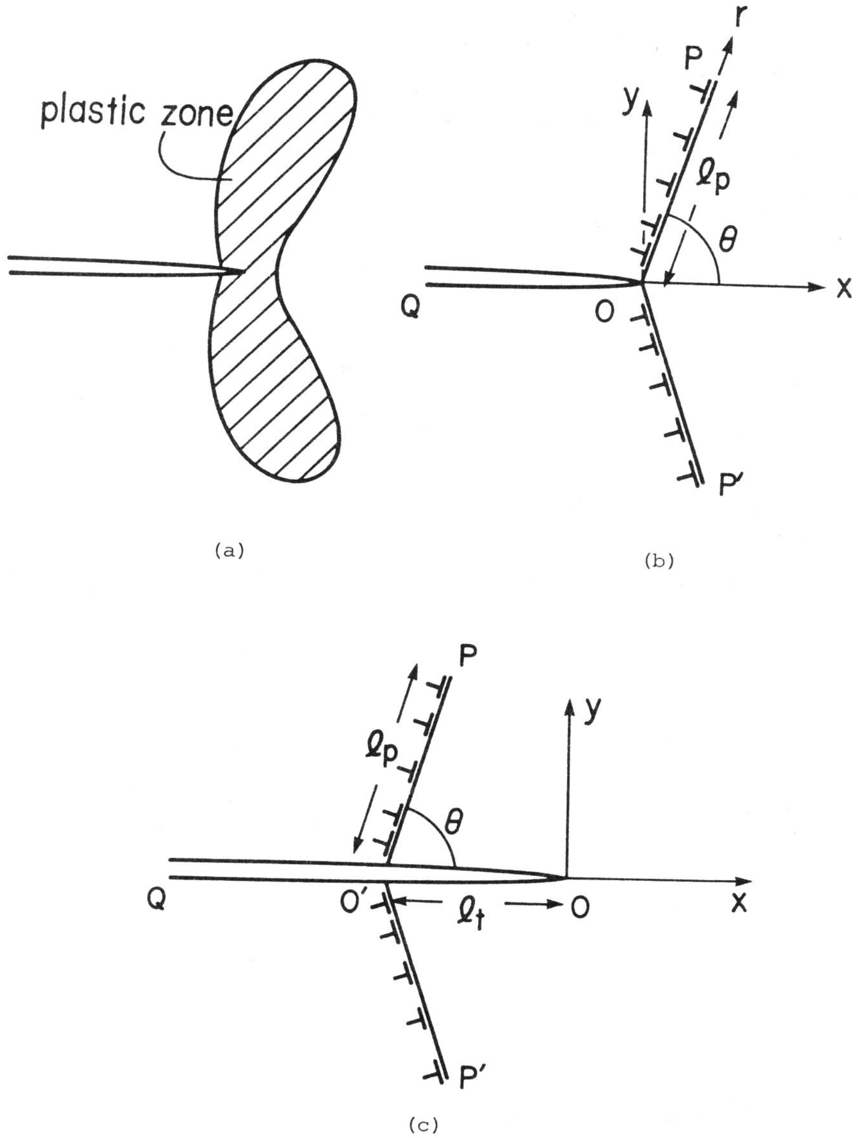

Figure 1 (a) The plastic zone at the tip of a semi-infinite crack in plane strain, (b) planes OP,OP′ of the plastic flow at the tip of a semi-infinite crack OQ, and (c) the extended crack OQ with the residual strain along the plastic zones O′P,O′P′ at the original crack tip O′.

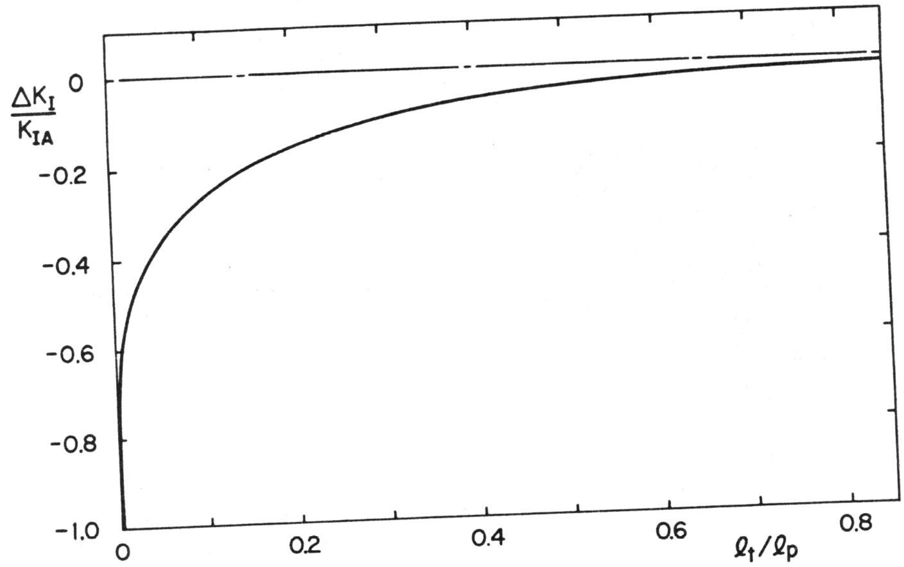

Figure 2 The stress intensity factor at the tip of the extended crack due to the residual strain.

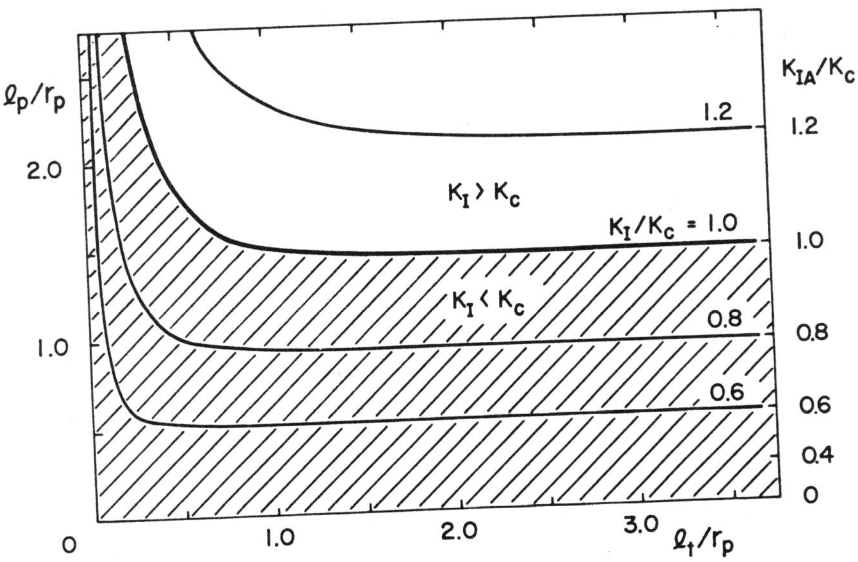

Figure 3 Lines for constant value of the stress intensity factor at the tip of the extended crack.

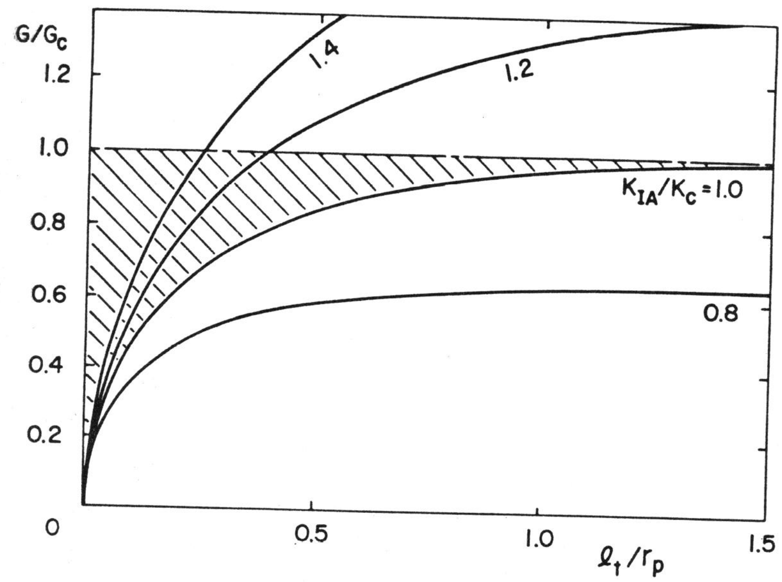

Figure 4 The energy release rate vs. the extended crack length for the indicated applied K-value.

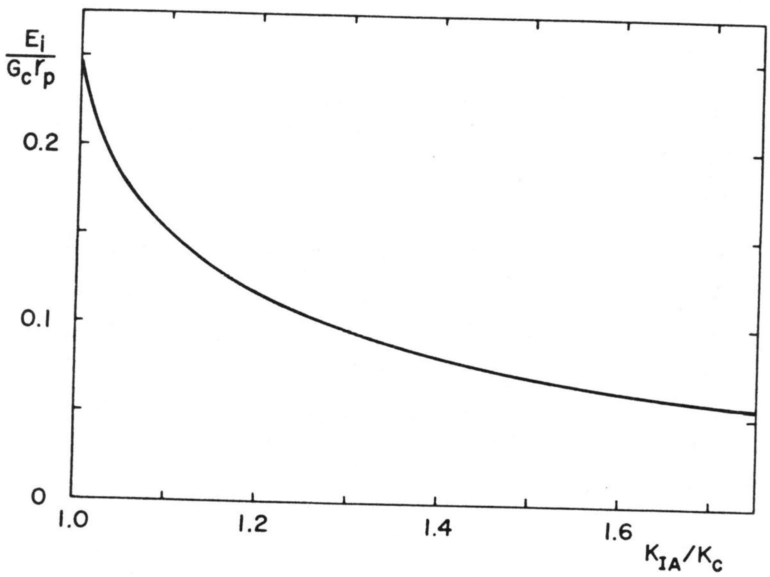

Figure 5 The energy required for the crack extension vs. the applied K-value.

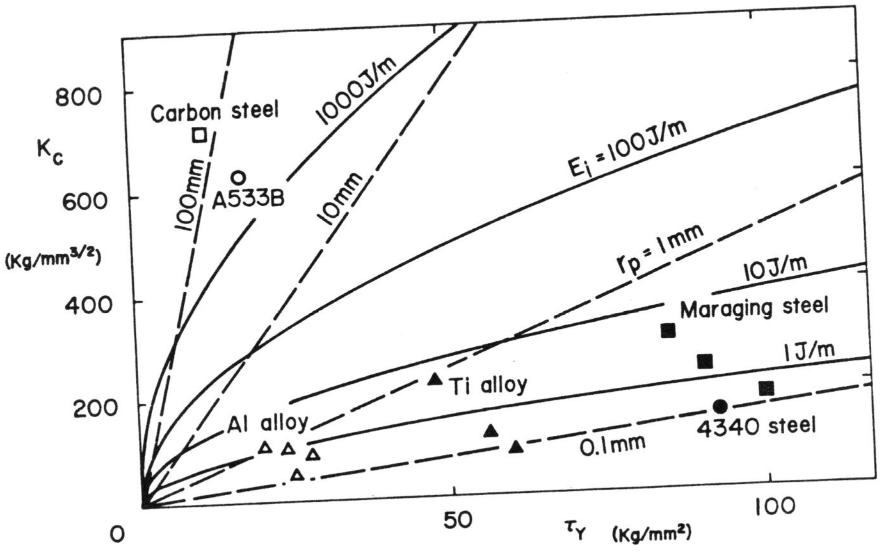

Figure 6 The characteristic length and the maximum energy for the crack initiation together with data for typical metals.