AN EQUIVALENT PLASTIC ZONE CORRECTION FACTOR FOR THE DETERMINATION OF R-CURVES

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In this paper an equivalent plastic zone correction factor, r_{peq} is proposed. Considering thin Alclad 2024-T3 sheets, the plastic zone size at the crack tip has been visualized and determined by the photo-stress coat technique. Also, Finite Element Method calculations, involving both elastic and elasto-plastic analyses, were performed to determine the same. An equivalent plastic zone correction factor is defined, computed and compared with Irwin's Plastic zone correction factor, r_p. Crack growth resistance curves have been drawn using the above data and these are compared.

INTRODUCTION

It is well known that in thin sheets, the size of the plastic zone at the tip of a crack is large at higher stress levels. Under such circumstances, the plastic zone correction factor plays an important role for evaluation of the effective crack length, which is essential for R-curve generation. The normal practice to account for the effect of the crack-tip plastic zone is to use the plastic zone correction factor, r_p due to Irwin (1), which is obtained from an empirical relation. In the present work an equivalent plastic zone correction factor, r_{peq} is calculated, based on the area of the plastic zone at the tip of a crack and considering r_{peq} to be the radius of an equivalent circular area. The extent of the plastic zone is determined theoretically, from FEM analysis, by plotting the yield locus in front of the crack tip from the stresses at the Gauss points in all elements surrounding the crack tip. In the experimental method, the plastic zone was observed on photo-stress coating and photographed in colour.

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The area of the plastic zone obtained by both menthods is calculated and transformed to an equivalent circular area with radius, r_{peq}. This r_{peq} is employed as the equivalent plastic zone correction factor for evaluating the effective crack length and the R-curve generated accordingly.

THEORETICAL ANALYSIS

For the theoretical analysis, centre cracked tension specimens of Alclad 2024-T3 with effective size of 280 mm x 140 mm x 1 mm and a centre crack to width ratio, $a_o/W = 0.25$ are used. Taking into account the symmetry of the specimens, only one quarter of the specimen was discretized with eight noded isoparametric quadratic quadrilateral elements for the elastic finite element analysis. The crack tip elements are degenerated to meet at the crack tip in the form of radially distributed triangles. The inverse square root stress singularity at the crack tip is achieved by simply shifting the mid-side nodes to one quarter position close to the crack tip, as suggested by Barsoum (2). The panel is subjected to uniform tensile stresses of magnitudes 98.10 MPa, 147.15 MPa, 171.675 MPa and 196.20 MPa. By the displacement substitution method, following three point Gaussian numerical integration procedure, stresses, strains and displacements were computed at all Gauss-points. The yield loci surrounding the crack as resulting from the Tresca criterion are shown in Figure 1.

In thin sheet fracture, it is expected that there is a considerable plastic deformation at the crack tip and thus an elastic analysis of the continuum may not depict the real picture of the stress field surrounding the crack tip. So, an elasto-plastic finite element analysis has also been carried out with an identical panel subjected to the same stress levels. The linearized total strain method of Zienkiewicz (3), as adopted by Egeland (4) was used. The applied method assures that each integration point follows the material stress-strain curve, which requires an iterative solution procedure. Iteration was continued until a good degree of convergence was achieved. The resulting yield loci determined using the Tresca yield criterion are given in Figure 2. The area enclosed by the yield locus for a particular stress level gives the plastic zone area. The equivalent plastic zone radius, r_{peq} was calculated from the total area of both the lobes of the plastic zone at the crack tip, to be used as the plastic zone correction factor. A normalized plot of r_{peq}, in the form of r_{peq}/a_o versus σ/σ_s, obtained by both elastic and elastic-plastic finite element analyses is shown in figure 3.

EXPERIMENTAL TECHNIQUE

For the experimental determination of the crack tip plastic zone shape and size, a photo-stress coating technique was applied. Centre cracked tension panels of Alclad 2024-T3 having effective size of 280 mm x 140 mm x 1 mm with a fatigue precrack length to sheet width ratio of 0.25 were studied under different stress levels. A photo-stress coating type PS-1B and adhesive type PC-1, supplied by M/s Photolastic Inc., U.S.A. were used in the experiments.

In the present case of plane stress conditon, $\sigma_3 = 0$. For the ductile material, following the Tresca yield criterion, plastic deformation occurs when the largest of $|\sigma_1 - \sigma_2|$, $|\sigma_2|$ and $|\sigma_1|$ becomes equal to the yield stress, σ_s. After careful analysis of the crack tip region, it was found that $|\sigma_1 - \sigma_2|$ attains the absolute maximum value. Hence, combining the stress-optic law and the Tresca yield criterion, the expression for the fringe order, N, denoting the yield locus becomes:

$$N = \sigma_s/E \cdot f_\varepsilon \tag{1}$$

For direct observation of the plastic zone, the value of the fringe order, N, was calculated apriori from the known values of the material yield stress, σ_s, and the strain-optic coefficient, f_ε, of the coating material. A further correction according to the method suggested by Zandman et al. (5) was incorporated in the computation of N in order to account for the reinforcing effect of the coating. The pre-calculated value of N was set on the Uniform-field compensator of the Reflection Polariscope and the fringe patterns at different stress levels were viewed through a telemicroscope. The particular fringe corresponding to the yield locus appeared black, while the other fringes appeared as the usual yellow-red-green loops. Coloured snaps were taken on high speed films, like Fuji 400 ASA, using a camera-adaptor fitted to the telemicroscope. The area bound by the black fringe is the plastic zone area surrounding the crack tip. Plastic zones at different stress levels are shown in Figure 4. The equivalent plastic zone radii, r_{peq} were computed as described earlier. The normalized plot of r_{peq} is also shown in Figure 3.

GENERATING OF R-CURVES

Crack growth resistance curves have been drawn as the effective stress intensity factor, K_{eff} versus the effective crack growth, Δa_{eff}. The physical crack lengths were measured by the Direct Current Potential Drop (DCPD) method due to Johnson (6).

The plastic zone correction has been incorporated by using both Irwin's method and the present photo-stress coating method, and the effective crack lengths have been calculated accordingly. The ASTM recommended finite width correction factor, Y, for a CCT-panel is applied for the effective crack length, a_{eff}:

$$Y_{eff} = 1.77 + 0.23\,(a_{eff}/W) - 0.51\,(a_{eff}/W)^2 + 2.27\,(a_{eff}/W)^3 \qquad \text{for } 0 \leq a_{eff}/W \leq 0.7 \qquad (2)$$

This modified value Y is used to calculate the effective stress intensity factor, K_{eff}. The R-curves, thus generated, are shown in Figure 5. The critical conditions for failure in terms of K_{eff} and Δa_{eff} were determined by drawing tangents in accordance with the energy balance principle.

DISCUSSIONS AND CONCLUSIONS

The experimental and theoretical values of the equivalent plastic zone radii, are found to be in close agreement. It is also observed that up to a stress ratio $\sigma/\sigma_s = 0.45$ Irwin's plastic zone correction factor agrees reasonably well with the one proposed here. But at higher stress ratios, Irwin's relation underestimates the size of the crack tip plastic zone. Hence, the presently proposed equivalent plastic zone correction factor appears to offer a more realistic estimate of the effective critical crack length. Besides, the R-curve can now be drawn purely experimentally, with the measurement of physical crack length by DCPD method and the application of r_{peq} as obtained by the photo-stress coating method.

From the R-curve presented in this paper it is clear that the one drawn with photo-stress coating r_{peq} as the plastic zone correction factor lies below that drawn using Irwin's plastic zone size. Also, from the critical values of K_{eff} and a_{eff} as obtained from both the R-curves, it can be seen that the proposed plastic zone correction factor offers a more conservative estimate of the critical failure condition, in terms of K_{eff}, in a thin centre cracked panel.

SYMBOLS USED

a_o	initial crack length	mm
a_{eff}	effective crack length	mm
K_{eff}	effective stress intensity factor	$MN/m^{3/2}$
N	normal incidence fringe order	-
r_p	Irwin's plastic zone correction factor	mm
r_{peq}	proposed equivalent plastic zone radius	mm
r_{pmax}	maximum radius vector of plastic zone	mm
R-curve	crack growth resistance curve	-
W	width of the panel	mm
$\sigma_1, \sigma_2, \sigma_3$	normal (principal) stresses	MPa
σ_s	yield stress	MPa
θ_{max}	tilt angle of r_{pmax} from crack axis	degree

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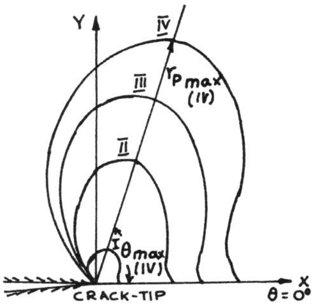
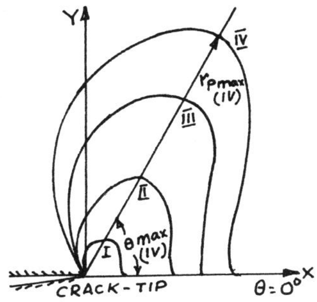




I : σ/σ_s = 0.316

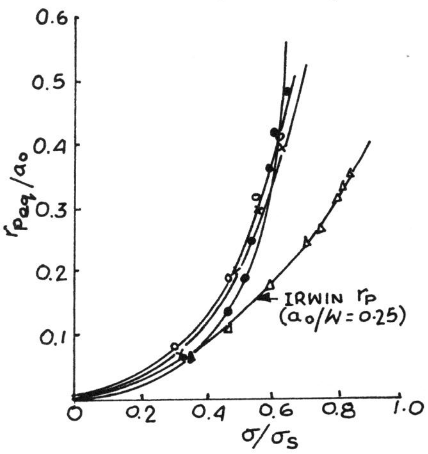
II : σ/σ_s = 0.475

III : σ/σ_s = 0.554

IV : σ/σ_s = 0.633

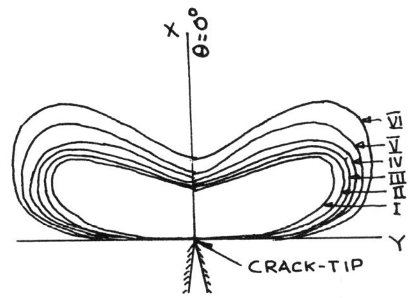
Figure 1. Plastic zones from elastic FEM analysis

Figure 2. Plastic zones from elasto-plastic FEM analysis



△ Irwin's r_p, based on DCPD test data
o present EPFM analysis
x present EFM analysis
● present photo-stress coating analysis

Figure 3. Comparison of normalized plastic zone sizes



I : σ/σ_s = 0.513

II : σ/σ_s = 0.541

III : σ/σ_s = 0.572

IV : σ/σ_s = 0.594

V : σ/σ_s = 0.619

VI : σ/σ_s = 0.664

Figure 4. Plastic zone sizes from photo-stress coating

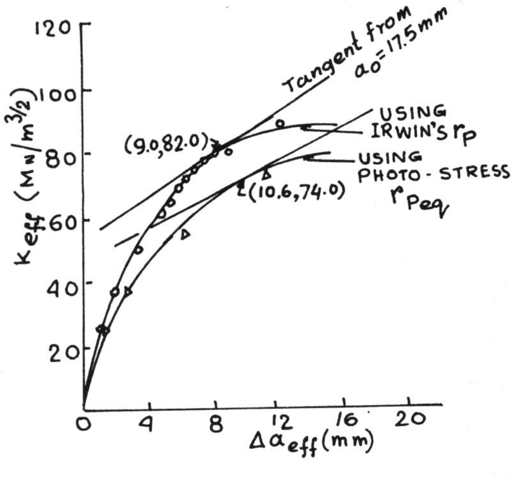


Figure 5. Comparison of R-curves using Irwin's r_p and photo-stress coat r_{peq}