COHESIVE CRACK TIP MODELLING OF PLASTIC FRACTURE

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The competition between plastic collapse at the ligament and brittle crack propagation is investigated on the basis of dimensional analysis and cohesive crack model. The transition from a collapse to another one is described by the brittleness number $s = K_{IC}/\sigma_y\sqrt{b}$, which is a function of fracture toughness, yield strength and specimen size. Size-scaled three point bending specimens of polypropylene were tested and analyzed. A simple plastic collapse at the ligament occurred for small size scales, whereas the transition from plastic collapse to brittle fracture is reproduced by the BCS crack model satisfactorily up to the asymptotic situation of very large specimens, for which LEFM is totally valid.

INTRODUCTION

Due to the different physical dimensions of strength $[F]\,[L]^{-2}$ and fracture toughness $[F]\,[L]^{-3/2}$, scale effects are always present in the usual fracture testing of common engineering materials. This means that, for the usual size scale of the laboratory specimens, the ultimate strength collapse or the plastic collapse at the ligament tends to anticipate and obscure the brittle crack propagation.
Such a competition between collapses of a different nature can be described through a cohesive crack tip modelling. The ductile-brittle transition when the specimen size increases is captured by the well-known BCS-model (Bilby et al (1), Heald et al (2)). The substantial assumption is the transition from "stress vs. strain" to "stress vs. displacement" constitutive law when the ultimate tensile strength is locally achieved.

The experimental results (Carpinteri et al (3)) obtained from size-scaled polymeric three point bending specimens (width = 1/2, 1, 2, 4, 8, 12 cm) are predicted theoretically. For each value of width, five different relative crack depths are considered: a/b = 0.1, 0.2, 0.3, 0.4, 0.5. Except for the larger specimens, the fracture process was stable and very ductile, but no necking near the crack was noticed. The plastic zone in front of the crack tip presented a strip-shape. A simple plastic col-

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lapse at the ligament occurred for small size scales ($b = 1/2$, 1, 2 cm), whereas the transition from plastic collapse to brittle fracture is reproduced by the cohesive crack model satisfactorily up to the asymptotic situation of very large specimens, for which LEFM is totally valid.

EXPERIMENTAL PROCEDURE (3)

The experimental material is polypropylene Moplen® D 60 P, originally provided in slabs 100 x 200 x 4 cm. The main properties of the material are:

Melt flow rate: 0.46 g/10′ (ASTM, D 1238-73)
Young's modulus: 1400 MN/m^2 (ASTM, D 790-71)
Yield strength: 33 MN/m^2 (ASTM, D 638-77)
Density: 0.912 g/cm^3 (ASTM, D 1505-68).

Three point bend specimens have been obtained from these slabs. Specimens maintained the original thickness of slab (4 cm), but their width b and length ℓ have been varied so that the constant ratio $\ell/b = 4$ always resulted. The following values of width b have been chosen: 0.5, 1, 2, 4, 8, 12 cm, which nearly constitute a geometric progression. The bending tests have been performed by a displacement controlled Instron machine. Thus the loading velocity was controlled so that all the utilized sizes were subjected to the same strain rate, by applying the formula:

$$\dot{\epsilon} = \frac{6 V_0 b}{\ell^2}$$

where: V_0 = velocity of the point of load application. Such a formula is strictly applicable to unnotched specimens. The strain rate was $\dot{\epsilon} \simeq 0.001 \ \mathrm{sec}^{-1}$. In the case of polymers it is important to work with constant strain rate, to avoid effects on yield strength and fracture mechanics parameters.

For every value b of the specimen width, five different relative crack depths have been utilized: $a/b = 0.1, 0.2, 0.3, 0.4, 0.5$.

The tests were carried out at 23 °C.

LIMIT ANALYSIS AT THE LIGAMENT

The competition between plastic collapse at the ligament and brittle crack propagation can be easily proved by considering the ASTM formula for the three point bending test evaluation of fracture toughness (Fig. 1):

$$K_I = \frac{P\ell}{t\, b^{3/2}} f\left(\frac{a}{b}\right), \qquad (1)$$

with:

$$f\left(\frac{a}{b}\right) = 2.9\left(\frac{a}{b}\right)^{1/2} - 4.6\left(\frac{a}{b}\right)^{3/2} + 21.8\left(\frac{a}{b}\right)^{5/2} - 37.6\left(\frac{a}{b}\right)^{7/2} + 38.7\left(\frac{a}{b}\right)^{9/2} .$$

At the crack propagation condition eq. (1) becomes:

$$K_{IC} = \frac{P_f \ell}{t\, b^{3/2}}\, f\left(\frac{a}{b}\right), \tag{2}$$

where P_f is the external load of brittle fracture. If both members of eq. (2) are divided by $\sigma_y b^{1/2}$ we obtain:

$$\frac{K_{IC}}{\sigma_y b^{1/2}} = s = \frac{P_f \ell}{\sigma_y t\, b^2}\, f\left(\frac{a}{b}\right), \tag{3}$$

where s is a dimensionless number able to describe the brittleness of the specimen (Carpinteri (4, 5)). Rearranging of eq. (3) gives:

$$\frac{P_f \ell}{\sigma_y t\, b^2} = \frac{s}{f\left(\frac{a}{b}\right)} . \tag{4}$$

On the other hand, it is possible to consider the non-dimensional load of plastic hinge formation at the ligament:

$$\frac{P_p \ell}{\sigma_y t\, b^2} = \left(1 - \frac{a}{b}\right)^2 . \tag{5}$$

Eqs (4) and (5) are plotted in Fig. 1 as functions of the crack depth a/b. While the former produces a family of curves by varying the brittleness number s, the latter is represented by a unique curve. It is easy to realize that plastic collapse precedes crack propagation for each crack depth when the brittleness number is higher than the critical value $s_0 = 0.75$. For lower s numbers plastic collapse anticipates crack propagation only for crack depths external to a certain interval. This means that real fracture phenomena occur only for sufficiently low fracture toughnesses, high yield strengths and /or large structural sizes. It does not matter the single values of K_{IC}, σ_y and b. What is important is only their function s.

Recalling eqs (4) and (5), we can obtain the ratio between fictitious and real fracture toughness, which is equal to the ratio between load of plastic collapse, P_p, and load of crack propagation, P_f, when $P_p < P_f$, and equal to unity when $P_p > P_f$:

$$\frac{K_{IC}^f}{K_{IC}} = \frac{P_p}{P_f} = \frac{1}{s}\left(1 - \frac{a}{b}\right)^2 f\left(\frac{a}{b}\right), \qquad \text{for} \qquad P_p < P_f , \tag{6-a}$$

$$\frac{K^f_{IC}}{K_{IC}} = 1\,, \qquad \text{for} \qquad P_p > P_f\,. \tag{6-b}$$

Combining the definition of brittleness number, eq. (3), and eqs (6), it results:

$$\frac{K^f_{IC}}{\sigma_y\, b^{1/2}} = \left(1 - \frac{a}{b}\right)^2 f\left(\frac{a}{b}\right)\,, \qquad \text{for} \qquad P_p < P_f\,, \tag{7-a}$$

$$\frac{K^f_{IC}}{\sigma_y\, b^{1/2}} = s\,, \qquad \text{for} \qquad P_p > P_f\,. \tag{7-b}$$

Eq. (7-a) is represented in Fig. 2 as a bell-shaped curve vanishing for $a/b = 0$ and $a/b = 1$. It presents a maximum for that value of crack depth for which the fracture curve $s = s_0$ is tangent to the plastic flow curve in Fig. 1. More precisely, for $s > s_0$ eq. (7-a) is valid for each crack depth a/b, whereas for $s < s_0$ eq. (7-a) is valid for external crack depths and eq. (7-b) for central crack depths.

Eq. (7-a) is represented also in Fig. 3 by varying the specimen width b. The dark shaded area is where the curves $a/b = 0.1$ to $a/b = 0.5$ are concentrated. It is a very narrow strip, specially for not too large sizes b. When $s > s_0$, the parabola (7-a) is replaced by the horizontal straight line $K^f_{IC} = K_{IC}$. The experimental points present a course which is only initially similar to that of eq. (7-a). This means that, only for small specimens (b = 0.5 / 1.0 / 2.0 cm) the collapse can be perfectly described by a plastic flow at the ligament. By increasing the size scale a transition occurs from plastic flow towards a true LEFM collapse. For b = 12 cm, however, the latter has not been reached yet, since the experimental points are still ascending. It is difficult to predict the true value of fracture toughness exactly. On the other hand, if the experimental work went on with larger specimens, it would be possible to standardize an extrapolation technique with a smaller number of specimens.

BCS - COHESIVE CRACK MODEL

An attempt is done to describe the ductile-brittle transition through the BCS-cohesive crack model (1, 2). The following expression for the fictitious fracture toughness is assumed:

$$K^f_{IC} = \sigma^f (\pi a)^{1/2}\, F(a/b)\,, \tag{8}$$

where σ^f is the nominal stress at failure and F is the shape-function in the Tada-Paris-Irwin notation (6). If it is recalled the equivalence:

$$F\left(\frac{a}{b}\right) = \frac{2 f(a/b)}{3\left(\pi \frac{a}{b}\right)^{1/2}}\,, \tag{9}$$

between Tada-Paris-Irwin function and ASTM function, the BCS fracture toughness:

$$K_{IC}^{f} = (\pi a)^{1/2}\, F\left(\frac{a}{b}\right) \frac{2}{\pi}\, \sigma_y \cos^{-1}\left\{\exp - \left[\frac{\pi K_{IC}^2}{8\sigma_y^2 a F^2(a/b)}\right]\right\}, \tag{10}$$

is transformed as follows:

$$\frac{K_{IC}^{f}}{\sigma_y b^{1/2}} = \frac{4}{3\pi}\, f\left(\frac{a}{b}\right) \cos^{-1}\left\{\exp - \left[\frac{9\pi^2 s^2}{32 f^2(a/b)}\right]\right\}. \tag{11}$$

Eq. (11) is plotted in Fig. 2 as a function of crack depth a/b and varying the brittleness number s. The experimental points are on the limit analysis curve for b = 1 and 2 cm, whereas they fall below for larger specimens.

Eq. (11) is represented also in Fig. 3. According to the BCS model, it is necessary to assume a true K_{IC} value to be inserted into eq. (11). The value $K_{IC} = 5.5$ $MN/m^{3/2}$ is that which best-fits the experimental results. The family of curves a/b = = 0.1 to a/b = 0.5 is more spread for small than for large sizes in this case. The opposite occurs for the limit analysis prediction. It is very clear from Fig. 3 that a simple plastic collapse at the ligament occurred for small size scales (b = 0.5 / 1.0 / 2.0 cm), whereas the transition from plastic collapse to brittle fracture is captured by the BCS model satisfactorily, specially for shallow cracks (b = 4.0 / 8.0 cm). The asymptotic situation of very large specimens is described by LEFM consistently (Carpinteri and Sih (7)).

SYMBOLS USED

a = crack length (cm)

b = beam width (cm)

K_I = stress-intensity factor ($MN/m^{3/2}$)

K_{IC} = fracture toughness ($MN/m^{3/2}$)

K_{IC}^f = fictitious fracture toughness ($MN/m^{3/2}$)

ℓ = beam span (cm)

P = external load (MN)

P_f = external load of brittle fracture (MN)

P_p = external load of plastic hinge formation (MN)

s = $K_{IC}/\sigma_y b^{1/2}$ = brittleness number

t = beam thickness (cm)

σ^f = nominal stress at failure (MN/m^2)

σ_y = yield strength (MN/m^2)

REFERENCES

(1) Bilby, B.A., Cottrell, A. H. and Swinden, K.H., "The spread of plastic yield from a notch", Proceedings of the Royal Society, Vol. A 272, 1963, pp. 304-314.

(2) Heald, P.T., Spink, G.M. and Worthington, P.J., "Post yield fracture mechanics", Materials Science and Engineering, Vol. 10, 1972, pp. 129 - 138.

(3) Carpinteri, A., Marega, C. and Savadori, A., "Ductile-brittle transition by varying structural size", Engineering Fracture Mechanics, Vol. 21, 1985, pp. 263-271.

(4) Carpinteri, A., "Size effect in fracture toughness testing: a dimensional analysis approach", Proceedings of the International Conference on "Analytical and Experimental Fracture Mechanics", Edited by G.C. Sih and M. Mirabile, Sijthoff and Noordhoff, 1981, pp. 785-797.

(5) Carpinteri, A., "Notch sensitivity in fracture testing of aggregative materials", Engineering Fracture Mechanics, Vol. 16, 1982, pp. 467-481.

(6) Tada, H., Paris, P. and Irwin, G., "The Stress Analysis of Cracks Handbook", Del Research Corporation, St. Louis Missouri, 1963, pp. 2.16 - 17.

(7) Carpinteri, A. and Sih, G.C., "Damage accumulation and crack growth in bilinear materials with softening", Theoretical and Applied Fracture Mechanics, Vol. 1, 1984, pp. 145-159.

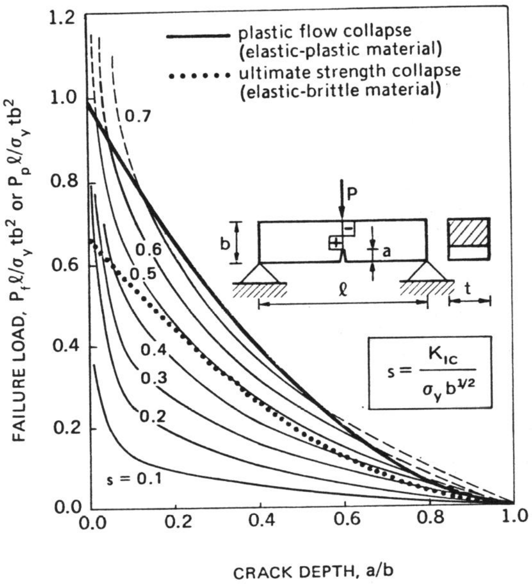

Figure 1 Interaction between plastic collapse and brittle crack propagation.

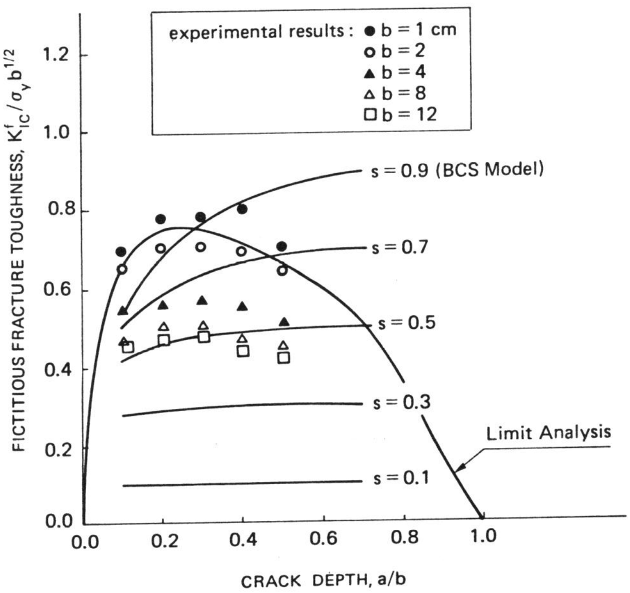

Figure 2 Fictitious fracture toughness vs. crack depth.

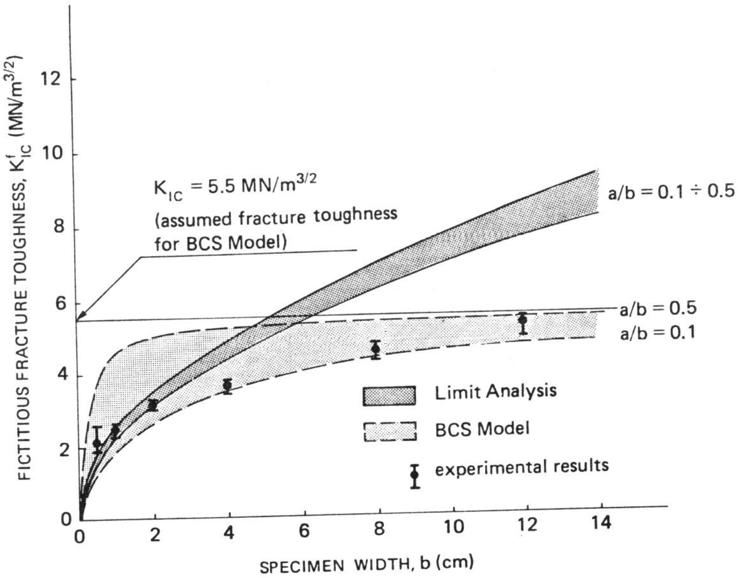

Figure 3 Fictitious fracture toughness vs. specimen width.