LEAK PREDICTION BY USE OF A GENERALIZED DUGDALE MODEL FOR SEMI-ELLIPTICAL SURFACE FLAWS IN PLATES UNDER TENSION LOADING

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By use of a Dugdale model generalized for semi-elliptical surface cracks in plates the external stresses are calculated for the state of ligament yielding and ligament rupture. For the prediction of the ligament rupture strain-hardening is included in the Dugdale model.

Good agreement with experiments is shown. The stress intensity factor related to the yield stress loading is calculated by use of the weight function method.

INTRODUCTION

The calculation of plastic failure in components containing surface defects or flaws is of great importance, since these components are made of highly ductile material. There are some different approaches to that problem. The simplest way is to calculate the plastic failure load from an equilibrium of forces as proposed by Harrison et al. (1), Harrop and Lidiard (2) or by Chell (3). More refined calculations are based on the use of the crack opening displacement (COD) or the J-Integral (4) to describe the onset of unstable crack extension. The most important methods to obtain them are FEM-calculations, the utilization of the line spring model (5) or a plastic strip model such as the Dugdale model (6).

Plastic strip models were used by Erdogan et al. (7,8) to calculate plastic limit loads and COD for through cracked and partly through cracked pipes. Mattheck et al. (9,10) developed a very simple method to calculate plastic yield loads, COD and J-integrals by use of a modified Dugdale plastic strip model. These calculations were carried out for the deepest point of a semi-elliptical crack only. Here a Dugdale plastic strip model is introduced that also takes into account the plastic strip at the sides of the crack. It is based on the calculation of averaged stress intensity factors as already used in (11).

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THE DUGDALE PLASTIC STRIP MODEL (6)

The Dugdale plastic strip model is an attempt to take into account the plastic behaviour near the tip of the crack. It is a combination of two elastic solutions to describe the plastic behaviour approximately. A fictitious crack is introduced by adding the length of the plastic zone d to the length of the real crack a_o. The fictitious crack is compressed by the constant flow stress σ_y over the region of the yield zone. Furthermore, the whole fictitious crack is loaded by the external stress σ (Fig. 1).

The length of the plastic zone can be calculated from the requirement of vanishing stress singularity at the crack tip. This can be achieved by choosing d in such a way that the stress intensity factor for the fictitious crack loaded by the external stress σ (denoted K_r) is equal to the stress intensity factor K_{Dug} for the same crack compressed by the constant plastic strip load σ_y over the length d of the yield zone:

$$K_r = K_{Dug} \tag{1}$$

So the problem of calculating plastic zone sizes is reduced to the calculation of the stress intensity factor K_{Dug}. This is done here by use of the weight function method for a generalized Dugdale model.

THE WEIGHT FUNCTION METHOD

The weight function method as described by Rice (12) allows to calculate the stress intensity factor for an arbitrary actual loading case from a given reference loading case. Cruse and Besuner (13) have generalized this method to calculate stress intensity factors in a weighted average.

The whole procedure of getting the weight function for a semi-elliptical surface crack is outlined in detail in (11). The basic equation for calculating stress intensity factors at the deepest point of a semi-elliptical crack loaded by σ_{new} (x,y) is

$$\overline{K_A} = \frac{H}{\overline{K_{rA}}} \int_S \sigma_{new}(x,y) \frac{\partial u_r(x,y,a,c)}{\partial S_A} dS \tag{2}$$

and at the two surface points

$$\overline{K_B} = \frac{H}{\overline{K_{rB}}} \int_S \sigma_{new}(x,y) \frac{\partial u_r(x,y,a,c)}{\partial S_B} dS \tag{3}$$

$\overline{K_{rA}}$ and $\overline{K_{rB}}$ are the reference stress intensity factors proposed by Newman and Raju (14) and averaged as follows

$$\overline{K_{rA}^2} = \frac{1}{\Delta S_A} \int_{\Delta S_A} K_r^2(\gamma)\ d\ [\Delta S_A(\gamma)] \tag{4}$$

$$\overline{K_{rB}^2} = \frac{1}{\Delta S_B} \int_{\Delta S_B} K_r^2(\gamma)\ d\ [\Delta S_B(\gamma)] \tag{5}$$

ΔS_A and ΔS_B are explained in Fig. 2. S is the crack area. The determination of u_r is described in detail in (11). Now σ_{new} (x,y) has to be specified as the strip load of the Dugdale model acting in the yield zone.

THE GENERALIZED DUGDALE MODEL FOR SURFACE CRACKS

Figure 3 shows the geometry considered. Applying the results from the foregoing sections the yield zones are calculated for an external stress σ from the requirements of vanishing stress singularity at the deepest point of the crack

$$\overline{K_{Dug}^A} = \overline{K_{rA}} \tag{6}$$

which results in d_A (σ). At the surface

$$\overline{K_{Dug}^B} = \overline{K_{rB}} \tag{7}$$

results in $d_B(\sigma)$.

$\overline{K_{rA}}$ and $\overline{K_{rB}}$ are again the reference stress intensity factors after Newman, Raju (14) expressed as weighted averages as proposed by Cruse and Besuner (13).

$\overline{K_{Dug}^A}$ and $\overline{K_{Dug}^B}$ are the stress intensity factors due to the strip load distribution (Fig. 3). For the value of the strip load $\sigma_f = 1.15\ \sigma_y$ is used. These stress intensity factors are calculated with Eq. (2) and (3) as follows

$$\overline{K_{Dug}^A} = \frac{4H\sigma_f}{\pi c\ \overline{K_{rA}}} \int_{y=0}^{c} \int_{x_1(y)}^{x_2(y)} \frac{\partial u_r(x,y)}{\partial a} dx\, dy \tag{8}$$

and

$$\overline{K_{Dug}^B} = \frac{4H\sigma_f}{\pi a\ \overline{K_{rB}}} \int_{y=0}^{c} \int_{x_1(y)}^{x_2(y)} \frac{\partial u_r(x,y)}{\partial c} dx\, dy \tag{9}$$

The values x_1 (y) and x_2(y) are defined by the inner and outer elliptical contours in Fig. 3.

The numerical procedure is the following:

1. Define the real crack a_o, c_o.
2. Define the yield zone d_A at the deepest point.
3. Make a plausible assumption for the initial value of the yield zone length d_B at the surface.
4. Calculate $\overline{K^A_{Dug}}$ with Eq. (8).
5. Calculate the external stress which is necessary to obtain the yield zone lengths d_A and d_B by use of Eq. (6).
6. Calculate $\overline{K^B_{Dug}}$ with Eq. (9).
7. Check whether Eq. (7) is satisfied. If not, correct d_B and return to point 4 until convergence is reached.

The result are the yield zone lengths d_A, d_B for an externally applied stress σ. This method includes the assumption of an elliptical contour of the yield zone between the two values d_A and d_B. This seems to be a reasonable approach to predict ligament plastification in the sense that $d_A = t-a_o$. But ligament yielding does not necessarily mean that the ligament has to fail by rupture.
Strain-hardening may increase significantly the external loads necessary to produce a leak.
Therefore, the next chapter will describe the effect of strain-hardening in the yield zone on the limit load which has to be applied for ligament rupture.

PREDICTION OF LIGAMENT RUPTURE WITH A CRITICAL YIELD ZONE STRESS DISTRIBUTION

Here a modification of the method described before is presented allowing to evaluate plastic limit loads while considering the effects of strain hardening. Figure 4 shows a plausible actual stress distribution along the crack periphery just before ligament rupture.

Here, the stress in the ligament at the deepest point of the crack and along the crack front is the true ultimate tensile strength σ_u. At the surface the stress in the plastic zone decreases linearly from σ_u to σ_y. The stress $\sigma(x,y)$ in the whole plastic zone is obtained by linear interpolation (see Figure 5) and can be described by the following equation

$$\sigma(x,y) = \sigma_u + \left[\sigma_y - \sigma_u + \frac{2}{\pi}(\sigma_u - \sigma_y)\arctan\left(\frac{x}{y}\right)\right] \cdot \sqrt{\frac{(x-x_i)^2 + (y-y_i)^2}{(x_o-x_i)^2 + (y_o-y_i)^2}} \qquad (10)$$

with the intersection points (x_i, y_i) and (x_o, y_o)

$$x_i = \sqrt{\frac{1}{\left(\frac{1}{a_o}\right)^2 + \left(\frac{x}{y}\right)^2 \left(\frac{1}{c_o}\right)^2}} \qquad (11)$$

$$y_i = c_o \sqrt{1 - \left(\frac{x_i}{a_o}\right)^2} \qquad (12)$$

and

$$x_o = \sqrt{\frac{1}{\left(\frac{1}{a}\right)^2 + \left(\frac{y}{x}\right)^2 \left(\frac{1}{c}\right)^2}} \qquad (13)$$

$$y_o = c \sqrt{1 - \left(\frac{x_o}{a}\right)^2} \, . \qquad (14)$$

There were also other kinds of stress distributions tested and it was found that the external stress necessary for ligament rupture is not very sensitive to the kind of distribution at the surface if one keeps to the assumption that $\sigma(x,y=0) = \text{const} = \sigma_u$ at the deepest point of the crack. *)
For this reason the above stress distribution was chosen which can be computed easily. Now Eqs. (8) and (9) can be rewritten to read

$$\overline{K^A_{Dug}} = \frac{4H}{\pi c \overline{K^A_r}} \int_{y=0}^{c} \int_{x_1(y)}^{x_2(y)} \sigma(x,y) \frac{\partial u_r(x,y)}{\partial a} \, dx \, dy \qquad (15)$$

and

$$\overline{K^B_{Dug}} = \frac{4H}{\pi a \overline{K^B_r}} \int_{y=0}^{c} \int_{x_1(y)}^{x_2(y)} \sigma(x,y) \frac{\partial u_r(x,y)}{\partial c} \, dx \, dy \qquad (16)$$

The subsequent procedure is the same as before. For $\sigma(x,y)$ Eq. (10) is used here.

RESULTS AND DISCUSSION

Figure 6 shows a comparison of the measured external stresses σ_{exp}, which are necessary for plastification of the ligament, with the theoretically predicted stresses σ_{th}.

In Figure 7 the ligament rupture stresses from experiment σ_{exp} and theory σ_{th} are compared. Here strain-hardening is regarded. The agreement on the 45 degrees line is satisfactory. The hollow points are theoretical results determined by Hasegawa et al. (15). The experiments have been performed by Göring and Müller (16).

The material used was steel 90 MnV8 (1.2842) with a yield stress $\sigma_y = 407\ \text{Nmm}^{-2}$ and a true ultimate tensile strength $\sigma_u = 814\ \text{Nmm}^{-2}$.

*) The deepest point of the crack is at $x = a_o$, $y = 0$.

The measured crack geometries were between $a_o/t = 0.28$ and 0.81 and for $a_o/c_o = 0.24$ up to $a_o/c_o = 1.03$.

CONCLUSIONS

1. The plastification of the ligament can be described satisfactory without regard of strain-hardening of the material considered in the present paper.

2. To predict ligament rupture it is necessary to include the effect of strain-hardening. A generalized Dugdale model describes the leak loads very well.

3. The weight function method is a very effective way to describe also such complex failure mechanisms.

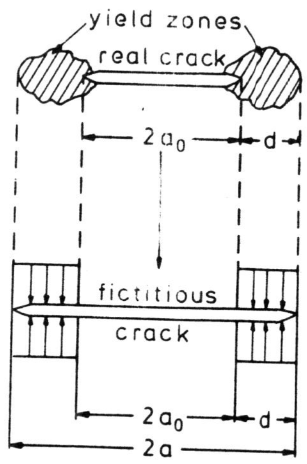

Fig. 1 Illustration of the Dugdale Model

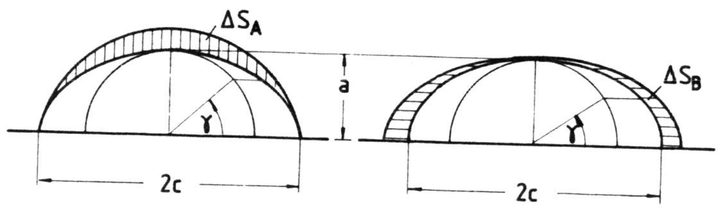

Fig. 2 Incremental changes in crack area for the calculation of averaged $\overline{K_A}$ and $\overline{K_B}$.

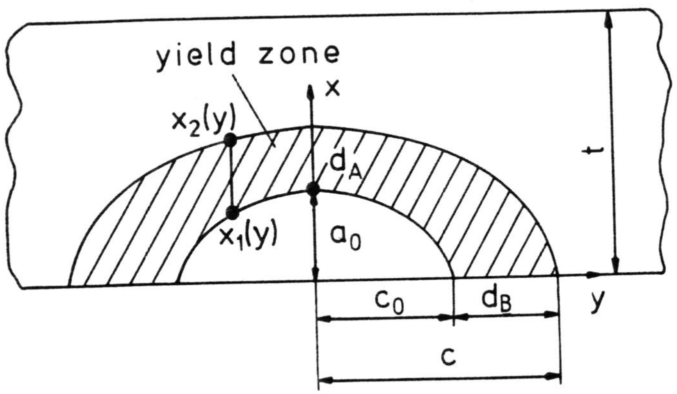

Fig. 3 Surface crack with semi-elliptical yield zone in the generalized Dugdale Model

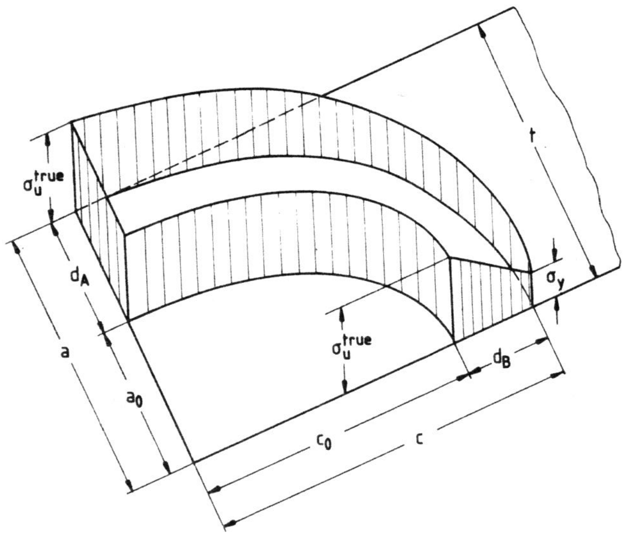

Fig. 4 Assumed yield stress distribution in the plastic zone ahead a semi-elliptical crack just before ligament rupture

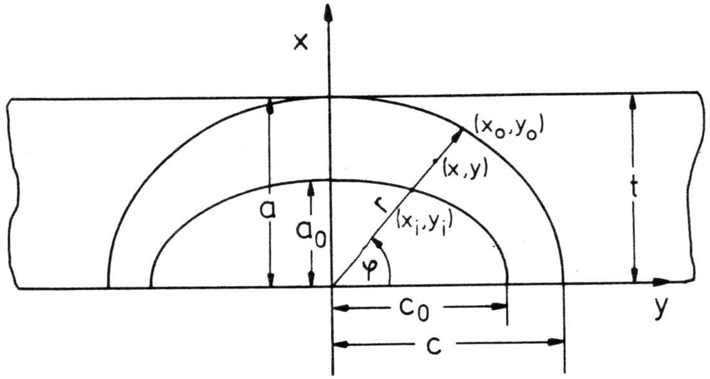

Fig. 5 Illustration of the symbols used for the description of the yield stress distribution varying linearly with φ and r.

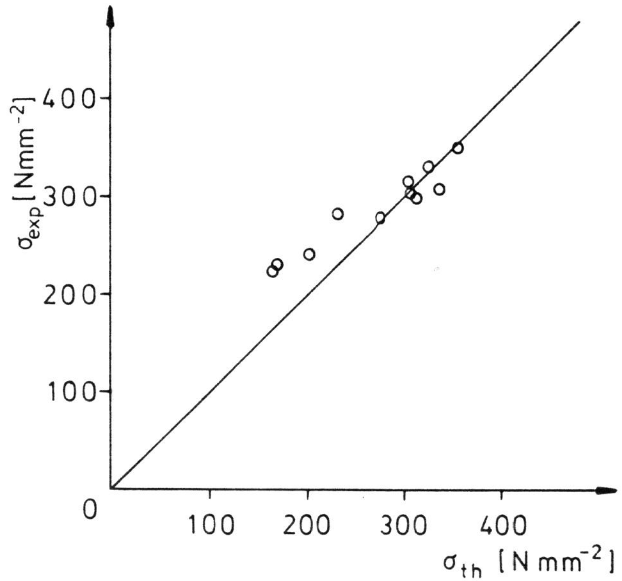

Fig. 6 Comparison of experimentally measured with theoretically predicted stresses for ligament yielding.

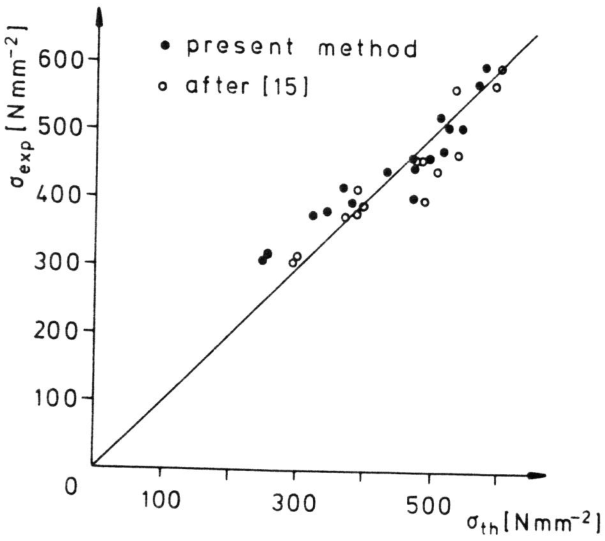

Fig. 7 Comparison of experimentally measured and theoretically predicted external stresses at ligament rupture

SYMBOLS

a_o, c_o: Semi-axis of the real crack (mm)

a, c : Semi-axis of the fictitious crack (mm)

d_A : Length of plastic zone at point A (mm)

d_B : Length of plastic zone at point B (mm)

$\overline{K_A}$: Stress intensity factor in weighted average at point A due to actual stress (Nmm$^{-3/2}$)

$\overline{K_B}$: Stress intensity factor in weighted average at point B due to actual stress (Nmm$^{-3/2}$)

$\overline{K_{Dug}^A}$: Stress intensity factor in weighted average at point A due to yield stress loading (Nmm$^{-3/2}$)

$\overline{K_{Dug}^B}$: Stress intensity factor in weighted average at point B due to yield stress loading (Nmm$^{-3/2}$)

$\overline{K_{rA}}$: Stress intensity factor in weighted average at point A due to external reference loading (Nmm$^{-3/2}$)

$\overline{K_{rB}}$: Stress intensity factor in weighted average at point B due to external reference loading (Nmm$^{-3/2}$)

K_r : Reference stress intensity factor (Nmm$^{-3/2}$)

K_{Dug} : Stress intensity factor of the Dugdale Model due to strip load (Nmm$^{-2/3}$)

σ_u : Ultimate tensile stress

σ_{th} : Theoretically predicted external stress (Nmm^{-2})

σ_{exp} : Measured external stress (Nmm^{-2})

σ_{new} : Actual applied stress (Nmm^{-2})

ΔS_A : Increment of crack area at point A (mm^2)

ΔS_B : Increment of crack area at point B (mm^2)

S : Crack surface (mm^2)

u_r : Reference displacement field (mm)

x, y : Cartesian coordinates (mm)

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