# FATIGUE LIFE PREDICTION OF PLAIN CONCRETE SUBJECTED TO UNIAXIAL TENSION AND TENSION-COMPRESSION

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From constant-amplitude tests on plain concrete subjected to uniaxial repeated tension and alternating tension-compression, a marked relation was deduced between cyclic secondary creep rate and time to failure. This relation is applied to improve the estimation of fatigue life of concrete subjected to program loading and variable amplitude loading. The tests are described and the results are presented and discussed.

## INTRODUCTION

Life, safety and durability of concrete structures are related to the initiation and propagation of cracks which mainly depend on the tensile properties of concrete. These properties, however, are affected by the type of loading such as dynamic loadings which are relevant to for instance off-shore structures, bridges and pavements. In that cases information is needed on fatigue behaviour of concrete.

During the last few years uniaxial fatigue tests on plain concrete have been performed in the Stevin Laboratory of the Delft University of Technology. Besides the effect of repeated tension also the effect of stress reversals on life has been studied. Details of these investigations have been reported in Cornelissen and Reinhardt (1), Cornelissen (2) and in Cornelissen and Reinhardt (3). The results of the constant-amplitude tests were presented in S-N diagrams in which the number of cycles to failure is related to the maximum stress-strength level of the loading cycles at given minimum stress-strength level. However, because of the inherent variability of concrete strength properties and consequently of the stress-strength levels, a lot of scatter was observed in these diagrams, as is illustrated in Fig. 1. This scatter results in unreliable fatigue life prediction.

In order to improve this prediction a method based on the development of cyclic strain during fatigue was applied. Also for the estimation of number of cycles to failure in program loading and variable amplitude loading tests, this method proved to be adequate.

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## EXPERIMENTS

### Specimens

The tests were carried out on tapered cylindrical specimens ($\phi$ 120 x 300 mm²). The concrete was composed of 325 kg/m³ rapid-hardening Portland cement type B (according to The Netherlands Standards), 1942 kg/m³ river gravel with maximum grain size of 16 mm. The water-cement ratio was 0.50. After a curing period of 14 days in water the specimens were stored in the laboratory ("drying" specimens) or wrapped in plastic sheeting ("sealed" specimens); they were tested in the fifth week after casting. Every batch consisted of twelve cylindrical specimens and six standard cubes (150 mm³). Six cylindrical specimens were used to determine the static tensile strength to which the stresses in the dynamic tests were referred. The compressive stresses were referred to the compressive cube strength. The average cube strength proved to be 47.3 N/mm² (v = 3.6%); the average tensile strength 2.45 N/mm² (v = 7.6%) for "drying" specimens, and 2.88 N/mm² (v = 3.7%) for "sealed" specimens.

### Testing equipment

The testing equipment which has been described in detail in (1), (2) and (3), consisted of a loading system and a control/measuring system (see Fig. 2).

The specimen was placed between two swivel heads, provided with pre-stressed spherical bearings to obtain a continuous loading signal in the tests with stress reversals. The capacity of the actuator was 100 kN. The longitudinal deformations of the specimen were measured with two LVDT's (HBM type W1T3) over a base of 254 mm.

The control/data unit based on a micro-computer generated the loading signal for constant-amplitude, program loading and variable amplitude loading tests. This unit was applied to record stress and strain data during the tests as well.

In Fig. 2 the temperature chamber in which the temperature was 21 °C (± 0.2 °C) is indicated too. This chamber was necessary because the development of the deformation (creep, expansion) is strongly dependent on the temperature.

## CYCLIC CREEP RATE

Various types of constant-amplitude tests have been performed including repeated tensile tests, tests with stress reversals, tests on "drying" and on "sealed" specimens. The frequency, however, was mainly 6 Hz. In all these tests the longitudinal strain was recorded from which the strains at maximum stress were derived. In Fig. 3 some typical examples are shown. As can be seen the maximum strain ($\varepsilon_{max}$) increases rapidly during the first 10% of total life, after that in the secondary part, there is a gradual increase of strain during about 80% of life, followed by a strong increase until failure.

For metals it is well known that a relation exists between the minimum of the strain rate in the secondary part of the creep curve and life attained (Taira et al. (4)). For concrete the strain rate in the secondary stage proves to be nearly constant. So for the constant-amplitude tests the secondary cyclic creep velocity was plotted against fatigue life. In Fig. 4 the relation is shown. The regression line based on about 150 results can



be expressed as:

$$\log t_f = -4.02 - 0.89 \log \dot{\varepsilon}_{sec} \qquad (1)$$

In this equation $t_f$ is in seconds and secondary creep velocity $\dot{\varepsilon}_{sec}$ is per second.

It can be concluded that a marked relation exists between log $\dot{\varepsilon}_{sec}$ and log $t_f$. Because $\dot{\varepsilon}_{sec}$ represents the actual state of the specimen under load, a better prediction of $t_f$ is possible than on the basis of stress-strength levels which have to be estimated. Equation 1 turned out to be independent of type of test, which could be verified for fatigue tests as well as for tensile creep tests (2). As indicated by Sparks (5), however, the relation is dependent on type of concrete.

Equation 1 will be applied to predict fatigue life in program loading and variable amplitude loading tests.

## PROGRAM LOADING TESTS

### General

In order to translate the results of constant-amplitude experiments into actual situations with complex loadings, Miner's rule is often applied. According to this rule the damage contribution M due to a number of n cycles is the sum of the damage contributions of each individual cycle. When the total damage becomes equal to unity it is presupposed that failure occurs. This rule can be written as a formula:

$$M = \sum_{i=1}^{n} \frac{1}{N_i} = 1 \qquad (2)$$

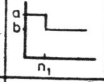
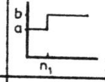
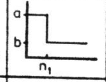
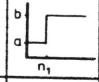
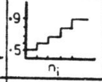
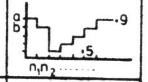
In this formula $N_i$ is the number of cycles to failure in a constant-amplitude test with maximum and minimum levels according to cycle i. Although there are some fundamental objections to this rule, a more practical rule is not available at present.

For concrete in repeated tension and in alternating tension-compression, Miner's rule was checked in program loading tests, in which specimens are subjected to a number of successive stress blocks characterized by minimum and maximum levels of the load cycles and a certain number of cycles per block. An overview of the different series investigated is presented in Table 1. It is remarked that in a given test only the maximum level was changed. Sequence effects were studied by means of starting a test with high maximum level followed by a lower maximum level and vice versa. The tests were executed on "sealed" specimens with loading frequence of 6 Hz.

### Miner's rule based on secondary creep rate

For 90 test results formula 2 was evaluated. The values of $N_i$ were taken from the appropriate S-N diagrams (1). This, however, resulted in considerable scatter in the Miner sums (values from 0.20 to 87.00) (3). As pointed out before a better estimation of life provides $\dot{\varepsilon}_{sec}$. For that purpose in each loading block i, $\dot{\varepsilon}_{sec}(i)$ was determined according to Fig. 5, and log $N_i$ and M were calculated using equations 1 and 2. The resulting Miner sums are presented in Table 2. Following this procedure a significant reduction of the scatter was realized, because the actual condition of the specimen was taken into account. That is also the reason why sequence effects are not expressed in the Miner sums of the various types of tests; the effect of preceding loading blocks is represented in $\dot{\varepsilon}_{sec}(i)$. It can be concluded that life prediction based on Miner's rule can be improved by the evaluation of secondary creep rate. It also emerges from the results that M = 1 forms a safe criterion in general.

Table 1. Summary of the investigated types of program loading tests.

| Series | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Minimum stress-level |   |   |   |   |   |   |
| 0.15 $f'_{cm}$ | a = 0.75* <br> b = 0.70 <br> $n_1$ = 510 | 0.70 <br> 0.80 <br> 510 | 0.70 <br> 0.40 <br> 510 | 0.40 <br> 0.70 <br> 327600 | 0.50-0.90 <br> $n_i$ = 4092 | a = 0.75 <br> b = 0.70 <br> $n_1 = n_2$ = 510 <br> $n_3$ .... = 4092 |
| 0.05 $f'_{cm}$ | a = 0.75 <br> b = 0.70 <br> $n_1$ = 510 | 0.70 <br> 0.80 <br> 510 | 0.70 <br> 0.50 <br> 510 | 0.50 <br> 0.80 <br> 327600 | 0.50-0.90 <br> $n_i$ = 4092 | a = 0.75 <br> b = 0.70 <br> $n_1 = n_2$ = 510 <br> $n_3$ .... = 4092 |
| 0.00 | a = 0.80 <br> b = 0.70 <br> $n_1$ = 510 | 0.70 <br> 0.80 <br> 4092 | 0.70 <br> 0.50 <br> 510 | 0.50 <br> 0.80 <br> 327600 | 0.50-0.90 <br> $n_i$ = 4092 | a = 0.80 <br> b = 0.70 <br> $n_1$ = 510 <br> $n_2$ .... = 4092 |
| 0.20 $f_{ctm}$ | a = 0.80 <br> b = 0.75 <br> $n_1$ = 510 | 0.70 <br> 0.80 <br> 510 | 0.70 <br> 0.50 <br> 510 | 0.50 <br> 0.80 <br> 327600 | 0.50-0.90 <br> $n_i$ = 4092 | a = 0.80 <br> b = 0.75 <br> $n_1 = n_2$ = 510 <br> $n_3$ .... = 4092 |
| 0.40 $f_{ctm}$ | a = 0.80 <br> b = 0.75 <br> $n_1$ = 510 | 0.80 <br> 0.90 <br> 510 | 0.70 <br> 0.60 <br> 510 | 0.60 <br> 0.90 <br> 327600 | 0.50-0.90 <br> $n_i$ = 4092 | a = 0.80 <br> b = 0.75 <br> $n_1 = n_2$ = 510 <br> $n_3$ .... = 4092 |

* Fraction of static tensile strength

Table 2. Ranges of Miner sums for program loading tests. (Miner sums based on secondary creep velocity).

| minimum stress level | series* | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 4 | 5 | 6 |
| 0.15 $f'_{cm}$ | 0.64-1.12 | 0.51-0.91 | 0.39-1.00 | 0.46-1.66 | 1.02-1.53 |
| 0.05 $f'_{cm}$ | 0.90-1.16 | 0.92-1.73 | 0.41-0.87 | 1.16-1.39 | 0.68-1.21 |
| 0.00 | 1.56-3.00 | 1.22-1.60 | 1.19-2.20 | 1.74-2.33 | 1.16-2.53 |
| 0.20 $f_{ctm}$ | 1.31-2.72 | 1.09-2.38 | 0.65-1.79 | 1.59-2.77 | 2.04-2.18 |
| 0.40 $f_{ctm}$ | 1.29-3.13 | 1.48-4.17 | 0.68-1.59 | 2.35-2.91 | 2.17-2.75 |

* see Table 1.

## VARIABLE AMPLITUDE LOADING TESTS

### General

A further step to simulate actual loading conditions is formed by variable amplitude loading tests, in which the amplitude of each loading



cycle may be different. The frequency, however, is held constant (in the present tests 6 Hz). The variation of the amplitude values was described by a probability density function namely a Rayleigh distribution of the peaks. The sequence of the cycles was of a pseudo-random character, because the "random" generator produced numbers in a arbitrary but reproducible way. The tests were executed with permutations of 1024 amplitude values, which were repeated until failure.

An overview of the types of tests is given in Fig. 6. Distinction can be made between three main types:

Type 1, comprising tests with constant maximum stress level.
Type 2, comprising tests with constant mean stress.
Type 3, comprising tests with constant minimum stress level.

For each type various stress levels as indicated were investigated; in general six tests per combination. Repeated tensile stress as well as stress reversals were realized. Sealed specimens were used for all these tests.

### Life prediction for variable amplitude loading tests

Miner sums were derived from the results of the variable amplitude tests according to formula 2. $N_i$ was estimated from the appropriate S-N diagrams. In Table 3 the calculated ranges of these Miner sums are presented for six test series. Besides the differences between these series, it can be seen that the results scatter considerably.

Table 3. Ranges of Miner sums for variable amplitude loading tests. (Miner sums based on S-N diagrams).

| Type* | min. | max. | Miner sum |
|---|---|---|---|
| 1 | 0.0 | 0.8 $f_{ctm}$ | 0.07- 0.43 |
| 1 | 0.1 $f'_{cm}$ | 0.7 $f_{ctm}$ | 0.01- 0.64 |
| 3 | 0.0 | 0.8 $f_{ctm}$ | 0.75-12.04 |
| 3 | 0.1 $f'_{cm}$ | 0.8 $f_{ctm}$ | 1.53- 5.33 |
| 3 | 0.2 $f_{ctm}$ | 0.8 $f_{ctm}$ | 0.11->58.83 |
| 3 | 0.1 $f'_{cm}$ | 0.7 $f_{ctm}$ | 1.67- 9.83 |

* see Fig. 6.

For a more reliable estimation of fatigue life again the development of maximum strain was taken into account. Because most damage will occur during the load cycles with maximum amplitude, the corresponding strain was recorded from which secondary creep rate was calculated as illustrated in Fig. 7. Some typical results were plotted against time to failure in Fig. 8, which shows that $\dot{\varepsilon}_{sec}$ determines time to failure for these types of tests too. Moreover the results indicate that the relation obtained (equation 1), describes the behaviour in an adequate manner, as denoted by the lines in this diagram. Obviously life is predicted by secondary creep rate irrespective of stress history. In this approach, however, it is assumed that creep deformation is mainly caused by the accumulation of damage such as initiation and propagation of cracks.

## CONCLUSIONS

1. Dispersion in S-N diagrams, caused by the variability of concrete strength properties, results in unreliable fatigue life prediction.

2. A marked relation exists between secondary creep rate and time to failure. This relation is independent of loading history.

3. Fatigue life prediction can be improved by the evaluation of secondary creep rate which represents the actual state of the specimen under load.

4. Miner's rule can be applied to estimate fatigue life of concrete subjected to repeated tensile or alternating tensile-compressive stresses, especially if partial damage is deduced from secondary creep rate. Program loading tests indicate that M = 1 is a safe criterion in general.

## SYMBOLS

$f'_{cm}$ = average compressive cube strength (N/mm²).
$f_{ctm}$ = average pure tensile strength (N/mm²).
$M$ = Miner sum.
$N$ = number of cycles to failure.
$n$ = number of cycles.
$t_f$ = time to failure (sec.).
$v$ = coefficient of variation (%).
$\varepsilon_{max}$, $\varepsilon_{min}$ = strain at maximum, minimum stress level.
$\dot{\varepsilon}_{sec}$ = secondary cyclic creep rate (per sec.).
$\sigma_{max}$, $\sigma_{min}$ = maximum, minimum stress level (N/mm²).

## LIST OF REFERENCES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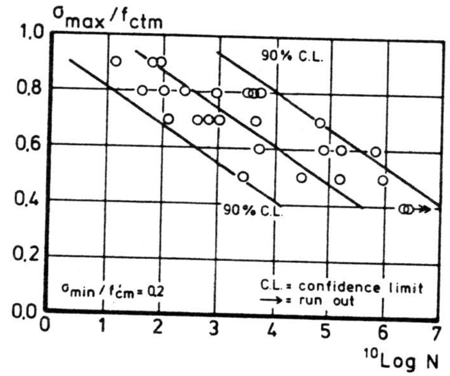


Fig. 1. S-N diagrams for concrete subjected to stress reversals ($\sigma_{min}$ = 0.2 $f'_{cm}$).

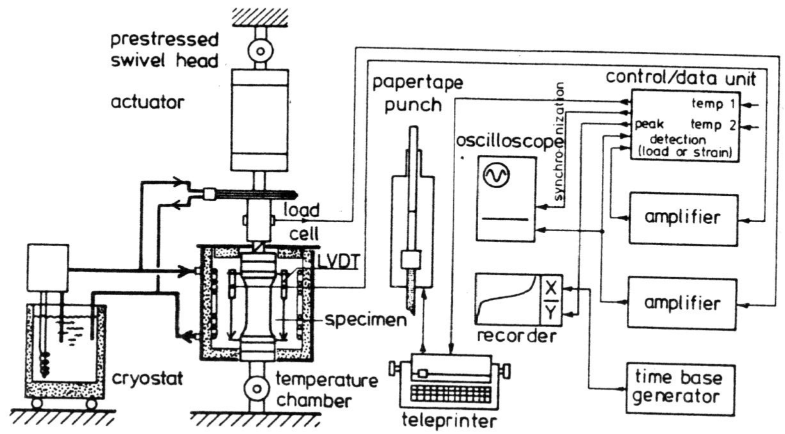


Fig. 2. Schematic view of testing equipment.

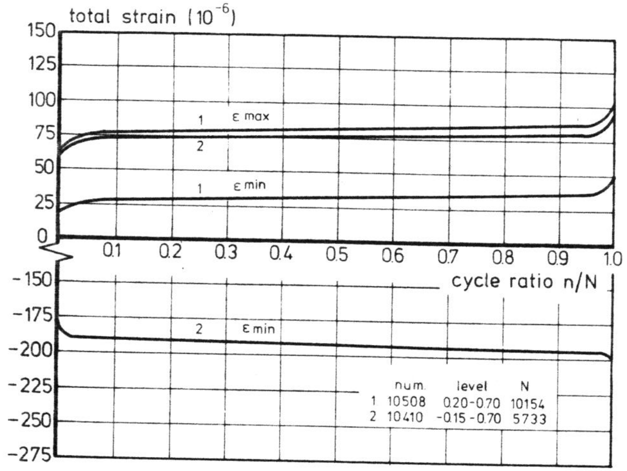


Fig. 3. Development of maximum and minimum strains for a repeated tensile test and a tensile-compressive test.

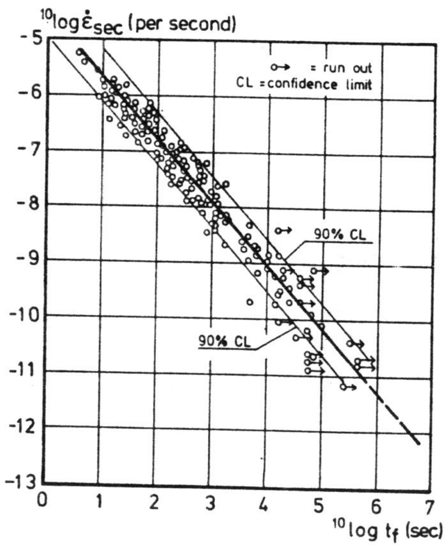


Fig. 4. Relation between secondary creep rate ($\dot{\varepsilon}_{sec}$) and time to failure $t_f$.



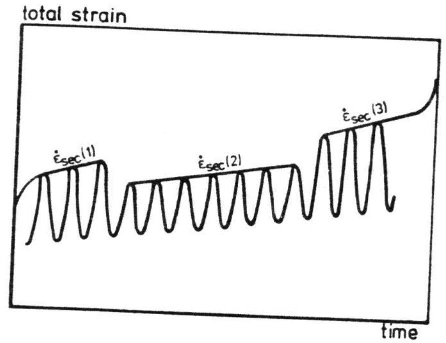


Fig. 5. Secondary creep rate in program loading tests.

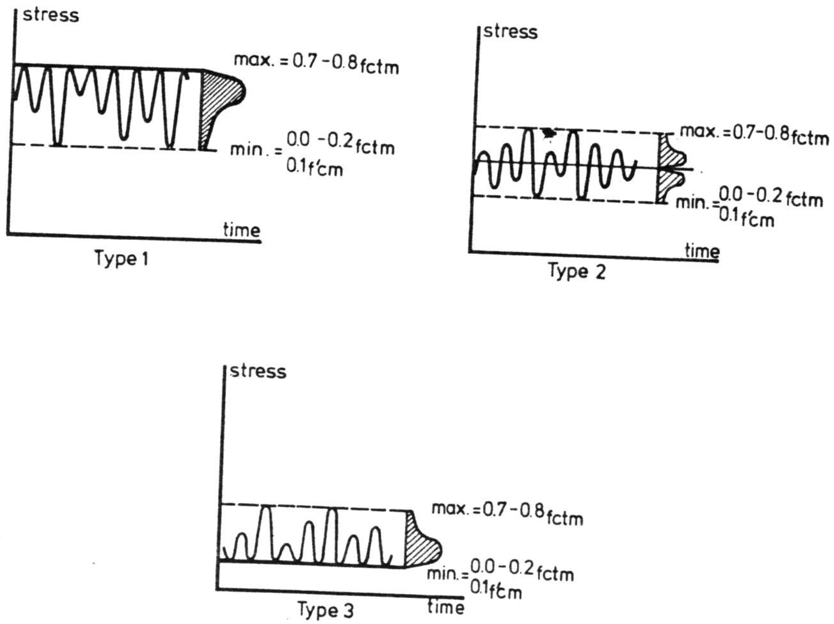


Fig. 6. Investigated types of variable amplitude tests.

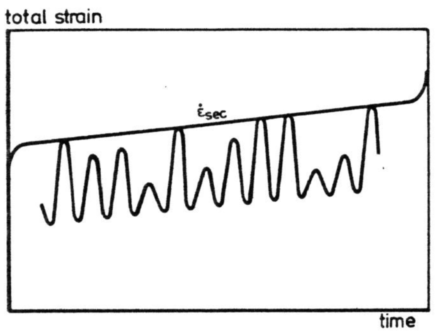


Fig. 7. Secondary creep rate in variable amplitude tests.

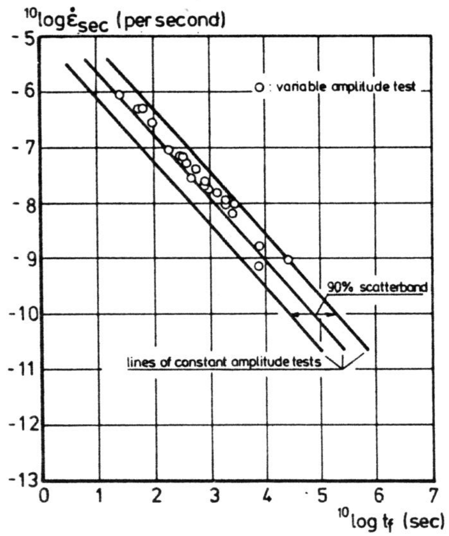


Fig. 8. Relation between secondary creep rate and time to failure for variable amplitude tests.