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ABSTRACT

An attempt to predict the deformed shapes of GRP rectangular plates with all four edges built-in and subjected to uniform pressure is described.

Using values of flexural and tensile modulus for each type of GRP board at 5×10^5 seconds and with a finite-element package 'PAFEC', a computer programme was prepared, from which the deformed shape of each board could be predicted.

In order to compare the predicted shape of the boards with that obtained experimentally, a rig was built and some tests were carried out.

1. INTRODUCTION

The lack of a consistent large displacement theory for composites and the complexity and number of possible solutions presented by those theories, as well as the occasional non-linear behaviour of the material under load, were the main reasons, for adopting a finite-elements package 'PAFEC' so as to solve the proposed problem.

The intention behind the work described in this paper was to show the practical and easily usable design tool, the objective being to determine the deformed shape of plates with built-in edges subjected to long-term lateral pressure.

2. MATERIALS CHARACTERISTICS

Four GRP boards (I-b, I-c, II-b and II-c) were used during these tests, and their characteristics are given in Table 1.

One PVC plate was also tested.

The overall dimensions and the average thickness of each board, together with the conditions of test are presented in Table 2.

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Table 1

Characteristics of Materials Received

Board	Ref.	Layers	Resin/Glass ****	Cure	Post-Cure
I, I-a I-b, I-c	BDV/F	7 300g/m ² cfm*	2.1 to 1	24h at RT***	16h at 40°C
II-a II-b, II-c	BDV/F/WR	(3 300g/m ² cfm (2 600g/m ² wr)**	1.3 to 1	24h at RT***	16h at 40°C

- * c.f.m. - continuous filament mat
 ** w.r - woven roving
 *** RT - room temperature
 *** Resin - polyester CRYSTIC 272 E

Table 2

Dimensions and Test Conditions of the GRP and PVC Components Used in Test to Prove the 'Design with PAFEC Programme'

Board	Dimensions (mm)	Average Thickness (mm)	Tested In	Total pressure N/m ²
I-b	546.1 × 381	5.630	Air	24481
I-c	546.1 × 381	5.259	Water	24481
II-b	546.1 × 381	3.401	Air	24232
II-c	546.1 × 381	3.587	Water	16850
PVC	546.1 × 381	6.221	Air	24481

3. THE TEST RIG

A rig for testing rectangular plates with built-in edges and subjected to uniform pressure was specially design and built (Fig. 1).

The rig has the overall dimensions of 546.1 × 381 (mm).

The uniform pressure was applied by quickly raising to the required height (water head) a water container (1) connected to the tank by a polyethylene pipe (8), a level pipe being attached to the water containers so as to permit easy checking of the water level in order to keep it constant.

As the boards were to be tested either in air or with water in contact with the upper surface, it was required that water should not touch the bottom surface of the boards, and to comply with this requirement a thin polythene film was fixed between the bottom face of the board (5) and the bottom frame (15).

The deformed shape of the plates was recorded by a measuring device formed by a transducer (3) and a multipot (9), the former being a linear variable displacement transducer LVDT.

4. TEST PROCEDURE

The procedure used during these tests was as follows:

1. Grid the plate.
2. Determine the board's thickness at each point.
3. Bolt together the frame (2), the board (5), the rubber gaskets, the bottom frame (15), the steel base-plate and the support structure (7).
4. Put the assembly on the support table (6) and check that it is level.
5. Fill the 'rectangular' tank formed by the board, the bottom frame (15) and the steel base-plate with water.
6. Check the initial contour.
7. Raise the container (1) to the required height, thus instantaneously applying the required pressure.
8. Leave the pressure on for a week, and record the deformed shape at prescribed intervals.
9. Remove the pressure and allow for 'recovery'.
10. Cut the board so that thickness measurements can be made with a micrometer, thus checking previous measurements made in step 2.

The central deflection of each plate as against elapsed time was determined.

The PVC board wastested merely in order to check the applicability of Timoshenko's theory to large displacements of a plate subjected to uniform pressure, since PVC can be considered a homogeneous material.

5. PREDICTION OF THE DEFORMED SHAPES USING 'PAFEC'

In the finite element technique, the elastic continuum is approximated by an assemblage of finite elastic element subjected to the constraints existing in the continuum. Then element's stiffnesses are computed and combined into a master-stiffness matrix which can be used to solve the problem.

A detailed explanation of the technique used can be seen in PAFEC 75 (1).

5.1 The Large Displacements Analysis

As the present analysis of the proposed problem will involve displacements greater than half of the thickness, these can be considered large enough to modify the shape of the structure and therefore alter the stiffness. Hence a large displacement analysis has had to be considered.

The 'PAFEC' analysis of this type of problem involves the use of an iterative process. The load is divided into a series of sufficiently small increments and these are applied one at a time.

After each has been applied, the deflections caused are calculated by using the linear theory.

5.2 The Use of the 'PAFEC' Package

In the present analysis, only a quarter of the plate was considered, owing to its symmetry.

It was decided to compare the experimental and theoretical data at 5×10^5 seconds, thus allowing for the inclusion of previous creep data obtained from 4-point bending and 'Heavy-Duty' tensile creep rigs by Marques (2) and Darlington and Marques (3).

The procedure used was as follows:

Step 1: Define the maximum pressure to be applied to each plate, and consider the restraints to be imposed.

Step 2: Define the percentage of maximum pressure to be applied so that the displacements obtained are considered small (Table 2).

At this stage the type of element should be considered. It was decided to use the 8-node facet shell element.

Step 3: Define the number of elements to be used.

The quarter plate was divided into 4 elements with a total of 21 nodes.

Step 4: The boards tested can be considered as quasi-isotropic, thence the following procedure:

4.1 Boards I-b and I-c

Using the 4-point bending and the tensile isochronous stress/strain curves at 5×10^5 seconds for specimens from Board I-a in air or in water in contact with one face only (Fig. 2 and Fig. 3) determine the initial 'flexural' and 'tensile' modulus.

As the 'tensile' and 'flexure' modulus for Board I-a are very similar, it was decided to use the value of the tensile modulus (considered to be more accurate) in all incremental steps.

After the final step the actual E_s (secant modulus) for each element will be determined and new computer runs made if necessary.

As new boards (I-b and I-c) were used for the tests described in 4, a correction factor was determined (2) and had to be applied to the modulus in air and with water in contact with one face only as determined for Board I-a (see Table 3).

Table 3

Young's Modulus (GN/m^2) at 5×10^5 Seconds, Used by 'Design with PAFEC Programme'

Step Board	4	8
I-b	4.742	4.742
I-c	4.666	4.666
II-b	6.138	6.729
II-c	6.100	6.942
PVC	2.64	2.64

4.2 Boards II-b and II-c

The 'flexure' and 'tensile' modulus can be determined by using the 4-point bending and the tensile isochronous stress/strain curves (linear stress/strain curves) at 5×10^5 seconds for Board II-a specimens in air and with water in contact with one face only (Fig. 4 and Fig. 5)

As there are significant differences between the 'flexure' and 'tensile' modulus for laminates of this type of construction, the 'flexure' modulus was used at this stage, since bending is the only effect to be considered.

A correction factor had also to be determined for the modulus in air and with the water in contact with one face only of Boards II-b and II-c.

The values of the flexure modulus at 5×10^5 seconds are presented in Table 3.

Step 5: Run the 'PAFEC' programme

Step 6: With the displacements obtained in step 5, a new shape of the boards can be defined.

The boards will no longer be flat, as the nodes co-ordinates begin to be non-coplanar, and 3-nodes shell element has to be used.

At this stage it was necessary to define a new mesh. The quarter plate was then divided into 8 elements with a total of 9 nodes.

Step 7: Define the percentage of the maximum pressure to be applied in each increment (Table 2).

Step 8:

8.1 Boards I-b and I-c

Enter as data with the value of the 'tensile' modulus determined as indicated in step 4.1.

8.2 Boards II-b and II-c

Using the 5×10^5 seconds tensile isochronous stress/strain curves ($T = 23^\circ\text{C}$), determine the tensile creep modulus for specimens tested in air and with water in contact with one face only.

Apply the correction factors for Boards II-b and II-c determined in step 4.2, thus determining the tensile creep modulus in air and with water in contact with one face for Boards II-b and II-c respectively.

At this stage, membrane effects begin to show. As it was not possible to dissociate bending and membrane effects with the 'PAFEC' Large Displacement module, an approximation had to be made.

From previous trials (2) it has been found that for pressure of similar order of magnitude as the ones used here, the membrane effect would account for about 30% of the total effect. Hence the value of 'modulus' used to insert in the 'PAFEC' data was determined as follows:

$$E_{\text{PAFEC}} = \alpha \times E_{\text{tension}} + (1-\alpha) \times E_{\text{Bending}} \quad (1)$$

With $\alpha = 0.30$

The values of the Young's modulus used are presented in Table 3.

Step 9: Run the 'PAFEC' programme, using the large displacements module.

With this module, the displacements are calculated for the pressure corresponding to the size of each increment and are then added together.

After this run, it could be checked whether or not the use of initial tensile modulus for Boards I-b and I-c was appropriate.

It was found that the stresses could be considered in the linear region of the isochronous stress/strain curves for Boards I-b and I-c. Therefore, the use of 'the initial tensile modulus' considered in step 4 and 8 for Boards I-b and I-c was an acceptable approach.

With Boards II-b and II-c it was considered that the membrane effect would count for 30%, hence the modulus was used as expressed in equation 1. The membrane effect was reassessed by comparing the middle surface stresses with the total stresses at the top and bottom surface. The comparison made enabled new values for α (in equation 1) to be determined for each element. The new values of Young's modulus to be used in step 9 are presented in Table 4. For Boards II-b and II-c another computer run (correspondent to step 9) was the carried out.

For the PVC Board the procedure used was identical to the one described for Board I-b.

Table 4

Element Membrane Effect (%) and Young's Modulus (GN/m^2) - Reassessed Values

Element Board	1-2 B% E	3 B% E	4-8 B% E	5 B% E	6 B% E	7 B% E
II-b	37 6.87	62 7.39	52 7.12	62 7.39	42 6.98	42 6.98
II-c	32 6.94	59 7.73	40 7.2	48 7.42	32 6.94	40 7.2

6. COMPARISON BETWEEN THE EXPERIMENTAL AND PREDICTED VALUES

6.1 Prediction for Large Displacements of Plates by Timoshenko's Theory

The maximum displacement of a plate with all the edges built-in can be predicted by using Timoshenko's large displacement theory (Timoshenko and Krieger-(4)) which assumes the material to be homogeneous and isotropic and does not allow for thickness variations in the plate.

The deviation between Timoshenko's prediction and the experimental values are presented in Table 5. The position of the nodes is indicated in Fig.6.

Table 5

Deviation (%) Between the Predicted and Experimental Values

Node Board	1	2	3	4	\hat{i}^*	Maximum Deflection (mm) (Exp)
I-b	-6.5	-11.7	-15.7	-19.6	-25	6.974
I-c	+2.1	-6.9	-10.0	-15.6	-23	7.277
II-b	-7.6	-12.2	-16.3	-17.5	-31.6	9.938
II-c	+11.2	-11.3	-4.3	-10.7	-16.5	6.594
PVC	+1.5	-6.7	-9.84	-13.7	-16.0	8.020

* Timoshenko's Prediction

It is apparent that the best approximation was obtained with the PVC Board, which is the closest to a homogeneous and isotropic material.

It can be said that in general the larger percentage of thickness variation, the greater is the discrepancy between the theoretical and actual values.

Table 5 also presents the value of maximum deflection at 5×10^5 seconds obtained experimentally.

6.2 Prediction with the 'PAFEC' Programme

The deformations of the central line parallel to X axis of the different plates were plotted.

The experimental curves showed different deflections for symmetrical points which was due to the different thicknesses for nodes symmetrical in relation to X,Y axes.

The experimental and predicted values can readily be compared by an analysis of Table 5.

It is apparent that in the main reasonable agreement has been obtained for the points of maximum deflection. However, for points closer to the edges the prediction tends to deviate more from the experimental values.

An increased thickness variation towards the plate's edges, the general trend being a reduction towards the corner, can account for one of the reasons why the theoretical and actual values are further apart towards the edges.

In general, it can be said that reasonable predictions of the deformed shapes of GRP rectangular plates were obtained, allowing for the fact that the built-in edges may permit very slight rotation, owing to relaxation in the clamping system. The fact that a correction factor had to be used based in the test of a specimen taken from an area a good way from the centre of the plate is another error-effect to be discounted in judging the precision of the method used for predicting the deformed shapes of rectangular plates with edges built-in.

7. CONCLUSIONS

The deformed shape of GRP plates with clamped edges and subjected to uniform pressure was measured experimentally for the case of central deflection $\gg \frac{1}{2}$ plate thickness. Satisfactory prediction of this behaviour was obtained using the creep data for GRP samples previously obtained, and allowing for the change from bending to membrane stresses where necessary in the 'Design with PAFEC computer programme'.

The complete computer runs required the step by step change of type of finite element, type of analysis (i.e. small to large displacement analysis), and the use of different modulus values in each step. It is considered that these procedures developed and used in the 'Design with PAFEC computer programme' can now be applied with some confidence in the prediction of the performance of other GRP components.

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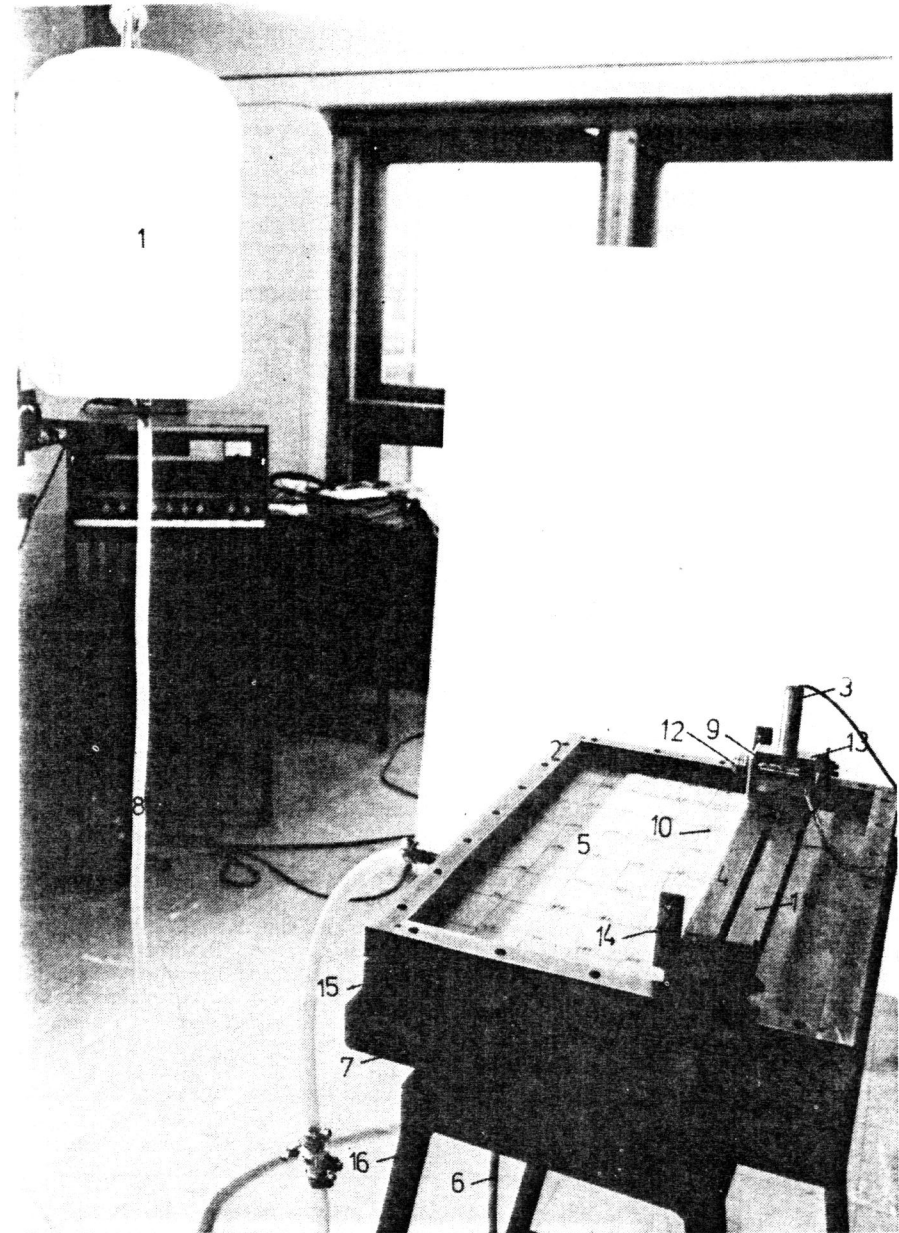


Fig. 1 - Caption on next page

Fig. 1 - Rig to test GRP panels with edges built-in and subjected to uniform pressure

- 1 - Water container
- 2 - Upper frame
- 3 - Transducer
- 4 - Sliding bar
- 5 - Board in test
- 6 - Drainage pipe
- 7 - Angle structure
- 8 - Pipe
- 9 - Multipot
- 10 - Nylon line
- 11 - Sliding bar
- 12 - Pulley
- 13 - Slide block
- 14 - Fixed peg
- 15 - Bottom frame
- 16 - Support

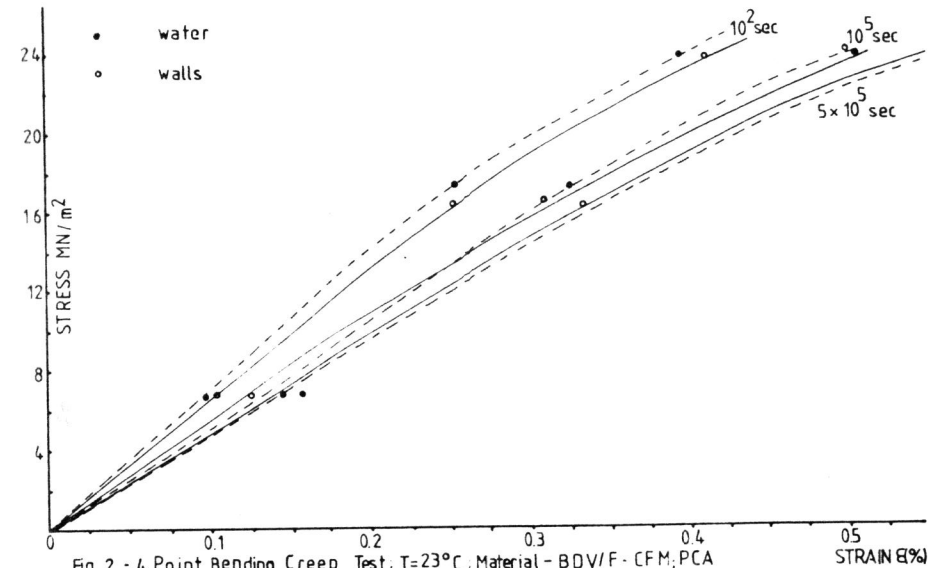


Fig. 2 - 4 Point Bending Creep Test, $T=23^{\circ}\text{C}$, Material - BDV/F-CFM; PCA
Isochronous stress/strain curves

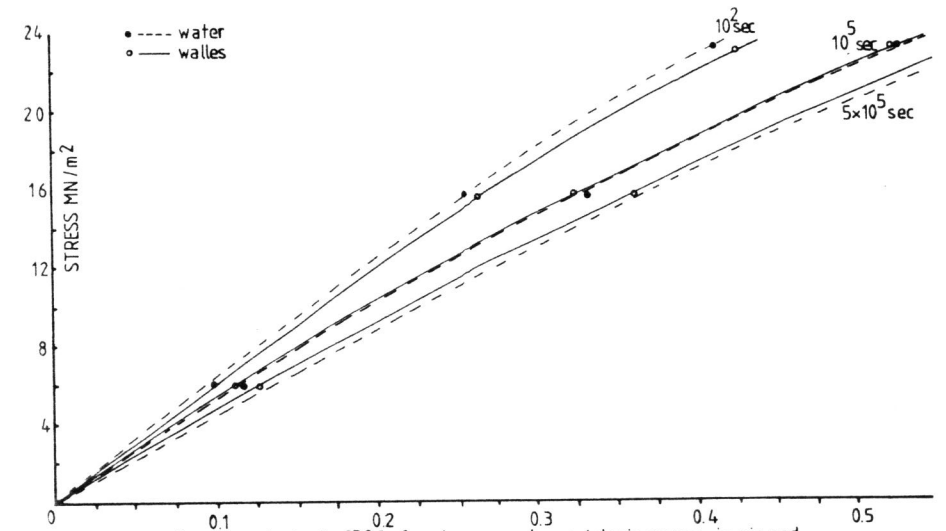


Fig. 3 Tensile creep test - $T=23^{\circ}\text{C}$. Isochronous stress/strain curves, in air and with one face in contact with water for specimens from Board I-a

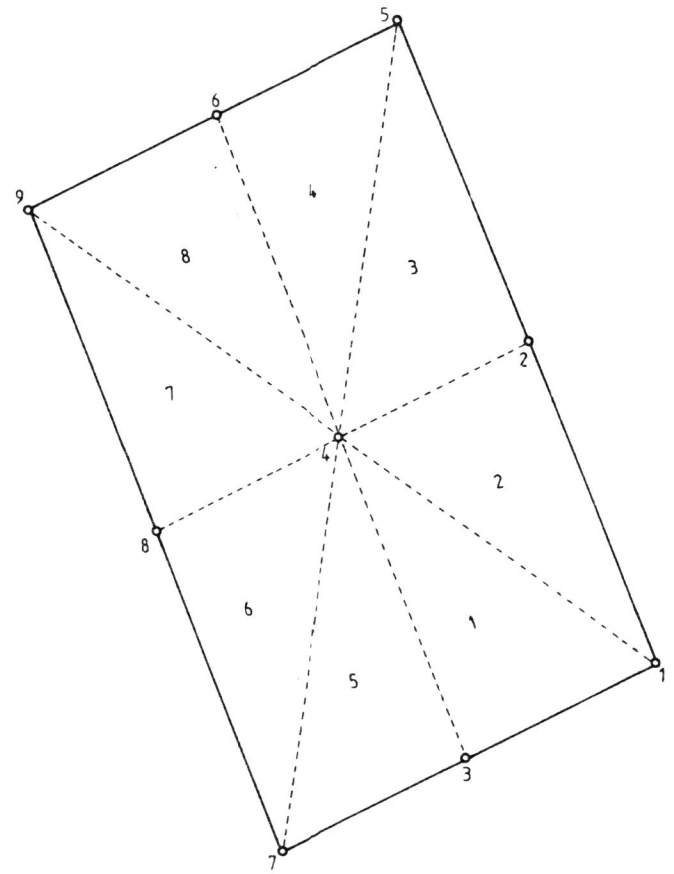
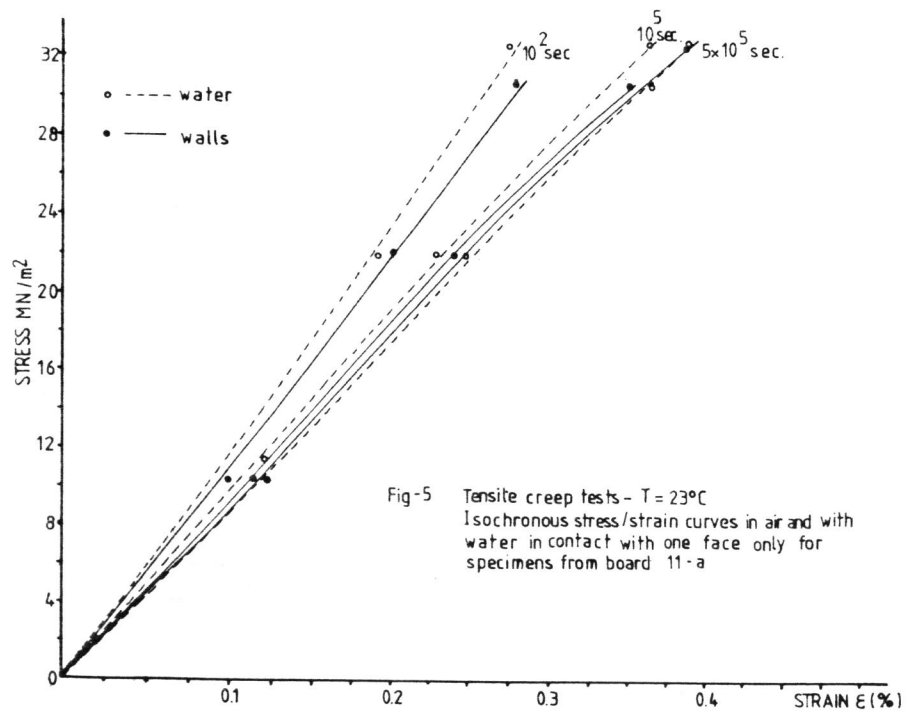
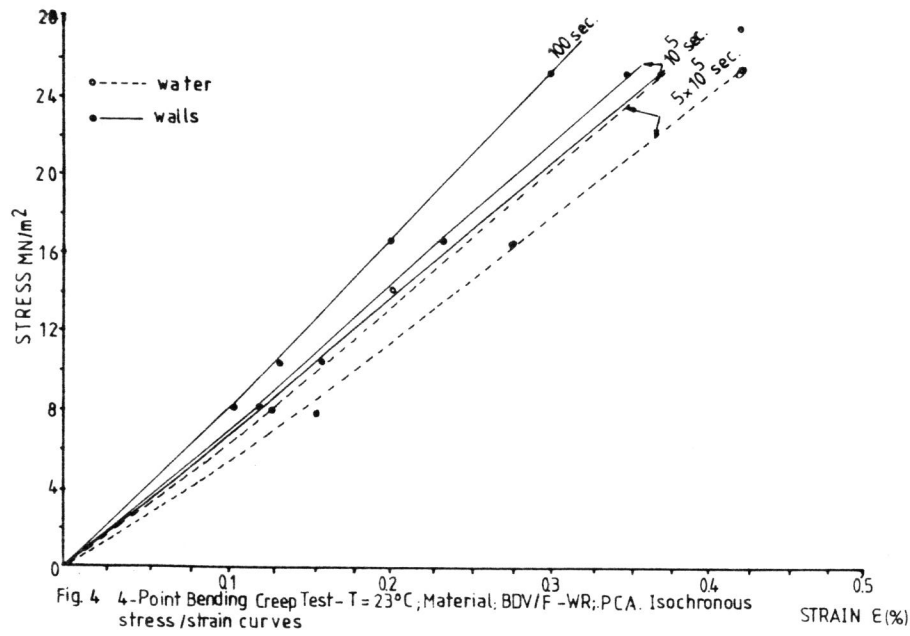


Fig. 6 - PAFEC mesh used in step 6 (1/4 plate)