CORRELATIONS BETWEEN THE ANISOTROPIES OF DEFORMABILITY, FAILURE AND THERMAL EXPANSION OF A GRANITE

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SUMMARY

This work was intended to assess whether the thermal expansion coefficient can be expressed by spatial laws as it occurs with most properties of rock meterials such as the modulus of deformability and the ultimate stress, and besides to correlate surfaces that define the anisotropies for the three properties mentioned.

Thus by means of laboratory tests on granite specimens obtained following nine spatial directions, values were obtained that made it possible to determine parameters defining the anisotropies for those three properties.

1. INTRODUCTION

Experience has shown that most properties of rock materials present anisotropic characteristics that can be expressed by spatial laws from the 2nd to the 8th degree. The anisotropies of deformability, permeability and compression ultimate stress have made the object of numerous studies, and significant correlations have been found between the principal directions of surfaces expressing these anisotropies as well as between values of their semi-axes. On the other hand, everything points to the existence of a close correlation between the above parameters, the fabrics of the material under study and the axes of symmetry of the crystals forming it; deviations found can be ascribed to phenomena subsequent to the formation of the crystals, such as tectonic movements and internal stresses.

The anisotropy of the thermal properties of materials has received little attention. In order to contribute to improving knowledge on this subject, LNEC (Laboratório Nacional de Engenharia Civil, Portugal) launched some studies for this purpose some time ago.

The considerations hereinafter are based on results obtained in laboratory tests carried out on specimens cut from a granite block that was taken out of the Cabril dam site, and concern the modulus of deformability, uniaxial compression ultimate stress and coefficient of thermal expansion.

2. ANISOTROPIES OF DEFORMABILITY, FAILURE AND THERMAL EXPANSION

From a granite block extracted from the Cabril dam site, prismatic specimens with the approximate size of 5 x 5 x 14 cm^3 were cut following nine

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directions in space (Fig. 1). The modulus of deformability, coefficient of thermal expansion and ultimate stress were determined on each specimen.

The modulus of deformability was calculated on basis of values obtained in uniaxial compression tests (Fig. 2), in which the maximum test load was 15.0 MP_a and unit strains produced were evaluated by means of strain gauges with 6 cm measuring base.

For the determination of the coefficient of thermal expansion use was made of an oven in which the specimens were subject to temperature variations, both these variations and length variations being measured (Fig. 3). The temperature variation which was about 60° C, was evaluated by means of thermocouples and a potentiometer, and the variation of length was measured by a suitable apparatus equipped with a 0.001 mm deflectometer.

As experience has clearly showed that the anisotropies of igneous rocks with reference to deformability and failure are well defined by spatial laws represented by 2nd degree expressions, of the ellipsoid type (1), an attempt was made to verify if the thermal expansion anisotropy of that type of rock complied with a law of the same type:

$$Ax^2 + By^2 + Cz^2 + 2Dxy + 2Exz + 2Fyz - 1 = 0 \qquad (1)$$

Thus by using the values of each of the three properties in the different directions (Tables of Figs. 4 to 6), the most probable values of the coefficients of expressions defining the anisotropies studied were obtained through the application of the least-square method.

Once the equations of ellipsoids are known relative to the reference base Oxyz, by means of an adequate rotation, these equations were reduced on the principal axes (2):

$$a\,X^2 + b\,Y^2 + c\,Z^2 = 1 \qquad (2)$$

From these equations reduced to the principal axes, the normal equations (3) were written:

$$\frac{X^2}{\alpha^2} + \frac{Y^2}{\beta^2} + \frac{Z^2}{\gamma^2} = 1 \qquad (3)$$

The ellipsoids of anisotropies (Figs. 4 to 6), represented by sections (ellipses) made in the ellipsoids by the reference planes and the principal planes, together with values of the semi-axes, permit to visualize spatially the surfaces defining the anisotropies under consideration and assess to what extent and how the ellipsoids depart from a spherical surface taken as reference.

The absolute and relative deviations (Tables of Figs. 4 to 6) calculated from the values obtained through an experimental approach and through a theoretical approach and also the coefficients of variation, inform as to the extent in which the experimental and the theoretical values agree, i.e., the degree of validity of the hypothesis formulated to define the anisotropy.

The coefficients of bulk anisotropy, a_m, and of maximum anisotropy, a_M, (Figs. 4 to 6) respectively express the mean decrease of a given property with reference to the isotropic material whose value equals the maximum presented by that property (semi-major axis of the ellipsoid) and the maximum decrease of the same property. Those coefficients are defined by

$$a_m = \frac{R}{R_e} \quad ; \quad a_M = \frac{R}{r} \qquad (4)$$

where:

R - radius of the sphere circumscribing the ellipsoid (semi-major-axis)

R_e - radius of the sphere whose volume equals the ellipsoid

r - radius of the sphere inscribing the ellipsoid (semi-minor-axis).

Deviations between experimental and theoretical values and the coefficient of variation show that for the rock tested also the anisotropy relative to thermal expansion is spatially well represented by a 2nd degree expression, of the ellipsoid type. Surfaces defining the anisotropies of deformability, failure and thermal expansion for the rock tested present slight eccentricities.

3. CORRELATIONS

Once anisotropies for some given properties of a rock material are quantified, it matters to investigate whether phenomena that generate those anisotropies influenced those properties differentially or if the parameters defining the different types of anisotropies studied are to same extent related to the formation of the material and to structural discontinuities of the rock mass to which the material tested belong, such as jointing.

The following development concerns possible correlations between anisotropy surfaces themselves and these surfaces and rock mass jointing.

a) Between the ellipsoids of anisotropy.

To correlate between one another the surfaces defining the anisotropies referring to the three properties studied, for each set of two ellipsoids were determined the angular differences between the homologous principal axes and the relations of the corresponding semi-axes. These determinations were obtained through linear transformations, composed of a rotation and an autometric transformation.

For the three correlations, in Fig. 7 are presented those linear transformations consisting of the rotation matrix [R] and the deformation matrix [D], which respectively indicate the rotation that leads to the coincidence of the homologous principal axes and the relations of the corresponding semi-axes.

Values in Fig. 7 show that no marked parallelism exists between the principal axes of the three surfaces of anisotropy and, as regards the relation between values of analogous semi-axes, that the ellipsoids of deformability and of thermal expansion can be considered practically homotetical upon coincidence of those axes.

b) Between jointing and the principal axes of the anisotropy ellipsoids.

To assess to what extent phenomena that originate jointing of the rock mass from which the test block was extracted had generated the anisotropies under study, the orientations of the principal axes of the anisotropy surfaces were correlated with the attitudes of the joint sets that form the rock mass jointing. The points and great circles that represent them (Fig. 7) are shown in sterographic representation. In that figure are represented the directions of the principal axes of the ellipsoids for the three charac teristics studied, and the most probable attitudes and corresponding normals of the five sets that form the rock mass jointing.

In the same Fig. – in wich the weight of each joint set is I (25 %); V (20 %); I_1 (20 %); I_2 (13 %); V_1 (12 %) – the principal axes of the three ellipsoids are found to be practically parallel to the planes of the joint sets, deviations ranging from 0.5° to 15°. The semi-axes of maximum and intermediate values tend to be co-planar with the attitudes of the joint sets of major weight, whereas those of minimum value tend to be co-planar with the attitudes of joint sets of less importance.

4. CONCLUSIONS

The analysis of results hereinbefore presented leads to the following conclusions:

1 - Deviations between values obtained through an experimental approach and through a theoretical approach as well as coefficients of variation corroborate that for the rock tested, the anisotropy of thermal expansion can spatially be well represented by 2nd degree expressions, ellipsoid type, as it occurs with anisotropies of deformability and failure.

2 - Surfaces defining anisotropies relative to the three properties studied do not present large deviations with reference to spherical surfaces, which indicates that the anisotropies of the properties under study are not much marked.

3 - Although the homologous principal axes do not present marked co-linearity, the three surfaces of anisotropy to same extent tend to homotety after making those axes coincide; homotety is practically reached with parameters that define the surfaces of anisotropy for deformability and thermal expansion.

4 - The principal directions of the three surfaces of anisotropy in study are significantly correlated with the attitudes of the sets that form the jointing of the rock mass; their semi-major axes are practically co-planar with the attitudes of the joint sets of more relative importance and the minor ones are practically co-planar with the attitudes of the sets of less relative importance.

5 - It is of interest to proceed with such studies on igneous rocks that present larger anisotropies, and also with sedimentary and metamorphic rocks which are known to present a higher degree of anisotropy.

Study of the Anisotropy

Sample orientation

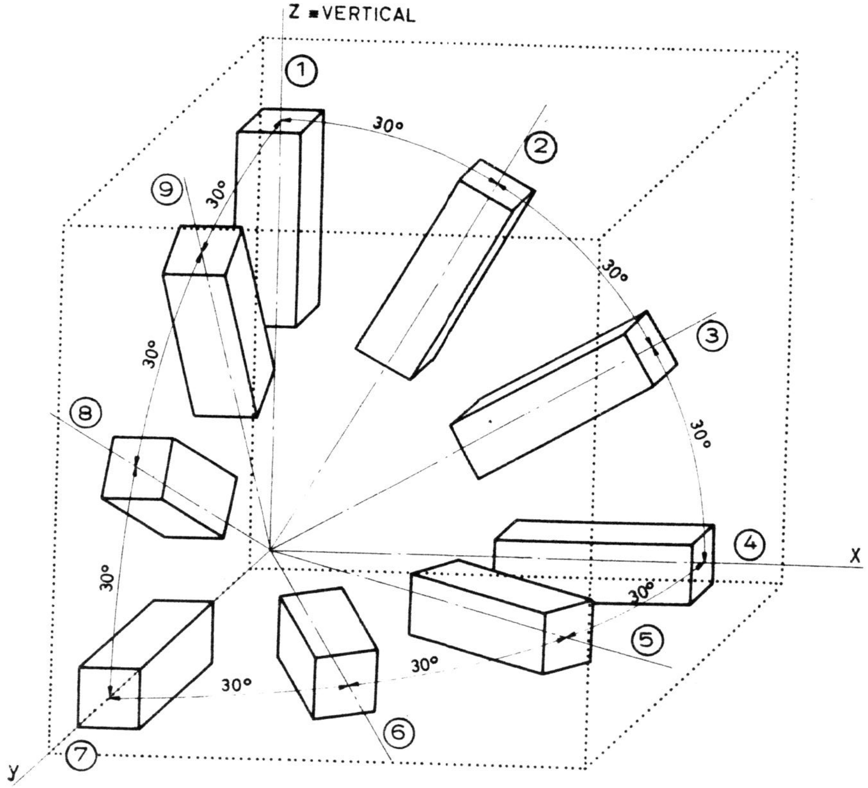


Fig. 1

Fig. 2

Study of the Anisotropy

Uniaxial compression tests

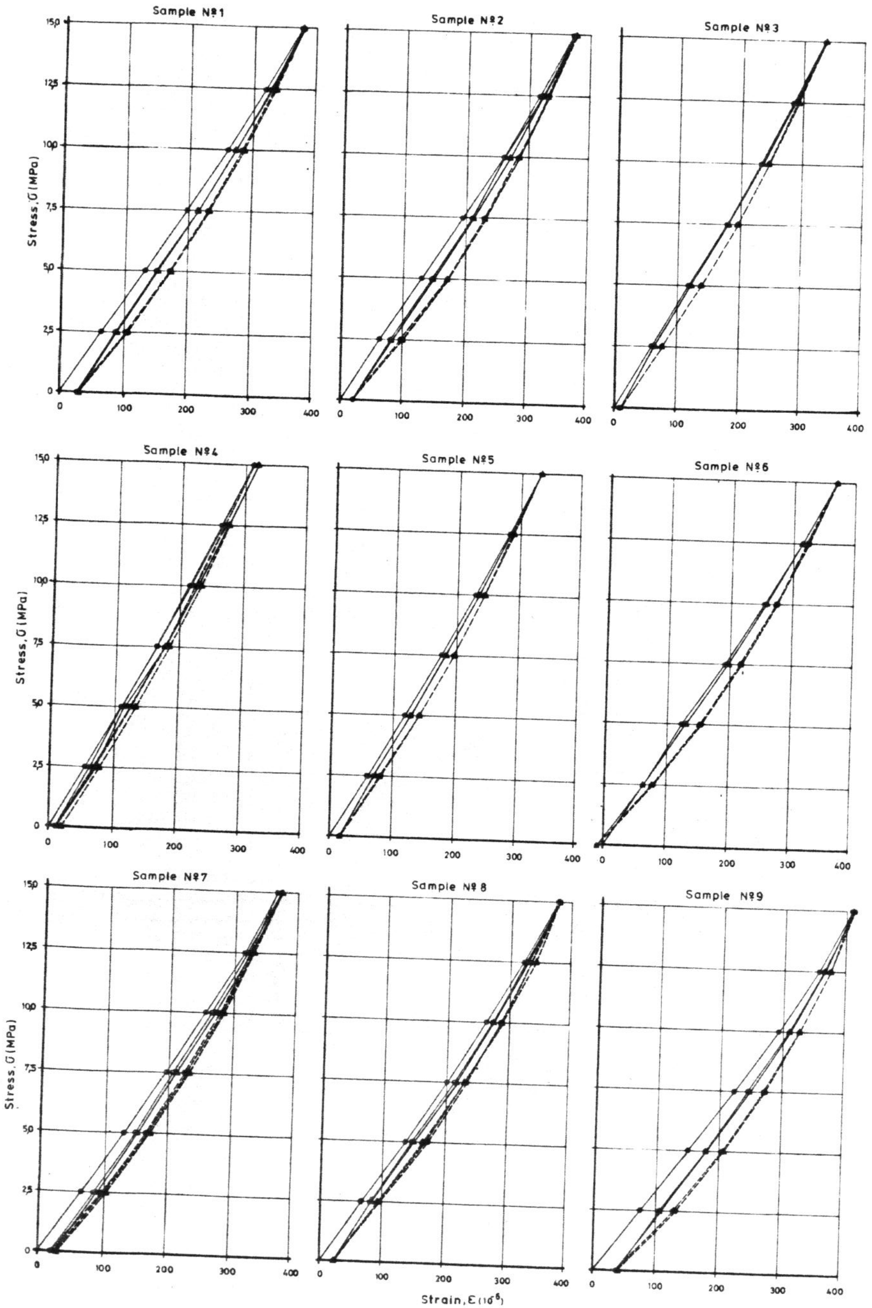


Fig. 3

Study of the Anisotropy

Thermal expansion tests

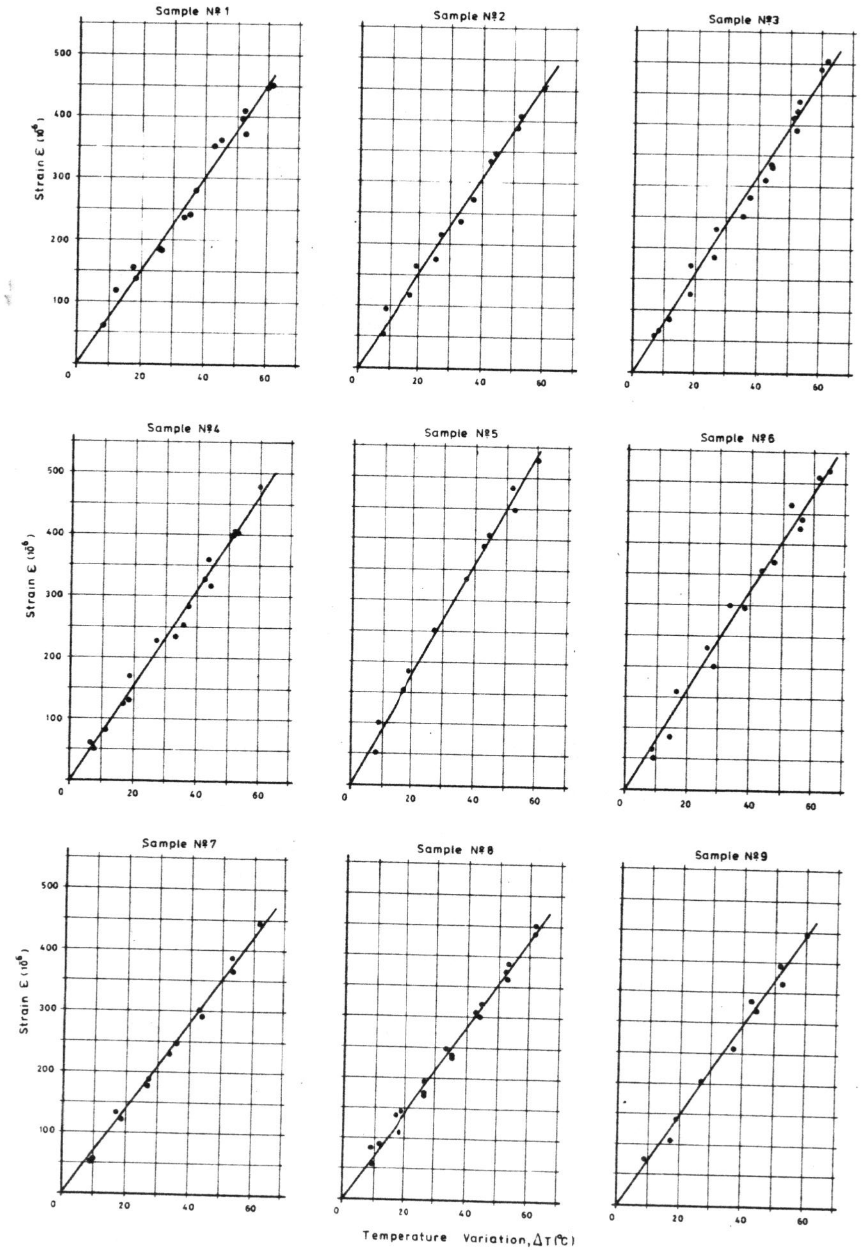


Fig. 4

Anisotropy of the Deformability Modulus

Normal Equation

$$\frac{X^2}{50{,}2^2}+\frac{Y^2}{38{,}9^2}+\frac{Z^2}{42{,}5^2}=1$$

Coefficiente of Variation

$\delta = 4{,}4\ \%$

Anisotropy Ratios

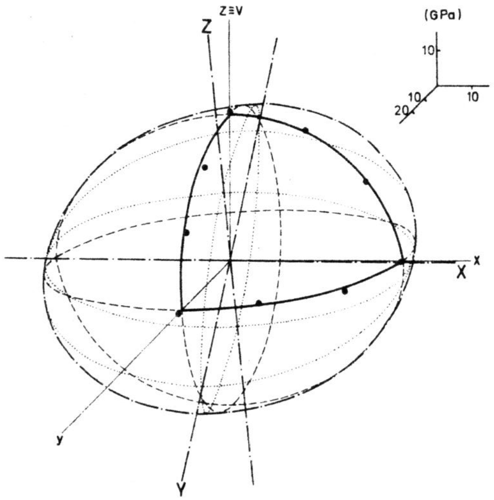
$a_m = a_s = 1{,}15$

$a_M = 1{,}29$

Rotation Matrix

(xyz — XYZ)

$$[S] = \begin{bmatrix} -0{,}985 & +0{,}156 & +0{,}067 \\ -0{,}038 & -0{,}735 & +0{,}677 \\ +0{,}209 & +0{,}691 & +0{,}691 \end{bmatrix}$$



Experimental and Theoretical Values

Orientation	Value (GPa)		Deviation	
	Experimental	Theoretical	Absolute (GPa)	Relative (%)
1≡Oz	42,6	40,7	+1,9	+4,5
2	42,3	41,9	+0,4	+1,0
3	46,0	46,3	-0,3	-0,6
4≡Ox	49,2	49,9	-0,7	-1,4
5	47,3	45,8	+1,5	+3,2
6	39,5	41,6	-2,1	-5,3
7≡Oy	42,6	40,7	+1,9	+4,5
8	41,9	42,6	-0,7	-1,7
9	39,3	42,6	-3,3	-8,4

Fig. 5

Anisotropy of the Ultimate Stress

Normal Equation

$$\frac{X_1^2}{103{,}3^2}+\frac{Y_1^2}{126{,}0^2}+\frac{Z_1^2}{108{,}9^2}=1$$

Coefficient of Variation

$\delta = 3{,}0\ \%$

Anisotropy Ratios

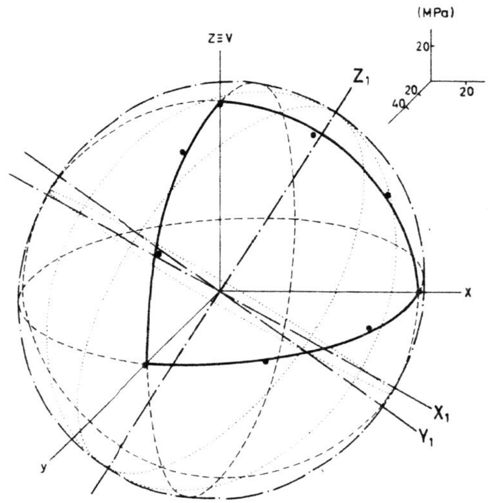
$a_m = a_s = 1{,}12$

$a_M = 1{,}22$

Rotation Matrix

($xyz - X_1Y_1Z_1$)

$$[T] = \begin{bmatrix} -0{,}611 & +0{,}507 & +0{,}608 \\ +0{,}644 & +0{,}765 & +0{,}009 \\ +0{,}460 & -0{,}397 & +0{,}794 \end{bmatrix}$$



Experimental and Theoretical Values

Orientation	Value (MPa)		Deviation	
	Experimental	Theoretical	Absolute (MPa)	Relative (%)
1≡Oz	106,0	106,7	-0,7	-0,7
2	105,0	106,2	-1,2	-1,1
3	110,0	108,9	+1,1	+1,0
4≡Ox	113,0	112,5	+0,5	+0,4
5	121,0	123,0	-2,0	-1,6
6	127,0	125,2	+1,8	+1,4
7≡Oy	119,0	116,1	+2,9	+2,5
8	107,0	115,2	-8,2	-7,1
9	114,0	110,3	+3,7	+3,4

Anisotropy of the Thermal Coefficient of Expansion

Fig.6

Normal Equation

$$\frac{X_2^2}{8{,}94^2}+\frac{Y_2^2}{6{,}39^2}+\frac{Z_2^2}{7{,}55^2}=1$$

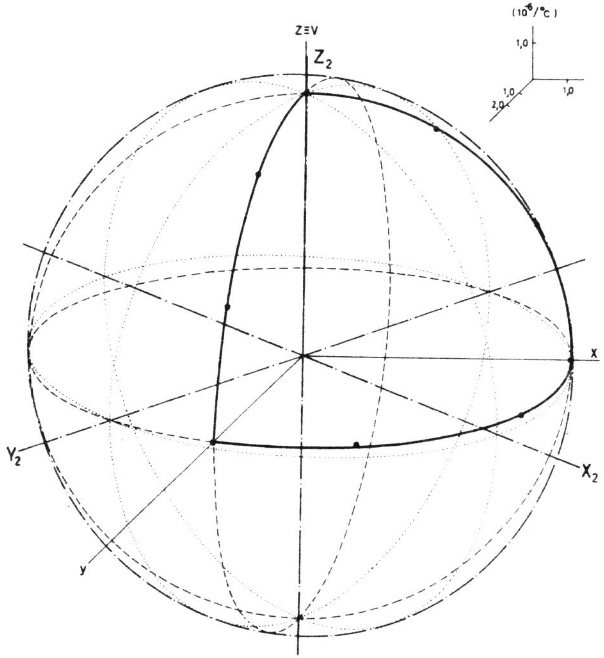
Coefficient of Variation

$\delta = 1{,}3\ \%$

Anisotropy Ratios

$a_m = a_s = 1{,}18$

$a_M = 1{,}40$



Rotation Matrix

$(x\,y\,z - X_2 Y_2 Z_2)$

$$[V] = \begin{bmatrix} -0{,}826 & -0{,}562 & +0{,}042 \\ +0{,}563 & -0{,}826 & +0{,}022 \\ +0{,}022 & +0{,}042 & +0{,}999 \end{bmatrix}$$

Experimental and Theoretical Values

Orientation	Value $(10^{-6}/°C)$		Deviation	
	Experimental	Theoretical	Absolute $(10^{-6}/°C)$	Relative (%)
1 ≡ Oz	7,55	7,55	0	0
2	7,66	7,67	-0,01	-0,14
3	7,82	7,81	+0,01	+0,13
4 ≡ Ox	7,72	7,83	-0,11	-1,40
5	9,08	8,91	-0,17	-1,91
6	8,03	8,22	-0,19	-2,31
7 ≡ Oy	7,06	6,95	+0,11	+1,58
8	7,08	7,08	0	0
9	7,39	7,37	+0,02	+0,27

Fig. 7

Stereographic Representation

Correlation between E and σ_{ult}

Rotation Matrix $[R] = [S]\cdot[T]^{-1} = \begin{bmatrix} +0{,}722 & -0{,}514 & -0{,}462 \\ +0{,}062 & -0{,}581 & +0{,}812 \\ +0{,}643 & +0{,}669 & +0{,}370 \end{bmatrix}$

$(X Y Z - X_1 Y_1 Z_1)$

Strain Matrix $[D] = \begin{bmatrix} 486 & 0 & 0 \\ 0 & 309 & 0 \\ 0 & 0 & 391 \end{bmatrix}$

$(X Y Z - X_1 Y_1 Z_1)$

Correlation between E and λ

Rotation Matrix $[R] = [S]\cdot[V]^{-1} = \begin{bmatrix} +0{,}729 & -0{,}682 & +0{,}067 \\ +0{,}473 & +0{,}601 & +0{,}645 \\ -0{,}532 & -0{,}438 & +0{,}724 \end{bmatrix}$

$(X Y Z - X_2 Y_2 Z_2)$

Strain Matrix $[D]$ (Em 10^7 MPa.°C) $\begin{bmatrix} 562 & 0 & 0 \\ 0 & 609 & 0 \\ 0 & 0 & 563 \end{bmatrix}$

$(X Y Z - X_2 Y_2 Z_2)$

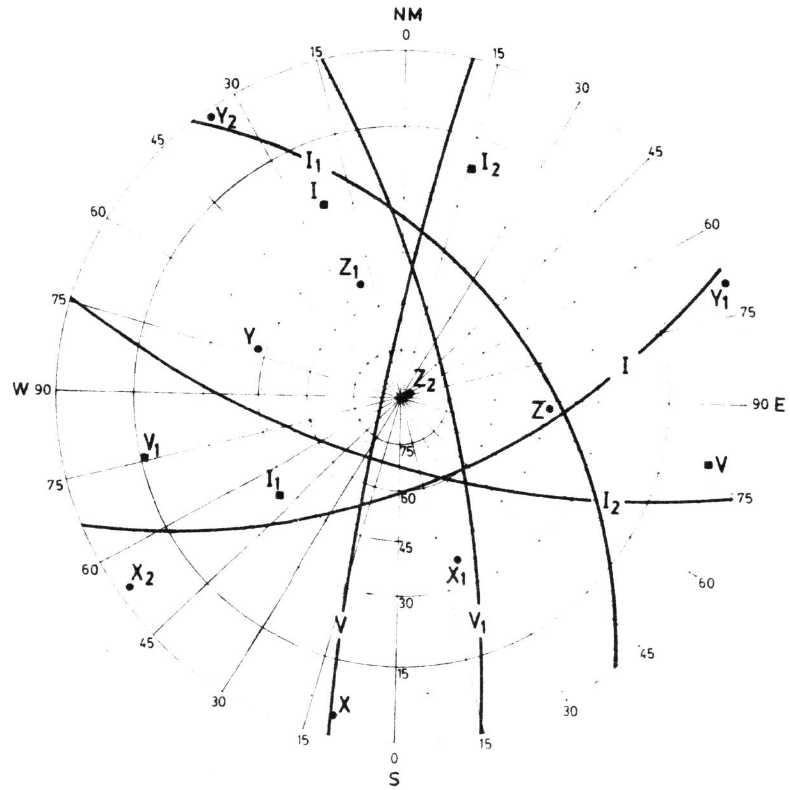
Correlation between σ_{ult} and λ

Rotation Matrix $[R] = [T]\cdot[V]^{-1} = \begin{bmatrix} +0{,}245 & -0{,}749 & +0{,}615 \\ -0{,}961 & -0{,}269 & +0{,}055 \\ -0{,}123 & +0{,}604 & +0{,}787 \end{bmatrix}$

$(X_1 Y_1 Z_1 - X_2 Y_2 Z_2)$

Strain Matrix $[D] =$ (Em 10^5 MPa.°C) $\begin{bmatrix} 116 & 0 & 0 \\ 0 & 197 & 0 \\ 0 & 0 & 144 \end{bmatrix}$

$(X_1 Y_1 Z_1 - X_2 Y_2 Z_2)$



- (X, Y, Z) — Axes of the E - Ellipsoid
- (X_1, Y_1, Z_1) — " " " σ_{ult} - "
- (X_2, Y_2, Z_2) — " " " λ - "
- ■ — Normal to the most frequente attitud of a joint set
- ⌐ — Most frequent attitud of a joint set