RELIABILITY OF WELDED STRUCTURES: A SENSITIVITY ANALYSIS

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The probability of failure of a pressure vessel containing elliptical surface cracks was assessed. A sensitivity analysis of the influence of distribution of weld metal toughness, level of residual stresses and defect growth by fatigue was performed. A comparison of crack growth calculated according to PD 6493 methodology and a solution which assumes defect growth in both depth and surface direction was carried out, and its influence on the probability of failure was assessed.

INTRODUCTION

The study of the reliability of welded structures has received particular attention in the last few years due to safety and economical risks associated with a structural failure. The failure of a welded structure can occur by several modes, but analysis of past service failures has revealed that most of them were due to brittle fracture and/or fatigue, associated with the presence of weld defects. Probabilistic fracture mechanics provides a means of quantifying the probability of occurrence of those failures, Johnston (1), Lidiard (2), the result relying entirely on the existence and quality of data available on the governing parameters. According to this methodology "failure" is defined as occurring when the size of a weld defect exceeds the tolerable defect size as calculated by a fracture mechanics analysis. Assuming that both defect size and tolerable defect size are distributed variables, the probability of failure is calculated from the interaction of the two distributions, as illustrated in fig. 1. This function defines the fraction of a population of structures which will fail at a given time when the distributions are as given. The growth of defects by fatigue can alter significantly the initial defect distribution, Dufresne (3), as shown in fig. 1, increasing the failure probability.

In the present paper a study on the major parameters affecting the probability of failure was carried out, namely the distribution of weld metal toughness, the level of residual stresses and defect growth by fatigue. A comparison of crack growth and its influence in the probability of failure was carried using the PD 6493 methodology, PD 6493 (4), and a solution based

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on the Raju - Newman work, Scott et al (5), which assumes crack growth in both directions.

ANALYSIS METHODOLOGY

The purpose of the present work was to perform a sensitivity analysis of various parameters controlling the reliability of a welded structure. The study was conducted by considering a welded pressure vessel containing elliptical surface defects in the cilindrical part. It should be noted that the values of probability of failure obtained are not true failure probabilities since the analysis was based on various assumptions (eg.distributions used) and was limited to surface defects only, located in the cilindrical part of the vessel. The overall dimensions of the pressure vessel considered and the geometry of the initial surface defects are illustrated in fig. 2. The calculations were performed on the basis of the following assumptions:

Base material - HII DIN 17155 steel (σ_{ys} = 240 MPa; σ_{UTS} = 410 MPa)
Weld Metal - Deposited with AWS E7016 Electrodes(σ_{ys}= 405 MPa; σ_{UTS}= = 520 MPa)
Wall thickness = 50.5 mm
Design Temperature = -10°C
Membrane Service Stress = 120 MPa
Stress fluctuation = 30% of membrane service stress
Nº cycles per year = 3 x 10^5

The weld metal toughness and initial defect distribution are (1) the major parameters controlling the probability of failure, as shown in fig. 1. In the present study a Weibull distribution proposed by Rogerson et al (6) for surface breaking defects was used. The probability density function for defect height is as follows:

$$\eta(x) = \beta/\eta\ ((x-\gamma)/\eta)^{\beta-1}\ \exp\ (-(x-\gamma)/\eta)^{\beta}) \qquad (1)$$

with β=0.85, γ=0.1, η=1.75.

This distribution was based on the analysis of non destructive examination records and does not take into account the reliability of NDE examination, which was not considered in the present study.

The toughness distribution used was obtained performing a statistical analysis of CTOD test results on weld metal published by the authors elsewhere (7). The histograms of CTOD results are presented in fig. 3, for the two types of weld metal considered. Analytical (Kolmogorov - Smirnov) and graphical tests were performed to determine the distribution which would give the best fit.In both cases this was obtained with a lognormal distribution, with a probability density function as follows:

$$g(a) = 1/\ (a.S_L.\ \sqrt{2\pi}\ )\ .\ \exp\ (-\ ((\ln\ (a)\ -\ \bar{a}_L\ )\ /S_L)^2/2) \qquad (2)$$

The parameters S_L and $\bar{a}_L$ being indicated in fig. 3. The methodology used to transform the toughness distribution into a distribution of critical defects, was the CTOD design curve, as described in the PD 6493 document (4). The evaluated probabilities of failure of the pressure vessel for the service conditions indicated, assuming the two levels of notch toughness and two levels of residual stresses, are given in Table 1. The surface defects were assumed to follow the above mentioned distribution with a/2C ratio equal to 0.2.

TABLE 1 - Effect of notch toughness and level of residual stress on the probability of failure

Level of residual stress	Toughness distribution of weld metal (CTOD)	
	manufacturer A	manufacturer B
σ_R^{*} = 24 MPa	3.35 x 10^{-3}	1.11 x 10^{-3}
σ_R^{**}= 240 MPa	6.04 x 10^{-2}	1.87 x 10^{-2}
NOTE: * Post weld heat treated ** As welded		

The values presented in Table 1 refer to the initial conditions (moment the pressure vessel is put in service). Assuming that the vessel is subjected in service to pressure flutuations, defect growth by fatigue can occur, which will alter the initial defect distribution. In order to evaluate the significance of fatigue in the probability of failure, the growth of the initial defects was estimated by using the Paris law:

$$\frac{da}{dN} = C\ (\Delta K)^m \qquad \text{mm/cycle} \qquad (3)$$

with C= 4.12 x 10^{-14} and m = 3.25. The PD 6493 methodology was adopted. As it is known this methodology assumes that a surface defect grows in depth until reaching a semicircular shape, mantaining this shape till final fracture. An incremental (step by step) procedure was adopted in the fatigue calculations. The pattern of crack growth is illustrated in fig. 4 for three defects of different initial size. The influence of defect growth by fatigue on the distribution of defects present in the vessel after a given period of time in service is shown in fig. 5. This distribution was obtained by correcting the Weibull parameters to take into account the growth of the initial defects. In fig. 6 the variation of the probability of failure for a 20 year period is presented, for two levels of toughness of the weld metal.

In order to verify the possible influence of the crack growth pattern, it was decided to perform the fatigue analysis considering that the defect can grow in both depth and surface directions depending on the value of ΔK. The solution adopted for the calculations was that proposed by Scott et al(5), for uniform tension:

$$K_I = \left[M_f + \left[E\ (K)\ \sqrt{c/a} - M_f\right] \left(\frac{a}{t}\right)^p \right] \frac{\sigma_m}{E(K)} \sqrt{\pi a} \qquad (4)$$

E(K) being the elliptic integral of the second kind, and p and M_f being a funtion of a/2C and a/t, and having different values at the surface and deepest point. Using the above expression, ΔK values can be calculated for those points allowing the evaluation of the defect growth. An incremental procedure was adopted in the computer calculations. The pattern of crack growth in this case is also illustrated in fig. 4, where it can be observed that the cracks grow in both directions, with the aspect ratio a/2C increasing. The influence of defect growth by fatigue as estimated by using this procedure, on the defect distribution, is shown in Fig. 5. At the same time Fig. 6 shows the variation of the probability of failure for a 20 years period in service. The influence of the initial aspect ratio on the probability of failure was also studied for both procedures, by considering two different values of a/2C, 0.1 and 0.2, the results being presented in fig. 7.

DISCUSSION

The values of probability of failure, which as said before do not correspond to a complete quantitative assessment of fracture, were found slightly higher (≈ 10^{-3}) than the values usually associated (2) with this type of structures. This was mainly attributed to the low toughness values obtained on the weld metal. In fact the values of probability of failure obtained were found considerably dependent on the toughness level, for both the levels of residual stresses. This is easily explained since the critical defect sizes depend on the toughness of the material and of the state of stress on the area were the defects are located. The critical size of defects were evaluated with the use of the CTOD design curve (4), which is known to have a high degree of conservatism on the assessment of defects, Burdekin (8).As the toughness decreases and/or the residual stress level increases, the allowable defects became smaller, increasing the interaction area and the probability of failure. In the present study the variation of the notch toughness was found less important than the effect of the presence of residual stresses (which depends on the performance of a post weld heat treatment), as shown in Table 1.

The probability of failure was found to increase with the number of years in service, as a result of cyclic loading inducing defect growth by fatigue. This was observed (Fig. 6) for both levels of toughness considered and for both the methodologies of defects growth estimation used. For a given level of toughness and stress range it can be seen that the use of the PD 6493 methodology leads to lower values of probability of failure,after a given number of years in service. This can be justified due to the influence of the pattern growth on each methodology.

In fact the PD 6493(4) assumes that an initial defect with a given a/2C ratio grows in depth only, until it reaches the semicircular shape, mantainning this shape in subsequent growths, i.e., the a/2C ratio is then constant and equal to .5, as illustrated in fig.4 for some initial defects. On the contrary the Raju-Newman modified solution (5) assumes that the defect can grow in both directions depending on the level of ΔK on each location, resulting in a completely different pattern of crack growth (fig. 4). The influence of this effect on the defect parameter ($\bar{a}$) distribution can be observed on Fig. 5 for a given number of years in service (10 and 20 years in service).In the first stages of crack growth the influence of the methodology is not very significant, being more pronounced as the number of years in service increases, due to a large amount of crack growth that is observed by the use of the Raju-Newman solution.

The probability of Failure was also found to be dependent on the initial aspect ratio assumed. This effect is illustrated in Fig.7. where it can be observed that lower values at a/2C ratio leads to an increase in the values of probability of failure obtained, and at the same time that this type of defects are more severe in terms of fatigue loading. This aspect shows the limitation of the current published distributions of defect dimensions, which relate only to depth or length without information concerning the aspect ratio of the defects.

CONCLUSIONS

- The probability of failure was found to be dependent on the toughness and on the residual stress level, the effect of the residual stress level being more significant.
- Fatigue crack growth alters the initial defect distribution leading to an increase of the probability of failure with the number of years in service.
- The pattern of crack growth is revealed to be an importante factor on the estimation of failure probabilities, and so, methodologies that do not take into account the possibility of surface defects growing in both directions, as a function of ΔK, may lead to unconservative results of probability of failure.
- The assumptions on the initial aspect ratio of the surface defects affect the evaluated values of probability of failure, and so it is important that proposed defect distributions give reliable information on the correlation of depth/length of the defects which is lacking at the moment.

SYMBOLS

a = defect depth (mm)

$\bar{a}$ = defect parameter (PD 6493)

$\bar{a}_L$ = mean of the Lognormal distribution
c = defect length (mm)
C = Constant in Paris Law equation
da/dN = fatigue crack growth rate (mm/cycle)
K_I = Stress intensity factor, mode I ($N. mm^{-3/2}$)
ΔK = Stress intensity range ($N. mm^{-3/2}$)
m = Exponent in the Paris law equation
s_L = Standard deviation of the lognormal distribution
t = thickness
σ_m = membrane stress (MPa)
σ_{ys} = yield stress (MPa)
σ_{UTS} = Ultimate tensile stress (MPa)

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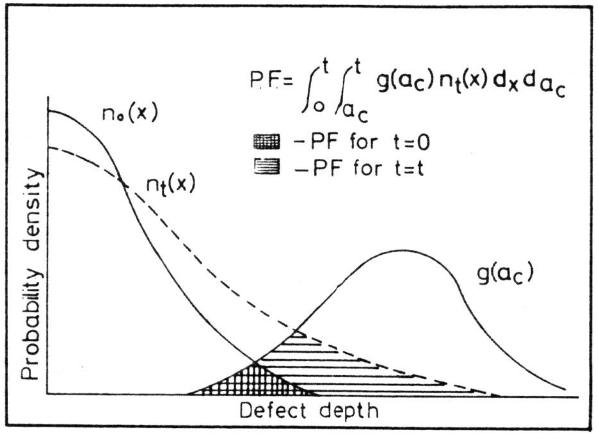


Fig. 1 Failure Probability calculated from the interaction of two distributions.

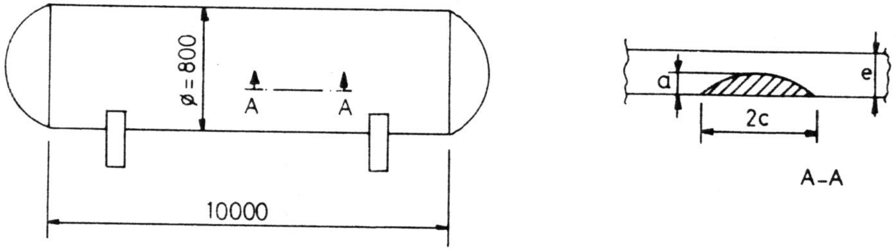


Fig. 2 Pressure Vessel geometry and type of defects considered.

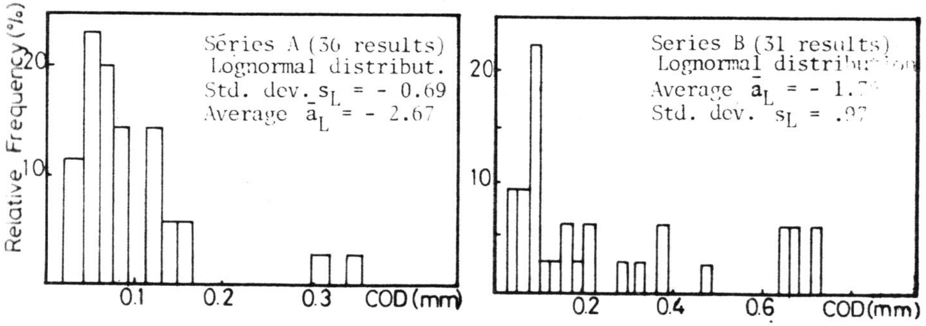
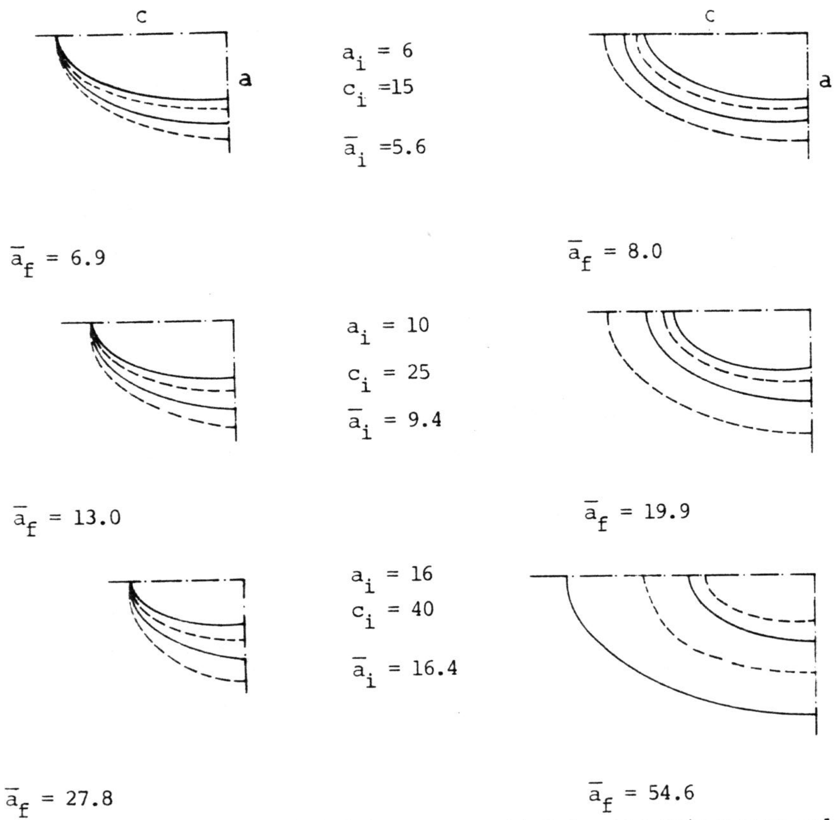
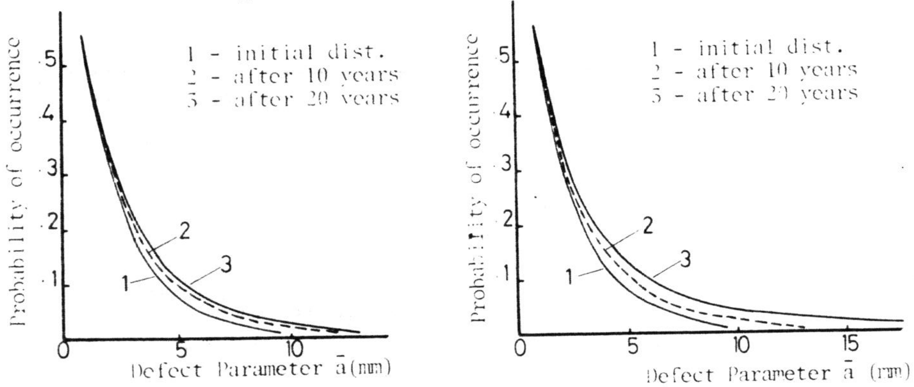


Fig. 3 Histograms of CTOD results obtained at -10°C in MMA welds with AWS E7016 electrodes from two different manufacturers.



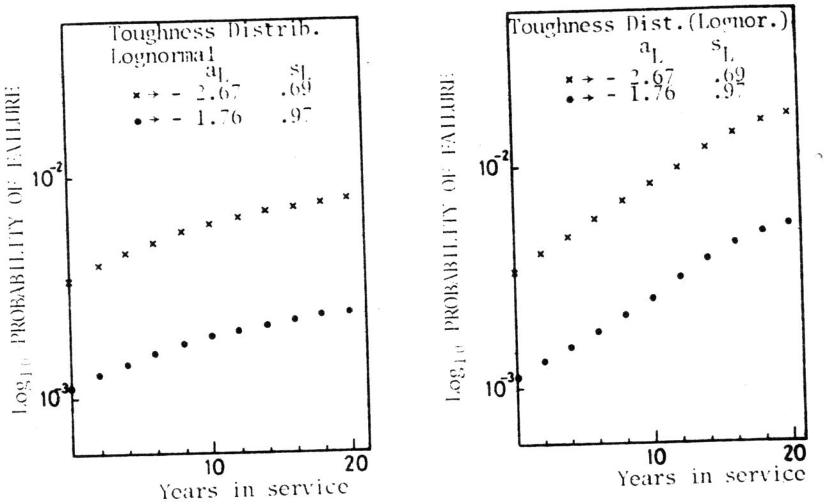
a) Using the PD 6493 methodology b) Using the Raju-Newman solution

Fig. 4 - Pattern of crack growth estimated for three different sizes of defects ($\bar{a}$ values represent the defect parameter corresponding to initial "a_i", and final "a_f", defect geometry).



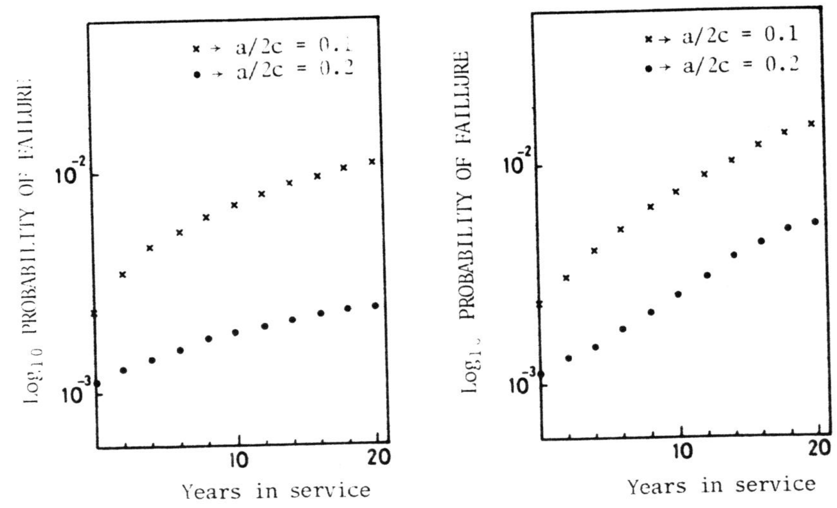
a) Using the PD 6493 methodology b) Using the Raju-Newman solution

Fig.5 - Influence of defect growth by fatigue on the defect distribution



a) Using the PD 6493 methodology b) Using the Raju-Newman solution

Fig. 6 Effect of fatigue on the probability of failure, considering two different methodologies to estimate the defect growth by fatigue and the two toughness distributions obtained. (a/2c=0.2)



a) Using the PD 6493 methodology b) Using the Raju-Newman solution

Fig. 7 Influence of the initial aspect ratio (a/2c) on the probability of failure (toughness distribution: $\bar{a}_L$= -1.76; S_L=.97).