INFLUENCE OF STRAIN RATE AND STRAIN CYCLING ON FATIGUE CRACK-TIP PLASTICITY

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A general relationship for the calculation of strain rates at the fatigue crack tip was derived. This relationship is applied to predict the effect of strain rate and strain cycling on the dimensions of the monotonic and cyclic plastic zones. Strain rates are independent of the applied ΔK or K_{max}, but depend on frequency and stress ratio R. The experimental values of the $\alpha^{\theta}/\alpha^{\theta}_{c}$ - ratios of the plastic zone size, obtained in plane strain conditions and quoted in the literature are predicted by this relationship, taking into account the high strain rates and also the strain cycling usually occurring within the plastic zones at the fatigue crack tip.

INTRODUCTION

Ahead of a propagating fatigue crack two types of plastic zones can be distinguished: the monotonic plastic zone and the reverse or cyclic plastic zone (Figure 1). The monotonic or forward plastic zone is generated by monotonic tensile deformations occurring during the load increase in a fatigue cycle and its size is controlled by the K_{max} value. The reverse or cyclic plastic zone is created during the unloading part of the cycle, by the compressive stresses generated by the elastic unloading of the material in the surrounding monotonic plastic zone. It is possible to consider a third highly deformed zone at the very crack-tip, where strains approach the value of ε_f, the fracture strain of the material. This small region where the cyclic fracture process occurs is called the "CTOD-affected"zone or process zone(1,2). In many studies the dimensions of the monotonic plastic zone, r_p^{θ}, have been calculated using the well known LEFM equation:

$$r_p^{\theta} = \alpha^{\theta} \left(\frac{K_{max}}{\sigma_{ys}} \right)^2 \qquad (1)$$

where σ_{ys} is the monotonic yield stress in uniaxial tension at usual strain rate values($\dot{\varepsilon} < 10^{-3}$ s^{-1} according to ASTM standard methods) and α^{θ} is a constant for each value of the polar angle θ.

For R=0 the dimensions of the cyclic plastic zone, r_c^{θ}, can be calculated using an equation similar to equation 1, but substituting the value of σ_{ys} by σ^{c}_{ys}, the theoretical cyclic yield stress:

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$$r_c^\theta = \alpha^\theta \left(\frac{K_{max}}{\sigma_{ys}^c} \right)^2 = \alpha_c^\theta \left(\frac{K_{max}}{\sigma_{ys}} \right)^2 \qquad (2)$$

In the absence of strain hardening, strain cycling and strain rate effects $\sigma_{ys}^c = 2\sigma_{ys}$ and therefore

$$\alpha_c^\theta = \frac{\alpha^\theta}{4} \qquad (3)$$

where α_c^θ is the theoretical constant for the cyclic plastic zone.

An analysis of the experimental results taken from the literature (2) and obtained in different materials showed a good agreement amongst the values of α_c^θ. However experimentally measured cyclic plastic zone sizes are smaller than the values which would have been obtained by this simplified analysis. The experimental $\alpha'^\theta/\alpha_c'^\theta$ - ratios seem to be somewhat higher than the previously suggested value of 4. These discrepancies between theoretical estimates and experimental observations, can be minimized considering the strain hardening or strain cycling which occurs in the material ahead of the fatigue crack tip and mainly in the cyclic plastic zone. For the correct evaluation of plastic zone dimensions the well documented effect of strain rate on the yield stress of metallic materials should be taken into account also. As, in most fatigue plastic zone studies, the strain rate and strain cycling effects have been omitted, the aim of the present work is to derive a relationship for the calculation of strain rates occurring at the fatigue crack tip. Finally, an attempt is made to justify the above-mentioned discrepancies in terms of strain rate and strain cycling effects, the latter ones through the cyclic stress-strain curve of the material. The analysis is illustrated using fatigue crack propagation data obtained by the authors in the medium strength steel BM 45 (1,2).

STRAIN RATE EFFECTS AND THE BEHAVIOUR OF STEELS

For most metals the yield stress is related to grain size by the well-known Petch equation:

$$\sigma_{ys} = \sigma_i + K_y\, d^{-1/2}$$

where σ_{ys} is the yield stress under uniaxial tension required to propagate yield from one grain to the next, σ_i is the stress required to move an unpinned dislocation, i.e. the friction stress resisting dislocation motion, d is the average grain diameter and $K_y = 2\sigma_n \ell^{1/2}$, where σ_n is the stress to nucleate slip in the next grain, at a distance ℓ from the grain boundary.

In steels, K_y has been found to be independent of temperature (3) and strain rate (4). Therefore the variation of σ_i with temperature and strain rate determines the observed variation of σ_{ys}.

Figure 2 summarizes experimental studies on the effects of both temperature and strain rate in mild steels (5). At room temperature the four regions in Fig. 2 reduce to three regions of strain rate behaviour as shown in Figure 3 (6).

THEORETICAL CALCULATION OF $\dot{\varepsilon}$ AT THE MONOTONIC AND CYCLIC PLASTIC ZONE BOUNDARIES

It is known that plane strain (Pε) conditions are very common and the most important in fracture problems. In the previous work (2) it was concluded that, in the steady state of fatigue crack growth in ductile metals, all the α_c^θ-values corresponded to Pε conditions. In region II of the fatigue crack propagation curve (da/dN, ΔK curve) the crack grows following a transgranular striation mechanism. The striations are always normal to the crack growth direction and generally, each striation corresponds to a load cycle as a result of alternating shear on the planes of maximum resolved shear stress (45º inclined slip planes). Therefore it is important to estimate the value of $\dot{\varepsilon}$ under these conditions at both monotonic and cyclic plastic zone boundaries (respectively Boundary 1 and Boundary 2 in Fig.1) and also to analyse the strain rate and strain cycling influence on the size of both plastic zones. For that purpose the Dugdale model (7) will be used.

According to that model the crack tip opening displacement (CTOD) is given by the equation

$$\text{CTOD} = \frac{K_{max}^2}{E\,\sigma_{ys}} \qquad \text{for } R = 0 \qquad (4)$$

Since this model is only meaningful for plane stress (Pσ) conditions, then

$$\text{CTOD (in P}\varepsilon) \approx \text{CTOD (equation 4)} \qquad (5)$$

as a result of experimental observations.

Considering that for a crack under Pε, plastic deformation is confined to two 45° inclined slip regions, the average shear strain, γ, associated with each inclined region is:

$$\gamma \approx \frac{\frac{1}{2}\,\text{CTOD (in P}\varepsilon)}{r_p^o} \qquad (6)$$

The monotonic plastic zone ahead of the crack tip represents the superposition of the two inclined regions A and B. Hence using the analysis of Hahn and Rosenfield (8) the tensile strain at boundary 1, ε_1, will be given by the equation

$$\varepsilon_1 \approx \frac{1}{2}\gamma^A + \frac{1}{2}\gamma^B \approx \frac{\frac{1}{2}\,\text{CTOD}}{r_p^o} \qquad (7)$$

Therefore substituting equation 4 we obtain

$$\varepsilon_1 \approx \frac{K_{max}^2}{2\, r_p^o\, E\, \sigma_{ys}} \qquad (8)$$

Assuming that r_p^o is constant with time the strain rate $\dot{\varepsilon}_1$ is

$$\dot{\varepsilon}_1 \approx \frac{K_{max}}{r_p^o\, E\, \sigma_{ys}}\, \dot{K} \qquad (9)$$

where $\dot{K}$ is the rate of change of the stress intensity factor with time.

Using the experimental value of $\alpha^o = 0.05$ for Boundary 1 in $P\varepsilon$ conditions (2), r_p^o is given by

$$r_p^o = 0.05\left(\frac{K_{max}}{\sigma_{ys}}\right)^2 \qquad (10)$$

combining equations 9 and 10 we obtain:

$$\dot{\varepsilon}_1 \approx 20\,\frac{\sigma_{ys}}{E}\,\frac{\dot{K}}{K_{max}} \qquad (11)$$

In constant-amplitude fatigue with triangular or sinusoidal load waves the average value of $\dot{K}$ can be taken as $\dot{K} = \frac{K_{max} - K_{min}}{t}$ or $\dot{K} = K_{max}\,(1-R)/t$, where $t = (2f)^{-1}$ is the half cycle period. Hence equation (11)for R=0 becomes:

$$\dot{\varepsilon}_1 \approx 40\,\frac{\sigma_{ys}}{E}\,f \qquad (12)$$

This equation implies that $\dot{\varepsilon}_1$ is independent of the applied K_{max} in the steady state of cyclic crack growth (Region II) but varies with test conditions, such as frequency f, stress ratio R, and the material parameters σ_{ys} and E.

To estimate $\dot{\varepsilon}_2$, the strain rate at Boundary 2, equation (9) may also be used substituting r_p^o by r_c^o. As a first approximation r_c^o may be considered one-fourth of r_p^o(equation 3). In that case $\dot{\varepsilon}_2$ will be 4 times higher than $\dot{\varepsilon}_1$ if R=0:

$$\dot{\varepsilon}_2 = 4\,\dot{\varepsilon}_1 \qquad (13)$$

However, a more accurate analysis should consider the strain cycling occurring in the cyclic plastic zone. Experimental measurements of microhardness in the cyclic plastic zone have indicated that the hardness varies inside this plastic zone (2). To illustrate this fact Figure 4 shows the hardness plot at the crack tip region obtained by the authors in fatigue tests of the medium strength steel BM 45 using CT specimens with 6 mm thickness. The hardness distribution is not constant in the cyclic plastic zone (Fig.4) and decreases from a maximum value close to the true fracture stress of the material, and attained at the crack tip, up to stress values close to the yield stress, near the cyclic plastic zone boundary (boundary 2 in Fig.1). Hence the strain behaviour at the cyclic plastic zone may be taken into account if σ_{ys}^c is substituted in equation 2 by $2\sigma'_{ys}$, the 0.2%' cyclic yield stress range corrected for strain rate and obtained from the cyclic stress strain curve of the material. The general equation of the cyclic stress - strain curve is

$$\frac{\Delta\sigma}{2} = k'\left(\frac{\Delta\varepsilon}{2}\right)^{n'} \qquad (14)$$

where k' is the cyclic strength coeficient and n' the cyclic strain hardening exponent of the material. Then the equation for $\dot{\varepsilon}_2$ and for R=0 becomes:

$$\dot{\varepsilon}_2 = \frac{K_{max}\,\dot{K}}{\alpha^o\left(\frac{K_{max}}{2\sigma'_{ys}}\right)^2 E\sigma_{ys}} = \frac{4}{\alpha^o}\;\frac{(\sigma'_{ys})^2(2f)}{\sigma_{ys}\,E} \qquad (15)$$

and

$$r_c^o = \alpha^o\left(\frac{K_{max}}{2\,\sigma'_{ys}}\right)^2 \qquad (16)$$

PLASTIC ZONE SIZE CORRECTIONS DUE TO STRAIN RATE AND STRAIN CYCLING EFFECTS

Using fatigue and tensile data obtained by the authors in the medium strength steel BM 45 (DIN CK 45 according to DIN 17200) (1,2) and presented in Table 1 a value of $\dot{\varepsilon} \simeq 1.5\ sec^{-1}$ was obtained applying equation (12).Hence even at the monotonic plastic zone(boundary 1 in Fig.1) very high strain rates are obtained. A plastic zone size correction is necessary to consider the new yield stress of the material which as refered to above will increase for that strain rate value. For similar steels the results available in the literature(5,6, 9,10) indicate an average of 20% increase (Fig.3) in the monotonic yield stress for that value of strain rate. Therefore equation(1) becomes

$$r_p^o = \frac{\alpha^o}{(1.20)^2}\left(\frac{K_{max}}{\sigma_{ys}}\right)^2 \qquad (17)$$

where the theoretical ratio $\frac{\alpha^o}{(1.20)^2}$ represents the experimental value $\alpha'^o = 0.05$ (Table 1).

At boundary 2 of the cyclic plastic zone $\dot{\varepsilon}_2$ can be calculated using equation (15), which resumes to the calculation of σ'_{ys}. As the experimental cyclic stress-strain curve of the material is not yet available a theoretical calculation is necessary,and this will be exemplified using the data for the BM 45 steel in Table 1. The value of n' can be taken as 0.15 as suggested in (11) since it varies in most metals between 0.1 to 0.2 regardless of their initial state.

Therefore in equation (14) only the value of k' needs to be calculated and this may be obtained from the coordinates of a particular point of the cyclic stress-strain curve(i.e.the0.2%offset yield strength).Landgraf et al(12) found that the fatigue ductility coefficient ε'_f can be approximated by

$$\varepsilon'_f = 0.002\,(\sigma_f/\sigma'_{ys})^{1/n'} \qquad (18)$$

where σ_f is the fracture stress. ε'_f is the plastic strain value at the first reversal ($2N_f$=1) according to the Coffin-Manson equation:

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f\,(2N_f)^c \qquad (19)$$

where $\frac{\Delta\varepsilon_p}{2}$ is the plastic strain amplitude, N_f is the number of cycles to failure and c is the fatigue ductility exponent.

Morrow (14) showed that c may be estimated from the cyclic strain hardening exponent as follow

$$c = -\frac{1}{1 + 5n'} \qquad (20)$$

Therefore for n' = 0.15 → c = - 0.57. Substituting in equation (19) and since $\Delta\varepsilon_p/2 = \varepsilon_f$(fracture strain) when $N_f = 1/4$,gives $\varepsilon'_f = 0.111$ for ε_f=0.165 (Table 1). Substituting now in equation (18) gives σ'_{ys}=350.33 MPa because

σ_f = 640 MP_a (Table 1). Therefore from equation (14), k' = 889.86 and the theoretical equation of the cyclic stress-strain curve of BM 45 steel will be

$$\frac{\Delta\sigma}{2} = 889.86 \left(\frac{\Delta\varepsilon}{2} \right)^{0.15} \tag{21}$$

The value of σ'_{ys} should now be corrected for strain rate. Hence equation (15) gives

$$\dot{\varepsilon}_2 = \frac{4}{0.05} \frac{(350.33)^2 (2x25)}{2.07 \times 10^5 \times 335} = 7.0 \ s^{-1}$$

and from the graph in Fig.3, $(\sigma'_{ys})_s = 1.3\ \sigma'_{ys}$. Therefore the ratio between the monotonic and cyclic plastic zones will be

$$\frac{r_p^o}{r_c^o} = \frac{\alpha'^o}{\alpha'^o_c}$$

where $\alpha'^o = \dfrac{\alpha^o}{(1.20)^2}$ and $\alpha'^o_c = \dfrac{\alpha^o}{\left(2x1.3x\ \dfrac{350.33}{335}\right)^2}$

$$\frac{r_p^o}{r_c^o} = 5.13 \quad \text{when the stress ratio is zero} \tag{22}$$

Therefore taking into account the strain rate and strain cycling at the plastic zones, a value of the ratio α'^o/α'^o_c higher than the theoretical value of 4 was found. The value of 5.13 is valid for fatigue crack propagation in the BM 45 steel at 25 HZ in air and needs to be confirmed with the cyclic stress-strain curve. Thus it seems that in strain rate sensitive materials it is possible to explain theoretically the experimental values of α'^o/α'^o_c close to 5 generally quoted in the literature (2) for Pε conditions.

CONCLUSIONS

Assuming that, in ductile metals, the cyclic plastic zone at a fatigue crack tip is generated and developed under plane strain conditions (Pε), and as a result of alternating shear on the planes of maximum resolved shear stress (45° inclined slip planes), the following equation for the calculation of $\dot{\varepsilon}$ at the cyclic plastic zone boundary for R=0 was derived

$$\dot{\varepsilon}_2 = \frac{4}{\alpha^o} \frac{(\sigma'_{ys})^2 (2f)}{\sigma_{ys} E}$$

This equation takes into account both strain rate effects and strain cycling in the material ahead of the fatigue crack tip. It shows that the strain rate increases with frequency and the dynamic yield stress of the material obtained from the cyclic stress-strain curve. Hence for higher frequencies significantly high strain rates can be obtained, and the dynamic yield stress of the material will be increased and less strain cycling will occur. Consequently high frequencies reducing the strain cycling or crack tip plasticity do not stimulate fatigue crack growth.

The above equation was applied to calculate, based on data previously obtained in the BM 45 steel, the ratio between the monotonic and cyclic plastic zones. Good agreement was found among the theoretical value (5.13) and the experimental results obtained by the authors and other values quoted in the literature.

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TABLE 1. Composition and tensile and fatigue data of BM 45 steel

Composition

Element	Weight (%)
C	0.45
Mn	0.74
Si	0.20
Ni	0.06
P	0.027
S	0.027
Cr	<0.03

Tensile and fatigue data(1,2)

σ_f (fracture stress) = 640 MPa

σ_{ys} (0.2% yield stress) = 335 MPa

$\varepsilon_{max} = \varepsilon_f$ (fracture strain) = 16.5%

E = 2.07×10^5 MPa

f (frequency) = 25 HZ

R (stress ratio) = 0.1

α'^{o} (plastic zone size factor) = 0.05

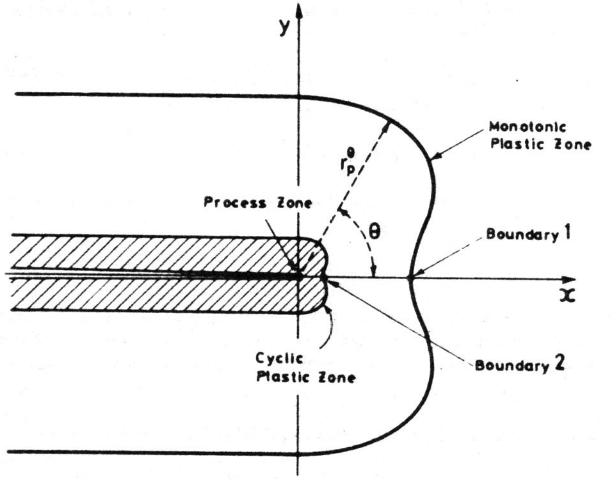

Figure 1 Schematic representation of the plastic zones around a fatigue crack in Pε conditions

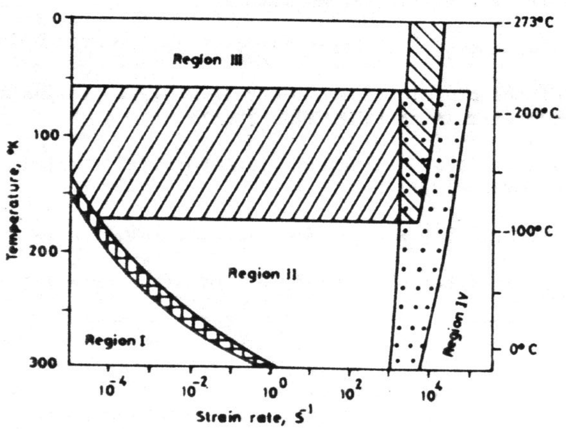

Figure 2 Regions of different strain rate sensitivity of yield stress of mild steels (after Rosenfield and Hahn (5))

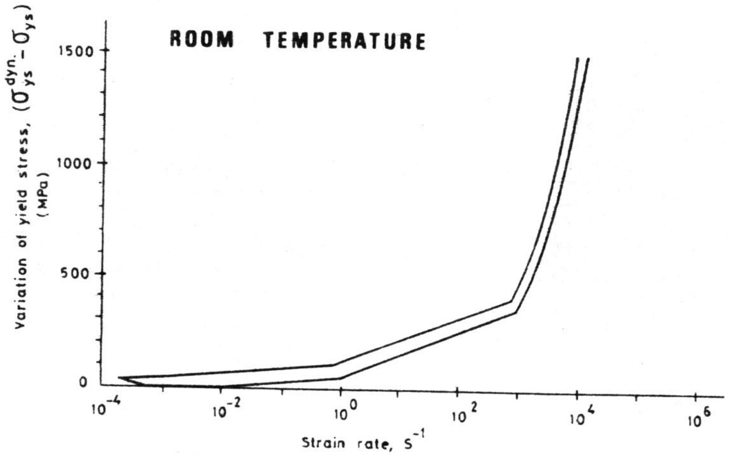

Figure 3 The variation of dynamic lower yield stress, σ_{ys}^{dyn} minus the monotonic yield stress, σ_{ys}, with plastic strain rates for a variety of mild steels (after Kanninen et al (6))

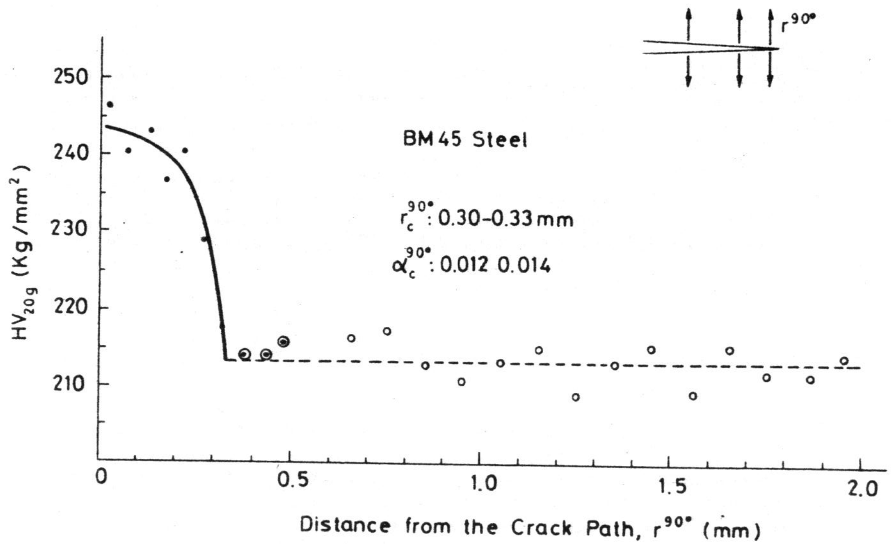

Figure 4 Cyclic plastic zone size determination by microhardness measurements on the specimen surface of BM 45 steel (2)