FATIGUE LIMIT ESTIMATION INCORPORATING RESIDUAL STRESSES ; A COMPARISON OF DIFFERENT FATIGUE CRITERIA

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Attempting to take into account the residual stresses in an estimation of fatigue limit leads to the problem of the calculation criterion. The state of stresses under load in fatigue may be monoaxial, while residual stresses are always multiaxial. If the designer of a part containing residual stresses wants to evaluate its fatigue limit, he must be able to include all stresses in his calculation.

Plane bending fatigue tests and measurements of stabilized residual stresses on induction-hardened parts, have shown that the Mises criterion currently used elsewhere does not correctly describe the true behaviour of a material. The criterion of CROSSLAND derived from the Mises criterion, which take into account hydrostatic pressure and the DANG VAN criterion based upon the maximum alternating shear stress and hydrostatic pressure, give a better correlation with experimental results.

KEYWORDS

Fatigue criterion, residual stresses, multiaxial stresses state.

INTRODUCTION

Although it is now known how to calculate the fatigue strength of part or a structure which is subjected to a uni-axial loading, the same may not be true for the more general case when the applied loading is multi-axial and the part is subjected to a complex static stress resulting from external loads or residual stresses introduced during manufacture.

However, simple methods exist which allow an approximate calculation to be carried out. They generally consist of the use of plasticity criteria ; such as that of Von Mises, and their extrapolation in fatigue. Although these methods, in simple loading, can result in a satisfactory approximation of the fatigue behaviour of the structure or the part, they are quite unable to take into account complex loading, and other criteria must then be used.

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In this study, it is therefore intended to examine the various possibilities which are offered to the engineer either for the calculation of the strength of a part which is subject to multi-axial fatigue loadings, or for taking residual stresses into account.

From the results of residual stress measurements and fatigue tests carried out on round bars of XC42 steel, superficially induction-heated and hardened it has been possible to demonstrate the inconvenient features of the MISES criterion, and the usefulness of those criteria which include the effect of hydrostatic pressure and the local shearing.

INFLUENCE OF RESIDUAL STRESSES ON FATIGUE LIFE IN THE CASE OF SURFACE TREATMENT

The macroscopic aspect of residual stresses means that the fundamental equations of elasticity can be applied to them, in particular the theory of superimposition. This means that, if a part is subjected to a residual stress field σR and to an operating stress σs, the part is in effect subjected to the loading $\sigma s + \sigma R$ (fig.1).

With regard to the strength of a structure in service, a residual stress will thus behave in the same way as a static or mean stress. The influence of the residual stress on the fatigue strength can then be quantified by using a HAIGH or GOODMAN endurance diagram, in which the mean stress for the fatigue loading σ_m is replaced by the new mean stress $\sigma'_m = \sigma_m + \sigma R$ (fig.1 and 2).
If $\sigma R > 0$ (as in the case of machining stresses due to grinding or turning), the alternating stress which the part can accept will be reduced. On the other hand, if $\sigma R < 0$ (as in the case of residual stresses produced by shotpeening, hammering, carburizing or induction hardening), the fatigue limit will be improved (fig.2).

When taking the residual stresses into account in a calculated prediction of fatigue life, the following considerations must be born in mind :

- When the residual stresses are subjected to cyclic loading they are greatly reduced. Depending on the applied loading, the type of treatment which produced the residual stresses, and the material concerned, this reduction may be total or partial. It is therefore important to know, for the part being considered, the nature of the reduction, in order to take into account the stabilised residual stress in the calculation. In order to estimate the reduction in the residual stresses, it is necessary to make measurements after fatigue testing, or to consult some existing published results.

- In practice, a uni-axial residual stress can not exist. Any surface treatment which generates residual stresses in a mechanical part will produce a bi-axial residual stress field at the surface and a tri-axial field in the sub-layers. If it is then wished to superimpose this complex residual : stress field on to a uni-axial alternating load, the application of a fatigue criterion is necessary. The use of only the residual stress which lies in the same direction as the applied fatigue stress can only be an approximation.

- The residual stresses are not the sole factor to be taken into account. A surface treatment, of whatever kind, will result in a change of :
  (i) the roughness,
  (ii) the superficial metallurgical structure, and
  (iii) the true loading, by the creation of residual stresses.

These three factors must, therefore, be taken into account in fatigue calculations.

RESULTS OF FATIGUE TESTS AND MEASUREMENTS OF RESIDUAL STRESSES ON CYLINDERS INDUCTRION-HEATED AND HARDENED

The fatigue tests involved repeated plane bending applied to cylinders, 36 mm in diameter made from XC42 steel, induction-heated and hardened. To eliminate the influence of the surface roughness, the test specimens were machined by fine turning, to give a surface roughness before treatment of 5 to 7 microns Rt (total height from highest peak to lowest valley)
The tables below give the chemical composition and the mechanical properties at annealed state and after water quenched.

- Chemical composition

C	Si	Mn	S	P
0.45	0.21	0.68	0.027	0.014

- Mechanical properties

State	Elastic limit $\sigma 0,02$ stress (MPa)	Ultimate strength Rm (MPa)	Elongation A%
Annealed	392	620	25
water quenched 52 HRC	1990	2150	1.5

For each treatment examined, measurements of the residual stresses where made on test specimens subjected to $5X10^6$ fatigue cycles, with an applied loading which corresponded to the fatigue limit for each treatment. Since the fatigue failures were initiated at the surface of the test specimens, the measurements were made at the surface. It is, in fact, the surface stress condition which, in this case, controls the fatigue strength. The measurements were made by means of X-rays. For each test specimen examined the longitudinal residual stress(σ_{Rl}) and the tangential residual stress (σ_{Rt}) were measured. (see table1).
In order to carry out a calculation of the fatigue strength in which the residual stresses can be taken into account, it is necessary firstly to know the fatigue strength of the steel forming the hardened case, in the absence of residual stresses. For this reason repeated tensile fatigue tests were carried out on test specimens of XC42 steel, uniformly hardened to 52HRC and approximating to the hardness of the treated case, obtained by induction hardening (54-56HRC). The ultimate strength of these uniformly-treated test specimens was then determined by means of a tensile test. From these results it was possible to draw the approximate endurance diagram (HAIGH type),which is probably close to the fatigue behaviour of a case obtained by induction hardening, with no residual stresses present.

Table I : Sammary of the results obtained for the four surface treatments

Ref. of treatment	Type and depth of treatment for 45 HRC	Surface hardness HRC	Fatigue limit at 5 X 10^6 cycles in MPa		Residual stress before fatigue MPa		Stabilited residual stress at the fatigue limit in MPa	
			σm	σa	$\sigma R1$	σRt	$\sigma R1$	σRt
A	2.7 mm (induction)	55-56	596	584	-338	-439	-128 -243	-468 -571
B	4.2 mm (induction)	55-56	623	610	-507	-633	-273 -341	-583 -676
C	4.7 mm (induction)	54-59	670	660	-770	-868	-655	-603
D	3.5 mm water-quenched after heating without stress relief annealing	60-61	780	750	-1043	-1072	-863 -777	-1132 -1156

CRITERIA DERIVED FROM THE MISES PLASTICITY CRITERION

As for incipient plastic deformation, fatigue cracks originates with shearing on crystallographic planes. It would thus appear correct to look for criteria which would attempt to take into account the maximum shear stress produced by the fatigue loading(s).
The MISES criterion, which only uses the octahedral stress, takes a mean value of the shear stress on differently-orientated grains and therefore gives fairly good results in the case of pure alternating radial loading.

KIOCECIOGLU'S criterion |1|

KIOCECIOGLU |1| has proposed the extension of the MISES criterion to fatigue for multi-axial loading alternating about a mean value.

In the case of fatigue cracks initiated at the surface, the stress condition is bi-axial, and σeqa and σeqm are then calculated from the relationships :

$$\sigma eqm = \pm (\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m}.\sigma_{2m})^{1/2} \qquad (1)$$

$$\sigma eqa = \pm (\sigma_{1a}^2 + \sigma_{2a}^2 - \sigma_{1a}.\sigma_{2a})^{1/2} \qquad (2)$$

The alternating stress obtained on $5X10^6$ cycles fatigue test (endurance limit) was plotted on the HAIGH diagram for each equivalent mean stress (fig.4). A very considerable scatter in the test results was noted, particularly in the case of those treatments which generate the highest residual compressive stresses.

The MISES criterion cannot therefore be used where there are high residual stresses or high mean stresses. On the other hand, it seems to be suitable for purely alternating combined loadings.

SINES' criterion (2)

Starting from the finding that a mean torsional stress has only a minor influence on the fatigue strength, SINES (2) has proposed a criterion which takes into account the effect of the different combinations of mean and alternating stresses. The criterion is a relationship between the alternating octahedral stress and the nominal mean octahedral stress :

$$\tau octa + \alpha \sigma octm = \beta \qquad (3)$$

The constants α and β are determined by fatigue tests for two values of the mean stress.
If the equivalent alternating stress in the MISES criterion sense and the mean hydrostatic pressure (pm) are used instead of the normal mean octahedral stress ($\sigma oct = pm = \frac{\sigma_{1m} + \sigma_{2m} + \sigma_{3m}}{3}$) the following equation may be written

$$\sigma eqa\ (MISES) + \alpha'.pm = \beta' \qquad (4)$$

CROSSLAND'S criterion (3)

CROSSLAND has proposed a criterion similar to that of SINES, but in which the maximum rather than the mean hydrostatic pressure is used. The criterion is written :

$$\tau octa + \alpha.\sigma oct\ max = \beta \qquad (5)$$

If the equivalent MISES stress and the hydrostatic pressure pmax are used, the relationship (5) becomes :

$$\sigma eqa + \alpha'.pmax = \beta' \qquad (6)$$

If the relationships (4) (6) are used to take the experimental results examined into account, figure 5, is obtained.

This representation provides a good correlation between the fatigue strength and the residual stresses. CROSSLAND'S criterion, as for that of SINES, clearly shows the improvement in the fatigue strength when the residual compressive stresses are increased. With these experimental results, an approximately linear limiting curve is obtained for the two criteria, well justifying a relationship of the type (4) or (6). The repeated tensiletest without residual stresses lies, on the limiting curve for the criterion. The difference between the two criteria reside in the value of σa corresponding to the pure alternating torsion (pm = pmax = 0)

FATIGUE CRITERIA MAKING USE OF THE MAXIMUM SHEAR STRESS

Criteria of FINDLEY (4) - MATAKE (5)

These various authors have separately proposed a criterion of the same type, making use, for the plane of maximum shearing, of :
- the alternating shear stress τa acting on this plane
- the normal shear stress σn acting on this plane

The criterion may then be written :

$$\tau a + B.\sigma_n < A \qquad (7)$$

The result obtained by means of this criterion on surface-hardened test specimens are plotted in figure 6(a).

The distribution of the experimental results reveals no linear correlation between the fatigue strength and the maximum normal stress because for pure alternating torsion (pmax = 0) the value of τa will be too high relating to the real value.

DANG VAN'S criterion (6)

The limiting curve for this criterion can be represented by a linear relationship of the form :

$$\tau a + \alpha . \text{pmax} = \beta \qquad (8)$$

The constants α and β are determined from reference fatigue tests. If, for example, the fatigue limits in pure alternating tension (σD) and in pure alternating torsion (τD) are known, the relationship (8) may be written :

$$\tau a + 3 \left(\frac{\tau D}{\sigma D} - \frac{1}{2}\right) \text{pmax} = \tau D \qquad (9)$$

The results obtained, using this criterion, on surface-hardened test specimens are plotted on figure 6(b). In the case of simple fatigue tests, as for plane bending for example, DANG VAN'S criterion gives the same results as CROSSLAND'S criterion.

CONCLUSION

Several criteria which may be used for the prediction of fatigue behaviour in complex stress conditions have been examined. From the results of fatigue tests and the measurements of residual stresses carried out on test specimens of XC42 steel superficially induction-hardened, it has been possible to draw the following conclusions :

- The MISES criterion cannot be used with loading which include mean or residual stresses. This criterion is acceptable only in the case of purely alternating loadings.

- The criteria of CROSSLAND and DANG VAN give a good estimation of fatigue strength when incorporating residual stresses. These criteria are similar and may be utilised in nonproportionnal loading after some modification.

- The comparison of FINDLEY'S and DANG VAN'S criteria shows that the fatigue behaviour of the parts studied is dependent on the maximum hydrostatic pressure, rather than the stress normal to the plane of maximum shear.

- The results presented and the criteria examined do not include the effect of the stress gradient (fatigue stress gradient or residual stress gradient). In attempting to predict the fatigue strength of parts with residual stresses and, for example, stress concentrations, other criteria should be used (7).

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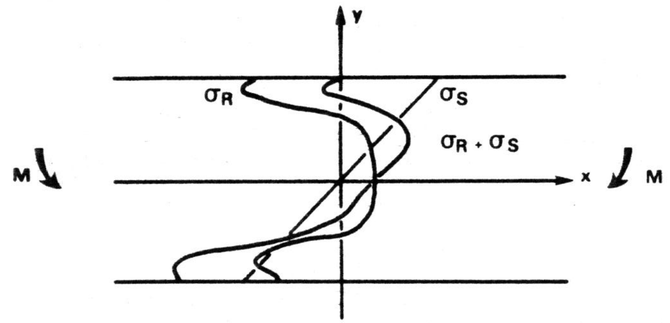


Fig. 1 : superimposition of residual stresses and operating stresses

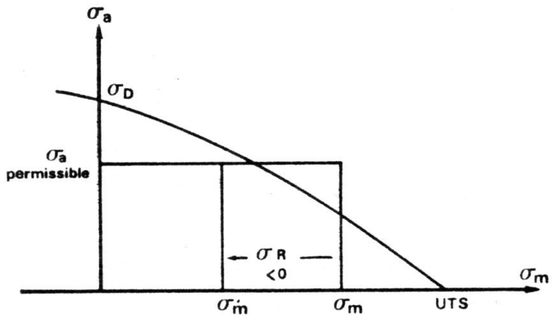


Fig. 2 : Use of the HAIGH diagram with residual stresses present

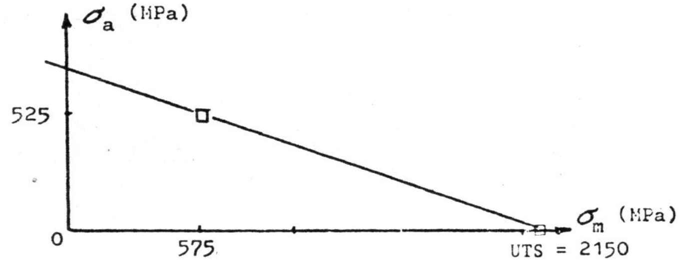


Fig. 3 : HAIGH endurance diagram for XC 42 steel with a hardness of 52 HRC

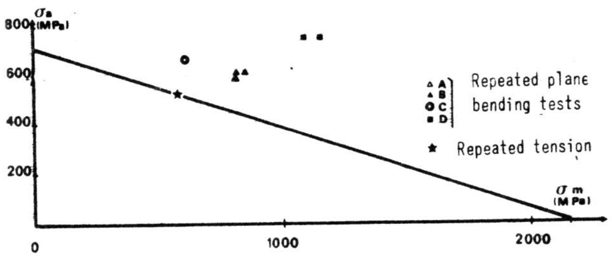
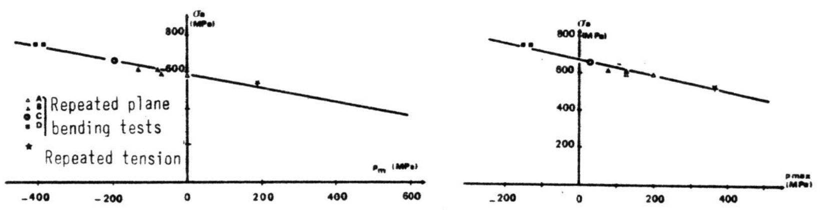


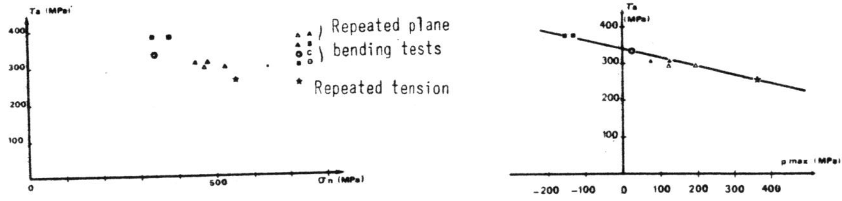
Fig. 4 : Comparison of the experimental results obtained by the MISES criterion with the reference endurance diagram.



(a)

(b)

Fig. 5 : Application of the experimental results with SINES criterion (a) CROSSLAND's criterion (b).



(a)

(b)

Fig. 6 : Application of the experimental results using FINDLEY's criterion (a), DANG VAN's criterion (b)