CRACKS IN THERMALLY SHOCKED PIPES

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By means of the weight functions method stress intensity factors were calculated for long axially orientated surface cracks in pipes subjected to thermal loading and internal pressure. Numerical values are given in dependence on crack depth and time for two different pipe diameters.

INTRODUCTION

In a pipe long axially orientated surface cracks may be considered as a conservative limiting case for semi-elliptical surface cacks with small a/c-ratios. Labbens et al. (1) treated this kind of problem by means of the finite element method with the aim of derivation of weight functions. Also by means of the finite element method Buchalet and Bamford (2) gained stress intensity factors for polynomial hoop stress distributions up to the third degree.

In this paper a long axially orientated crack located at the inner wall of a pipe is considered as shown in figure 1. The pipe is loaded by internal pressure and by thermal stresses. These are caused by a sudden cool down of the inner wall of the pipe, where the outer wall of the pipe is assumed to be perfectly isolated.

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METHOD OF WEIGHT FUNCTIONS

The fundamental equation of the weight functions method (3) may be written as

$$K_{new} = \frac{H}{K_R} \int_0^a \sigma_{new}(x) \frac{\partial u_R(x,a)}{\partial a} dx \qquad (1)$$

where K_R and u_R are the stress intensity factor and the crack opening displacement, respectively, of a reference solution with the same geometry and an arbitrary loading. The constant H is equal to $E/(1-\nu^2)$ in the case of plane strain problems. $\sigma_{new}(x)$ is the actual loading, for which K_{new} has to be calculated.

The reference stress intensity factor K_R is given by (2) as

$$K_R = \sigma_0 \sqrt{\pi a}\ F(a) \qquad (2)$$

The crack opening displacement field u_R is not given in (2). According to a proposal by Mattheck et al. (4) the following equation given by Petrosky and Achenbach (5) may be used :

$$u_R(x,a) = \frac{\sigma_0}{H\sqrt{2}} \left[4F(a)\sqrt{a}\sqrt{a-x} + G(a)\frac{(a-x)^{3/2}}{\sqrt{a}} \right] \qquad (3)$$

In the case of a constant reference loading σ_0 the unknown function $G(a)$ is found to be :

$$G(a) = \frac{5\pi}{a^2\sqrt{2}} \int_0^a [F(a)]^2\, a\, da - \frac{20}{3} F(a) \qquad (4)$$

By means of equations 1 to 4 the new stress intensity factors K_{new} may be calculated.

TEMPERATURE AND STRESS DISTRIBUTIONS

The hoop stress distribution $\sigma_{\varphi\varphi}$ acting as σ_{new} in our problem is well known from the corresponding boundary value problem of thermoelasticity. The following initial and boundary conditions have to be fulfilled.

For the temperature :

$$T(r,\tau \leq 0) = 0 \quad ; \quad R_i \leq r \leq R_a \qquad (5)$$

$$T(R_i,\tau > 0) = T_i \qquad (6)$$

$$\left. \frac{\partial T(r,\tau)}{\partial r} \right|_{r=R_a} = 0 \qquad (7)$$

For the stresses :

$$\sigma_{rr}(r = R_i) = -p_i \qquad (8)$$

$$\sigma_{rr}(r = R_a) = 0 \qquad (9)$$

R_i and R_a are the inner and outer radius of the pipe, τ denotes the time. The resulting hoop stress distribution is given by :

$$\sigma_{\varphi\varphi}(r,\tau) = \frac{R_i^2}{R_a^2 - R_i^2} p_i \left(1 + \frac{R_a^2}{r^2}\right) - \frac{E\alpha}{1-\nu} T(r,\tau)$$

$$+ \frac{E\alpha}{1-\nu} \frac{1 + R_i^2/r^2}{R_a^2 - R_i^2} \int_{R_i}^{R_a} x\, T(x,\tau)\, dx + \frac{E\alpha}{1-\nu} \frac{1}{r^2} \int_{R_i}^{r} x\, T(x,\tau)\, dx \qquad (10)$$

The temperature distribution in equation 10 has the following form

$$T(r,\tau) = T_i - \pi \sum_{n=1}^{\infty} \left[e^{-\kappa a_n^2 \tau}\, T_i\, J_1^2(a_n R_a) \qquad (11)\right.$$

$$\left. \frac{J_0(a_n R_i)\, Y_0(a_n r) - Y_0(a_n R_i)\, J_0(a_n r)}{J_0^2(a_n R_i) - J_1^2(a_n R_a)} \right]$$

where J_0, J_1, Y_0 and Y_1 are the Bessel and Neumann functions, respectively, of order zero and one. The eigenvalues a_n have to be calculated from

$$J_1(a_n R_a)\, Y_0(a_n R_i) - J_0(a_n R_i)\, Y_1(a_n R_a) = 0 \quad . \qquad (12)$$

Equations 10 to 12 were evaluated numerically for two pipe geometries with R_i = 50 mm and R_a = 55 mm, and R_i = 60 mm and R_a = 66 mm, respectively. The material parameters chosen were E = 200 000 N/mm^2, ν = 0.3, α = 1.2 · 10^{-5} K^{-1}, κ = 4.219 mm^2/s, and T_i was assumed to be equal to -120°C. The internal pressure p was chosen to 200 bar. Some results are given in figures 2 and 3.

RESULTS FOR THE STRESS INTENSITY FACTORS

The hoop stresses calculated for different times were now taken as σ_{new} in equation 1 to gain stress intensity factors K_{new} depending on time and crack depth. The evaluation was done numerically. Some results are shown in the figures 4 to 7. The stress intensity factors calculated for different times are presented in the figures 4 and 5 for the two pipe geometries considered in dependence on crack depth. The figure 6 gives the time dependence of the stress intensity factors with the crack depth as parameter for the pipe with R_i = 50 mm. For the same pipe geometry a comparison of the maximum values of the stress intensity factors with the stationary values (when the thermal stresses are zero) is shown in figure 7.

DISCUSSION OF THE RESULTS

As can be seen from figure 6, the stress intensity factors show a maximum value at relatively short times. For longer times they are decreasing monotonically until the stationary values are reached. The maxima lie between 0.3 s for small values of the crack depth and 0.6 s for a/t =0 .8 in the case of the pipe geometry with R_i = 50 mm and between 0.5 s and 0.8 s in the case of the pipe with R_i = 60 mm, respectively.

As figure 7 shows the thermal stresses are the reason for an increase of the stress intensity factor of between about 20 % in the case of large crack depths and about 150 % in the case of small crack depths (a = 1 mm) in comparison to the values with internal pressure loading only.

Although this increase disappears again for longer times, it should be taken into account in the case of fracture mechanics considerations of pipes which eventually may be subjected to a thermal shock loading.

NOTATIONS

a	= crack depth (mm)
R_i	= inner radius of the pipe
R_a	= outer radius of the pipe
K_R	= stress intensity factor of the reference solution ($Nmm^{-3/2}$)
u_R	= crack opening displacement of the reference solution (mm)
σ_{new}	= actual loading of the crack (Nmm^{-2})
K_{new}	= stress intensity factor to be calculated ($Nmm^{-3/2}$)
$T(r,\tau)$	= temperature (°C)
$\sigma_{\varphi\varphi}(r,\tau)$	= hoop stress (Nmm^{-2})
τ	= time (s)

REFERENCES

1. Labbens, R., Pellissier-Tanon, A., and Heliot, J., ASTM STP, 590, (1976) 368.

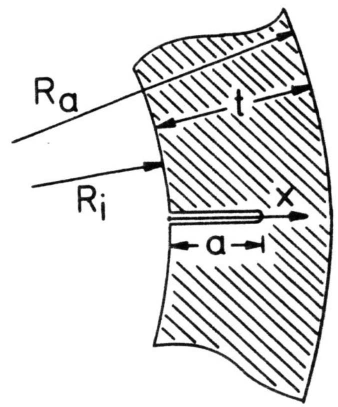
2. Buchalet, C.B., and Bamford, W.H., ASTM STP, 590, (1976) 385.

3. Rice, J.R., Int.J.Sol.Struct., 8, (1972) 751.

4. Mattheck, C., Morawietz, P., and Munz, D., Int.J.Fract., 23, (1983) 201.

5. Petroski, H.J., and Achenbach, J.D., Eng.Fract.Mech., 10, (1978) 257.

Fig.1 : Part of pipe cross section with axial crack



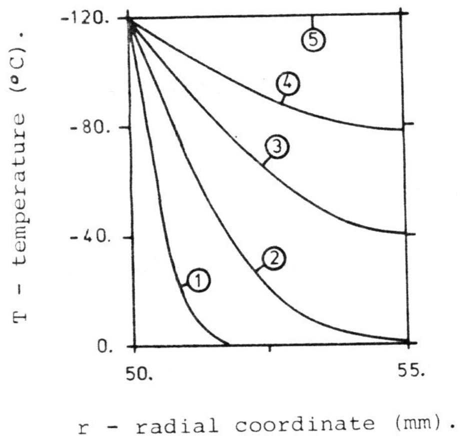
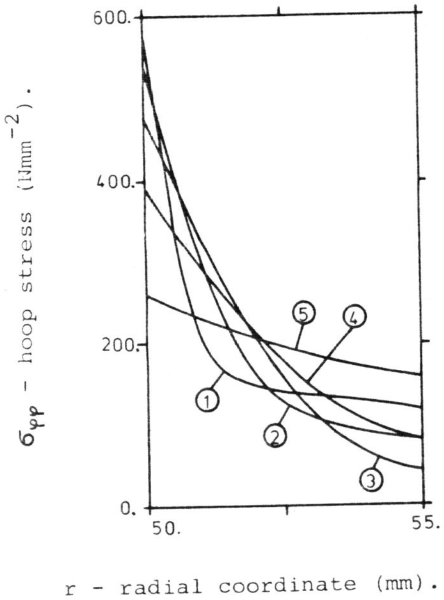


Fig.2 : Temperature distribution through the wall (time steps as in fig. 3)

Fig.3 : Hoop stresses through the wall (1:τ=0.05s, 2:τ=0.2 s, 3:τ=0.6s, 4:τ= 1.6s, 5:τ=4.8s)



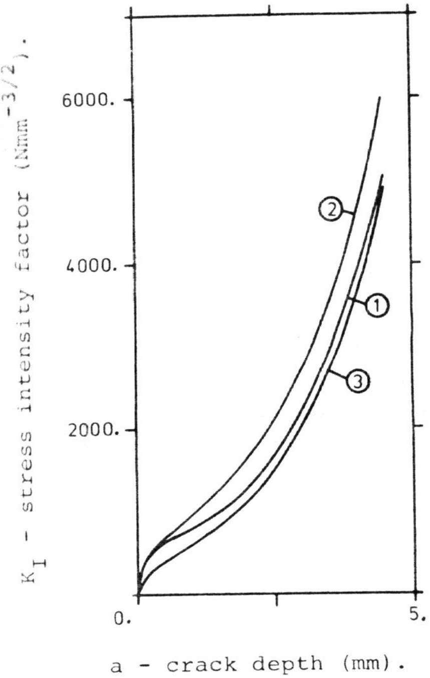
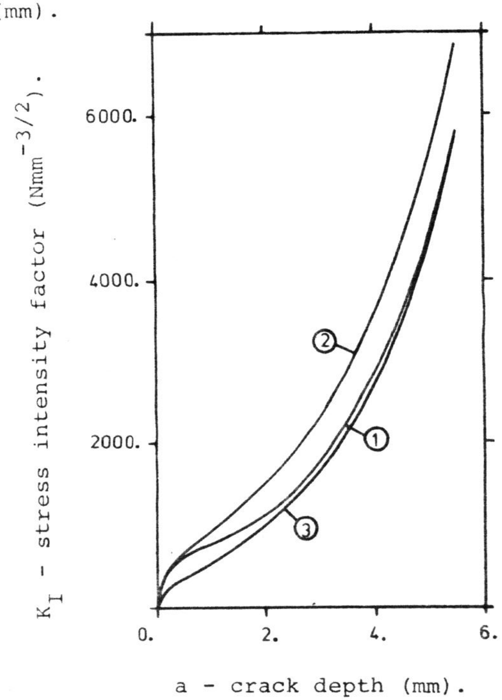


Fig.4 : Stress intensity factors in dependence on crack depth for the pipe with R_i=50mm (1:τ= 0.05s, 2:τ=0.6s, 3:τ =4.8s)

Fig.5 : Stress intensity factors in dependence on crack depth for the pipe with R_i=60mm (1: τ=0.05s, 2:τ= 0.8s, 3:τ=8s)



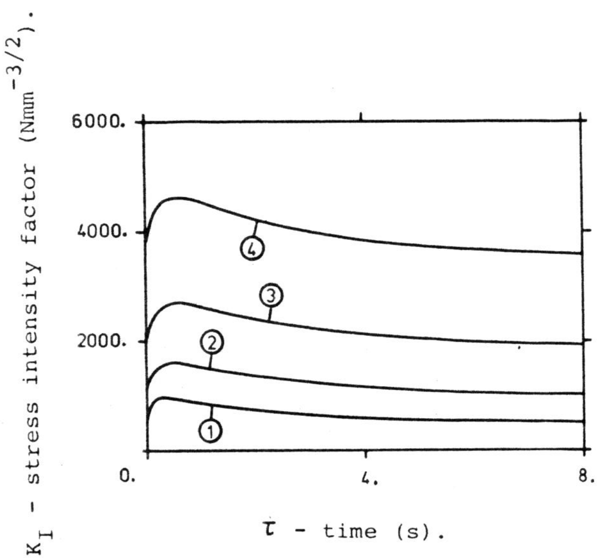


Fig 6 : Stress intensity factors in dependence on time for the pipe with R_i=50mm (1:a=1mm, 2:a=2mm, 3:a=3mm, 4:a=4mm, 5:a=5mm)

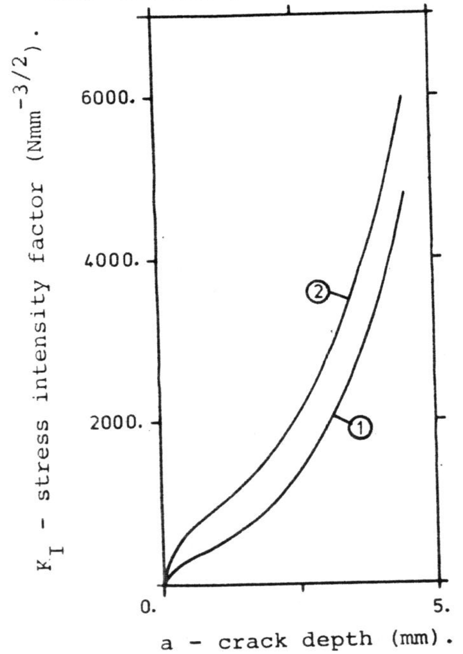


Fig.7 : Stress intensity factors in dependence on crack depth for the pipe with R_i=50mm (1:maximum values with thermal stresses for τ =0.6s, 2:without thermal stresses at $\tau \to \infty$)