A CRITICAL DEPTH CRITERION FOR THE EVALUATION OF LONG-LIFE FATIGUE STRENGTH UNDER MULTIAXIAL LOADING AND A STRESS GRADIENT

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The analysis of fatigue test data from technical literature has shown the possibility of using a critical layer characterizing the microstructural state of the material, and representing the elementary volume which contains the damage process during fatigue loading. Combining this parameter with a criterion for multiaxial fatigue gives a new intrinsic fatigue criterion of a material which should allow the computation of fatigue life of notched parts under multiaxial stresses. Further experimental work is needed to fully validate this model.

KEYWORDS

Fatigue criterion, notched parts, multiaxial stress state, long-life fatigue.

INTRODUCTION

Most fatigue cracks in mechanical parts initiate in stress concentration zones where the material is simultaneously submitted to a complex state of stresses and a stress gradient.

When designing notched parts, for example, submitted to multiaxial stresses the mechanical engineer is faced with two problems ; first he must have a long-life fatigue criterion for the material submitted to the complex stress state, allowing a forecast of behaviour from uniaxial fatigue testing results (under tension, rotating bending etc...) Then he must be able to introduce into this criterion the concept of stress gradient which, as is well known, influences considerably fatigue behaviour. A great deal of research has been done on both aspects of this problem, but there has been until now no general criterion available to take simultaneously into account the stress gradient and the multiaxial stress state.

Hereafter, we present research work aiming to establish the fundamentals of a general criterion. Analysing results of fatigue tests on notched specimens we have been able to illustrate the use of critical depth characterizing the microstructural state of material.

This new parameter should be representative of the elementary volume active during the damage process of fatigue crack initiation. Integrating this

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concept into a fatigue criterion under multiaxial stresses gives a general criterion which agrees well with experimental data.

DEVELOPMENT OF A FATIGUE CRITERION FOR MULTIAXIAL STRESSES AND A STRESS GRADIENT

The development of a long-life fatigue criterion of general validity should take into account several experimental facts :

1. The mechanism corresponding to the initiation of fatigue cracks is most often the shearing of crystallographic planes. It appears logical then to have a criterion relating the complex stress state to an alternating shear stress which might be the local shear stress in the most favourably oriented plane (TRESCA criterion), or the local octahedral shear stress in a (111) plane with respect to the three main directions (MISES criterion). DANG VAN has shown that in case of proportional loadings, this local shear has the value of macroscopic alternating shear |2|.

2. The hydrostatic pressure or octahedral normal stress strongly affects the fatigue strength. One may explain numerous testing results by taking it into account.

3. The criterion in question shall be valid in the case of proportional multiaxial loading, that is for in-phase varying stresses, but also in the more general case of non-proportional or out-of phase loading. For those reasons, we have selected DANG VAN's |2| and CROSSLAND's |3| criteria for the treatment of the experimental data. They are expressed by relationships (1) and (2) respectively

$$\Delta\tau + \alpha \; P_{max} < \beta \quad \text{(DANG VAN)} \qquad (1)$$

$$\Delta\tau_{oct} + \alpha' P_{max} < \beta' \quad \text{(CROSSLAND)} \qquad (2)$$

where $\Delta\tau$ is the amplitude of the local shear stress in the most favourable plane, $\Delta\tau$ oct the amplitude of the local octahedral shear stress, and Pmax the maximum hydrostatic stress produced by fatigue cycle. Both criteria are rather similar, and the may also be usable with some modifications in the case of general loading (non-proportional). For non proportional loading, the relationship suggested by FUCHS for the MISES criterion |4| may be used for the CROSSLAND criterion.

In view of introducing the concept of stress gradient into a multiaxial stress fatigue criterion, the following consideration should be taken into account : the fatigue behaviour of a material cannot be explained alone by the stress state at some particular position. The heterogeneity of metals and the stress distribution underneath the surface where the fatigue cracks initiate are active in any fatigue damage process. It is necessary to undertake any calculation on a volume large enough to represent the true behaviour of the material. This has brought about the idea of a volume or a distance which would characterize the material. About thirty years ago NEUBER and HEYWOOD have already used a characteristic length ρ in the definition of the fatigue notch factor Kf.
STIELER |5| suggested in 1954 the concept of "critical layer" in order to take into account the phenomenon of elastic shakedown at the notch tip. More recently, DEVAUX |6| using elastic linear fracture mechanics has shown that there was a possibility to predict crack initiation with a criterion based upon a distance "d" characterizing the material. MATAKE and IMAI |7| applying the FINDLEY criterion to torsion bending test results, calculated the mean value of stresses for half the thickness of a grain. Moreover, the analysis of the initiation and crack growth threshold of short fatigue cracks |8| |9| has shown the important effect of grain size, and emphasized the need to take into consideration the stress state within a sufficiently large volume, or at some defined distance from the surface. These various studies show that it should be possible to determine a critical depth (layer, thickness) characterizing each material, which would be used as basic information for calculation.

DEMONSTRATION OF THE CRITICAL LAYER

Several series of fatigue tests under tension-compression on cylindrical notched specimens have been analysed |10|, from data obtained from |13| |14| |15|,|16| applying both the DANG VAN and the CROSSLAND criteria. Each geometry has been analysed by the finite element method, and the variation of the principal stresses σ_1, σ_2 and σ_3 in relation to depth has been determined. Moreover, the distribution of the amplitudes of the shear stress τa and the maximal hydrostatic stress p max have been calculated for the fatigue limit.

$$\tau a = \frac{\sigma_{1a} - \sigma_{3a}}{2}$$

where σ_1a and σ_3a are the amplitudes of the two extremal principal stresses

$$P\ max = \frac{\sigma_{1max} + \sigma_{2max} + \sigma_{3max}}{3}$$

In order to get a very accurate distribution of stress in the neighbourhood of the notch, the finite element mesh was refined in the layer near the surface, the size of the elements being between 7 to 15 µm.

When plotting the calculated values for the surfaces of a specimen in a graph of τa versus Pmax the curves in fig. 1 are obtained. For each geometry the experimental data adheres fairly well to the linear relationship τa - pmax. It would seem that the DANG VAN criterion correctly accounts for the experimental data, but it cannot be considered as an intrinsic criterion, the relationship $\tau a - P_{max}$ being geometry dependent.

If in place of the values of τa and pmax calculated for the surface of the specimen we plot the calculated values for some definite distance from the surface (depth) which depends upon the material, for instance 50 µm for AFNOR 35 CD 4 steel with a tensile strength of 1000 MPa, the experimental data are regrouped and align themselves along a single straight line, whatever the specimen geometry. This line expresses the "behaviour law" of the material for unnotched specimens (fig. 2). The off-line values are experimental results for cases where a macroscopic plastic strain occured at the notch tip (hydrostatic pressure $> \frac{\text{cyclic yield stress}}{3}$).

The following table gives the depth of the critical layer CC, in µm for various metals. The data has been obtained from a small number of results and should be considered approximate, pending further work.

Metal references		Heat treatment	UTS (MPa)	CC (μm)
XC 38	|14|	annealed	585	70
35 CD 4	|13|	quenched and tempered	915 1000 1500	50 50 40
35 NCD 16	|14|	quenched and tempered	1270	50
TA 6 V	|15|	ann. 730°C, 2 h	1020	180
TA 6 VE2	|15|	quenched and tempered	1190	120
AU 4 G1	|16|	quenched and tempered	480	40
AZ 5 GU	|16|	T 7351	480	80
A7U 4 SG	|16|	T 651	480	80

In order to take into account the physical mechanisms of crack initiation, particularly shear of the crystallographic planes in a grain of metal, it would be advisable to consider mean values of shear $\overline{\tau a}$ and hydrostatic stress $\overline{pmax}$ from the surface to the critical depth. When retracing the graph of $\overline{\tau a}$ versus $\overline{pmax}$ for the critical depth giving the best alignment of the data, values of the foregoing table are obtained with an approximation ratio of 2 (fig. 3) due to the definition of the mean values of $\overline{\tau a}$ and $\overline{pmax}$.

The critical depth may be more accurately determined by plotting the graph $\overline{\tau a}$ - $\overline{pmax}$ for various depths, and selecting the one which gives the best correlation coefficient ρ, and the least standard mean square deviation σ^2 for the linear regression $\overline{\tau a}$ versus $\overline{pmax}$ (fig. 4).

When using the mean values $\overline{\tau a}$ and $\overline{pmax}$, the critical depth will be defined as the half-thickness of the volume element giving the best fit of the experimental data with the fatigue data obtained on specimens without stress concentration. If the CROSSLAND criterion is used in place of the DANG VAN criterion, plotting the representative points on a graph $\overline{\tau oct}$ - $\overline{pmax}$ will give the best alignment of data for the same values of the critical layer depth. The DANG VAN criterion will however give a better correlation for the alignment of the experimental data.
The value of the critical depth will be different according to the material concerned. The calculated critical depth depends upon the microstructure. It is approximately 70 μm for a XC 38 steel in the annealed condition, hence larger than for hardened and tempered 35 CD 4 and 35 NCD 16 steels whose microstructures are finer.
It should be noted that for a 35 CD 4 steel heat treated to a tensile strength of 1500 MPa, the value of 40 μm for the calculated critical layer depth is less than for the same steel heat treated to 1000 MPa, where it is 50 μm. It seems logical that the concept of a critical layer is related to microstructure. STIELER had already mentioned that the depth of that layer should be of the size of a grain, what would be justified if we consider that slip must affect at least one grain to initiate damage. Unfortunately, the microstructure of quenched and tempered steels is highly complex and various grains can be defined for these steels, all of different sizes (lats, plates, packets, prior austenite grains).

TAKING INTO ACCOUNT PLASTIC STRAIN AT THE NOTCH TIP

In the foregoing analysis, we have neglected the experimental results implying macroscopic plastic strain at notch tip, hence a MISES equivalent stress larger than the cyclic yield stress of the metal.

The plastic strain under cyclic stresses alters the stress state in the critical zone. As a result the maximal hydrostatic stress is altered due the plastic deformation, while the shear amplitude remains unchanged. Computation by the finite element method (CASTOR EVP 2 D program of CETIM) for elastoplastic conditions gives the new stress distribution. Introducing the data of simple tensile testing on steel we obtain a value inferior with respect to computation data for the elastic state (fig. 5 and 6). This first computation is however too short to take into account the cyclic plastic strain at the tip of the notch. An iterative computation including the strain-hardening law during a sufficient number of iterations must be made to get the stress state after elastic shakedown, but it is a long and costly procedure. J. Zarka and J. Casier |12| have suggested a simplified method which gives within a short time an approximate value of stresses under stabilized cycling. Introducing the data of the cyclic-hardening curve into the computation, the new procedure gives new values for pmax (fig. 5 and 6). Taking into account cyclic strain-hardening brings the outlying data closer to the intrinsic curve of the material. These data however still remain off the intrinsic behaviour law of the material. It may be assumed that in the case of confined plastic strain the data points representing the material behaviour are obeying a specific law in an approximate stress range of

$$pmax > \frac{\text{cyclic yield stress}}{3} .$$

Further work should define an equation for this behaviour law from an equation of elastic behaviour $\overline{\tau a} + \alpha \overline{pmax} = \beta$ and a relationship for cyclic strain hardening.

CONCLUSION

The analysis of fatigue tests under tensile-compression stressing on notched specimens has illustrated a critical depth characterizing the microstructural state of various metals. Introducing this parameter into criteria of fatigue under multiaxial stresses may explain the fatigue behaviour under complex stressing and a stress gradient.
The concept of "critical depth" combined with a fatigue criterion valid for non-proportional loading, such as the DANG VAN or the CROSSLAND modified criterion, should result in a long-life fatigue criterion of general validity for any geometry and loading. Further work should be done to check the validity of such criteria for non-proportional loading, and to try to relate the concept of a critical depth with the microstructure of the metal.

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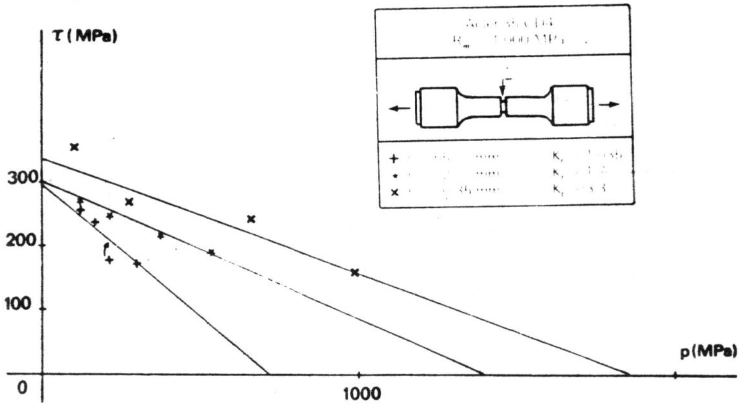

Fig. 1 : τa - pmax graph plotted with calculated data for the surface at fatigue limit.
r : radius at tip of notch

35 CD 4 - UTS = 1000 MPa - YS = 900 MPa.

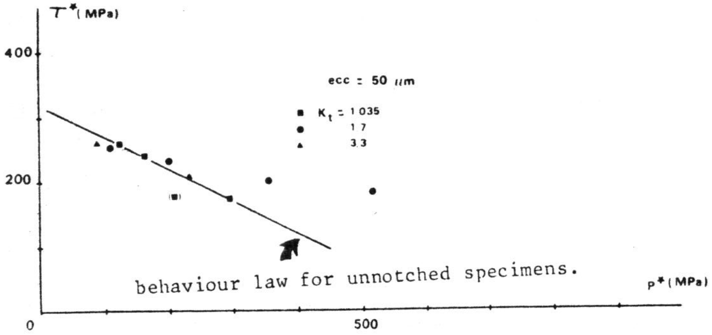

Fig. 2 : $\overset{*}{\tau a}$ - $\overset{*}{p}$max graph plotted with calculated data for a depth of 50 μm from the surface. Elastic analysis.

AFNOR 35 CD 4 Steel (U.T.S. = 1000 MPa).

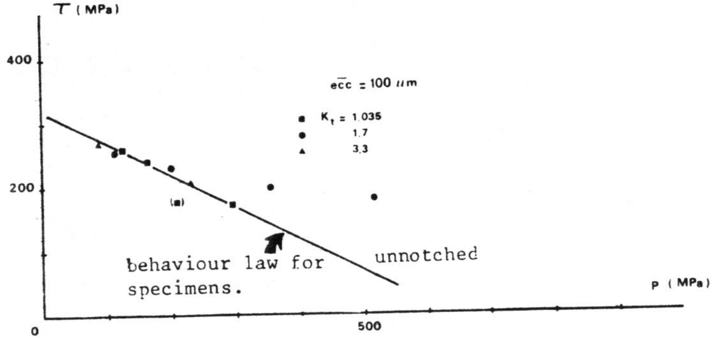

Fig. 3 : $\overline{\tau a}$ - $\overline{p\text{max}}$ graph plotted with mean values calculated for the depth of the critical layer. Elastic analysis.

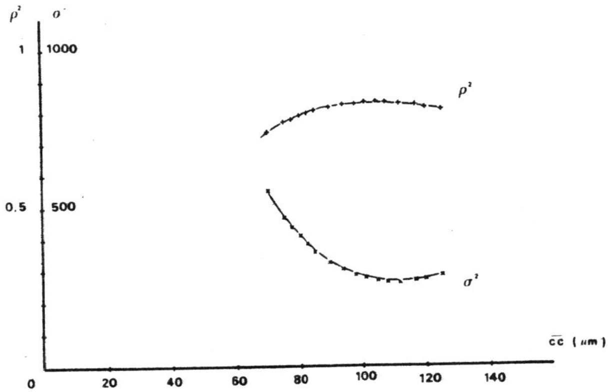

Fig. 4 : Determination of the critical layer depth for 35 CD 4 steel (U.T.S. 1000 MPa) with the correlation coefficient ρ and the standard mean square deviation σ^2.

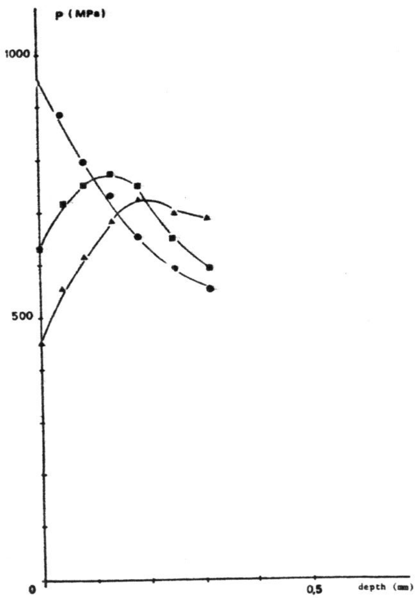

Fig. 5 : Variation of the maximal hydrostatic stress with depth. Computation taking or not into account cyclic strain. Mean stress = 500 MPa.Kt = 3.3.

- ● elastic analysis
- ■ elastoplastic analysis with simple tensile curve
- ▲ elastoplastic analysis with cyclic strain-hardening curve.

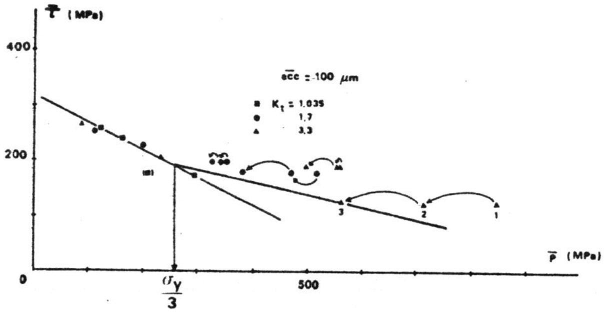

Fig. 6 : τ_a - p_{max} graph plotted with computed data when taking into account plastic strain at the notch tip.

1. elastic analysis
2. elastoplastic analysis with simple tensile curve
3. elastoplastic analysis with cyclic strain-hardening curve

σ_y : cyclic yield stress.