

MICROSTRUCTURAL SHORT CRACKS IN FATIGUE

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Recent work on short crack growth is reviewed. The differences between physically small cracks and microstructurally short cracks are discussed. A crack growth equation for short cracks is presented which includes a microstructural parameter  $d$ . A small crack growth equation is also presented expressed in terms of plastic strain range. A third equation for long crack behaviour in linear elastic fracture mechanics terms is also presented and reasons given why elastic parameters should not be applied to any other but long cracks subjected to low stress.

INTRODUCTION

It is not disputed that microstructurally short cracks can not be evaluated by linear elastic fracture mechanics (LEFM). It is equally obvious that long cracks at low stress levels have successfully been analysed by the LEFM approach. The problem is to find a compatible form of fracture mechanics that can bridge the gap between short and long crack growth behaviour.

In order to arrive at a solution acceptable to both researchers and engineering designers, current work on microstructurally short cracks will be briefly presented and the usefulness of the plastic shear strain range ( $\Delta\gamma_p$ ) and microstructural unit size,  $d$ , emphasized. The inability of LEFM parameters to quantify microstructurally short and physically small cracks will also be presented. The relative importance of short, small and long cracks in fatigue lifetime will also be discussed and finally the importance of the link between short and long cracks emphasized.

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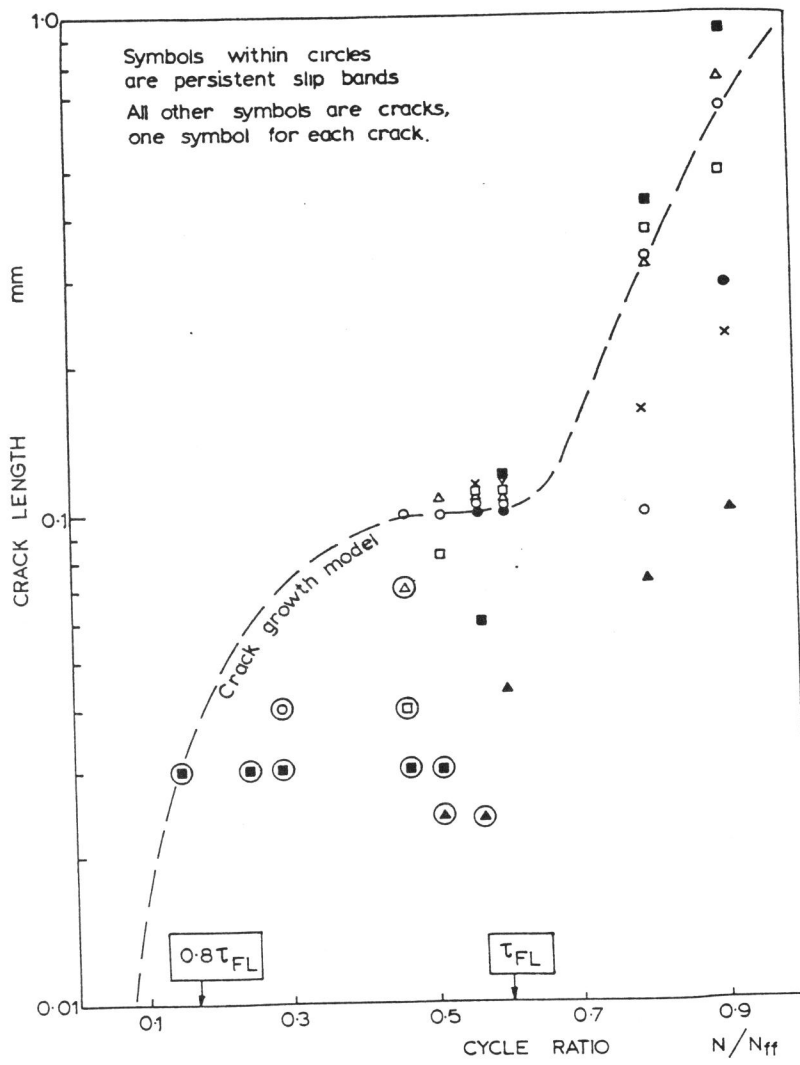


Fig 1. The accumulation of fatigue damage in an increasing shear stress range test.

MICROSTRUCTURALLY SHORT CRACKS

Fig 1. presents experimental data reported in the first european special technical publication on Short Cracks (1). The initial shear stress range level  $\Delta\tau_0$  was below the fatigue limit ( $\Delta\tau_0 \sim 0.7 \Delta\tau_{FL}$ ) and the development of slip bands and microcracks were recorded as a function of lifetime in a slowly increasing stress range test. Fatigue damage in the form of cracks was observed long before the stress range reached the level of the fatigue limit stress. Crack growth began quickly but then slowed down as the first barrier was reached, which, at these low stress levels was the first ferrite grain boundary. The largest ferrite grains of about  $100\mu\text{m}$  diameter produced the most dangerous cracks. Surface crack growth, for a plastic shear strain range  $\Delta\gamma_p$ , was given by

$$da/dN = A \Delta\gamma_p^\alpha (d - a) \mu\text{m/cycle} \quad [1]$$

with  $A = 6$ ,  $\alpha = 2.24$  and  $d = 100 \mu\text{m}$ .

In constant stress range tests, data similar to that given in Fig 1 was also recorded via a replication technique for the heavily banded 0.4%C ferrite-pearlite structure, see Fig 2. In this case however the major barriers to growth were the pearlite bands which had an average spacing  $d = 330\mu\text{m}$ . Since the fatigue limit stress  $\Delta\tau_{FL}$  was higher than  $\Delta\tau_0$  the ferrite grain boundaries did not present a significant barrier to growth in the constant stress range tests. The important point however is that crack growth as represented by equation [1] described the behaviour of microstructural cracks in both kinds of test by using the appropriate dimension  $d$ .

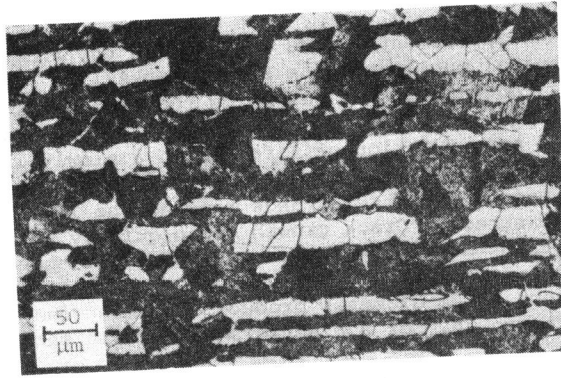
$$d = 100 \mu\text{m} \text{ for } \Delta\tau < \Delta\tau_{FL}$$

$$d = 330 \mu\text{m} \text{ for } \Delta\tau > \Delta\tau_{FL}$$

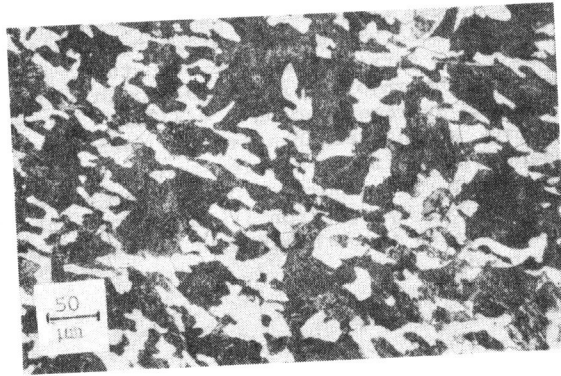
LONG AND SMALL CRACK GROWTH BEHAVIOUR

Davidson and Lankford (2) discussed the behaviour of small and long cracks in aluminium alloys and their results are summarized in Fig 3. They concluded that since small cracks fail to satisfy the similitude requirements which validate the use of the stress intensity factor range  $\Delta K$ , this factor cannot be used for small cracks in the same way that it is used for large cracks. The three diagrams of Fig 3 illustrate the fundamental difference in behaviour between small and large cracks. Such results can be appreciated if one examines the basis of similitude in linear elastic fracture mechanics analyses.

Very large cracks in engineering plant can be equated to smaller cracks in laboratory samples if and only if the elastic



Longitudinal Section



Transverse Section

Fig 2. The microstructure of the medium (0.4%) carbon steel used in the experiments.

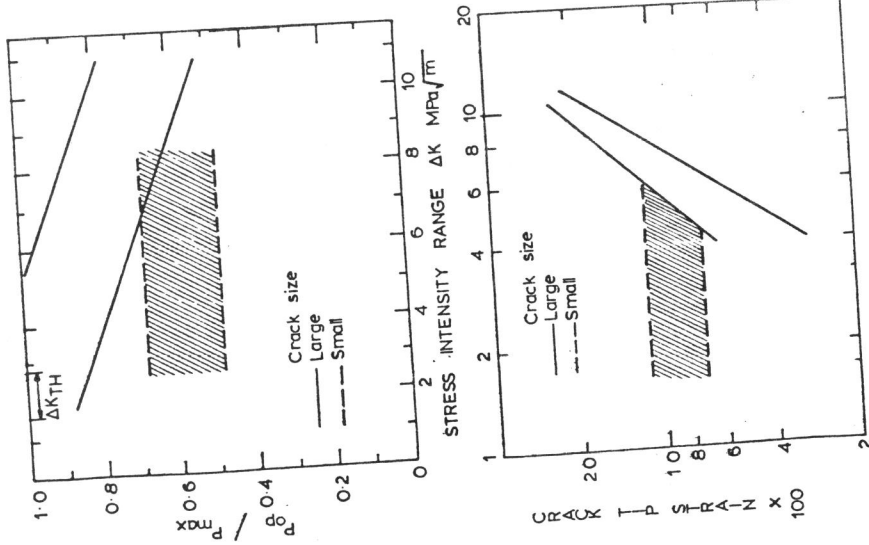
The mean and standard deviation distances are

Ferrite grain sizes:-  
 $35 \pm 26 \mu\text{m}$  (longitudinal) and  $38 \pm 23 \mu\text{m}$  (transverse)

Distances between Pearlite bands:-  
 $140 \pm 93 \mu\text{m}$  (longitudinal) and  $50 \pm 31 \mu\text{m}$  (transverse)

Fig 3  
Differences in growth behaviour between long (low stress) cracks and small (high stress) cracks in 7075 Aluminium Alloy; after (2)

(a) Opening load vs  $\Delta K$   
 (b) Crack tip strain vs  $\Delta K$   
 (c) Crack tip strain vs Crack tip opening.



stress intensity field at the tip of both cracks are similar. Significant loss of similitude occurs when stresses are increased to above 0.3 the yield stress of a material simply because the crack tip plastic zone now becomes so large as to significantly perturb the elastic stress field. Expressed another way the LEFM requirement of only small-scale yielding at the crack tip no longer applies. Even for conventional LEFM specimens errors of 1.5% and 7% occur for CCP and CT specimens respectively. An added complication is that of mean stress since this also controls the size of the plastic zone. Fatigue tests carried out by Hopper and Miller (3) showed that correlations for fatigue crack growth under biaxial fatigue conditions could be achieved only on a basis of similitude of plastic zone sizes and not by invoking LEFM parameters.

From the above remarks it can be appreciated that several zones of crack growth behaviour can be isolated and Fig. 4 illustrates the boundaries of six zones. Obviously at the common boundary between two zones, the laws describing the behaviour of cracks should display identical characteristics. A full description of the boundaries and zone types has been given by Brown (4).

In an endeavour to bridge the gap between physically long and microstructural short cracks two approaches naturally present themselves. The first is to modify equation [1] to exclude microstructural effects; this will be discussed later. The second is to modify LEFM analyses. This latter approach has gained much support because LEFM is now standard laboratory practice for quantifying crack growth behaviour. Therefore a modification of  $\Delta K$  in the form of the effective stress intensity factor  $\Delta K_{\text{eff}}$  is frequently used. This parameter takes into account the fact that cracks do not start to open on loading or to close on unloading at the minimum stress level. Much experimental evidence shows that crack closure effects are dominant at  $\Delta K$  values close to threshold but are less severe at higher maximum stress and  $\Delta K$  levels. By invoking  $\Delta K_{\text{eff}}$  arguments (i.e. the value of  $K_{\text{max}} - K_{\text{opening}}$  or the value of  $\Delta K$  to represent the period during which the crack is open) a lower threshold value is achieved, i.e.  $(\Delta K_{\text{eff}})_{\text{th}} < (\Delta K)_{\text{th}}$ ; this helps correlate some of the small crack data with long crack data.

There are however serious problems with the  $\Delta K_{\text{eff}}$  approach which limits its general applicability. This is not to say that the closure phenomenon is not real - it is and has been accurately measured by several reputable workers. Furthermore it satisfactorily explains why fatigue cracks behave in the manner they do. The problems, some but not all of which may be overcome eventually, are:-

1. For similitude conditions the closure effect requires to be the same, i.e. the same mechanism needs to be involved

to the same extent. As yet there has been no quantitative distinctions between closure due to (a) plasticity behind the crack tip, (b) residual compressive stresses ahead of the crack tip, (c) corrosion products wedging the crack open, (d) crack surface roughness due to crack path deviations and (e) microstructural variability. When stress levels and/or crack lengths change, the individual contributions of each of the above causes of closure will themselves change so destroying similitude conditions.

2. In tension type tests K factors are calculated on a basis of Mode I loading but as cracks grow from the surface to the interior the mode of crack advance also changes from mode II to mode I, i.e. the fatigue process changes from a mode II shear crack of the Stage I kind to a mode I tensile Stage II kind. At and close to the fatigue limit it is the short shear crack which dominates the early life period.
3. The designer faced with a specific crack length in a component or structure and a given loading pattern can determine  $\Delta K$  but can not calculate a  $\Delta K_{eff}$  value since this parameter relies on too many extrinsic factors.
4. The factor K is not valid at high stress levels, i.e. when  $\Delta\sigma > 2\Delta\sigma_{cy}/3$ , since the assumptions on which the mathematics of K calculations are based no longer hold; see (4). Even the use of the J contour integral is highly suspect since at high stress and small crack lengths the crack tip plastic zone is not contained at the crack tip by a surrounding elastic field but breaks out on to the specimen surface (5).
5. The factor  $K_{th}$  itself (disregarding  $\Delta K_{eff}$ ) is only an elastic approximation and can not be applied to practical situations in which components and structures are subjected to complex stresses due to combinations of bending, torsion and axial loading, i.e. mixed mode loading (6).
6. Fracture, unlike deformation, requires a two parameter description to adequately describe the growth behaviour of cracks (7). As stated previously  $\Delta K$  alone is insufficient particularly in biaxial situations (3).
7. Crack closure effects vary along the whole crack front from the surface (where closure is invariably measured) to the mid-section plane. Therefore the size of the component is important. A simple one-dimensional appreciation of crack closure across the entire crack front is not valid. Furthermore microstructurally short and physical small cracks have a two-dimensional shape, e.g. elliptical,

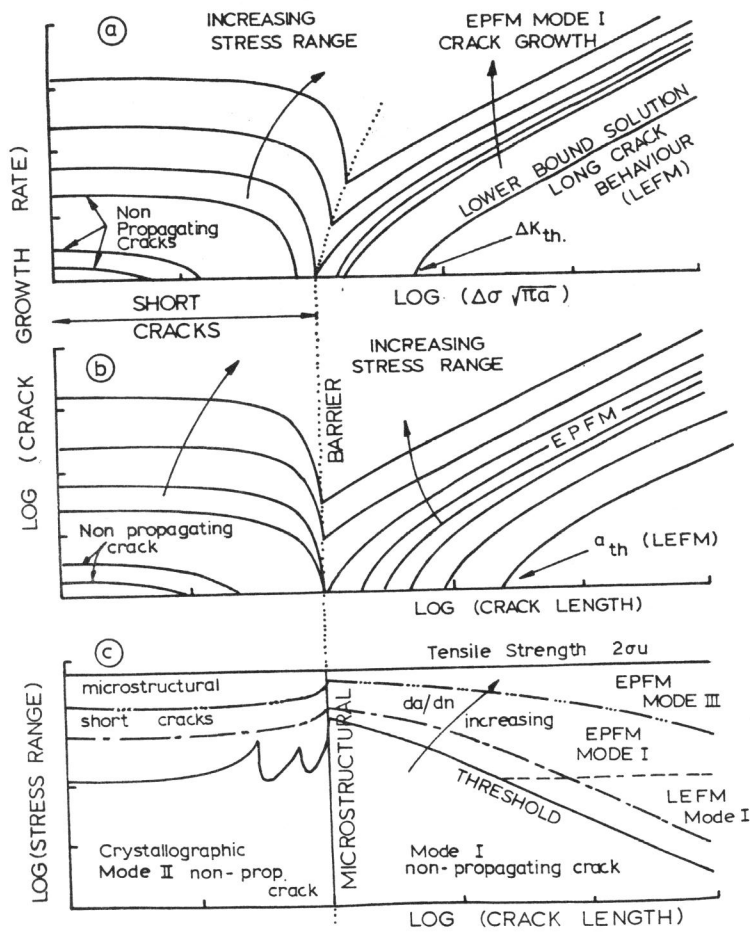


Fig 4. Short and Long Crack Growth Regimes.

- (a) crack growth rate as represented by continuum mechanics
- (b) crack growth rate as a function of crack length
- (c) a fracture mechanism map showing six zones.



that changes throughout the major part of lifetime when the cracks are short and then small. Hence closure must be a continuously varying function.

8. Short cracks that grow below threshold conditions and exhibit retardation as they approach threshold (prior to accelerating to failure) may well be equated to the long crack growth curve based on  $\Delta K_{eff}$  but this curve will not distinguish between cracks that are slowing down and those that are accelerating.

Despite these shortcomings crack closure concepts may be used for explaining long crack growth behaviour. However they will have little use in design calculations for several other reasons which will become apparent below.

At about the fatigue limit stress level for plain specimens the surfaces are at, or close to, the cyclic yield stress of the material and short and small cracks will be embedded in plastic zones of greater extent than the crack length or of similar extent respectively. This is the major reason for the breakdown of LEFM applicability. On Kitagawa-type plots (8) (see Fig 4c) this is largely disguised by the log stress axis of the plot. Furthermore the LEFM approach is only a lower bound solution and in many cases crack growth rates are significantly higher than threshold or general LEFM conditions. Tomkins' (9) has clearly shown in his theoretical model and further extensive experiments that crack growth can best be described by a strain based approach, e.g.

$$da/dN = f(\Delta \epsilon . a)$$

[2]

#### A Link between Short and Long Crack Growth Behaviour

This paper presents a strain based approach as an alternative to the  $\Delta K_{eff}$  approach. Here the total strain range  $\Delta \gamma$  may be divided into its elastic  $\Delta \gamma_e$  and plastic  $\Delta \gamma_p$  components each of which will dominate at low and high strain ranges respectively. It follows that a link between the traditional Basquin-Coffin-Manson, stress-strain parametric approach to fatigue failure can also be established. For elastically low-stressed bodies with long cracks  $\Delta K$  will be synonymous with a strain range intensity factor which has long been known to rationalize much conventional LEFM crack growth data of different materials (10). In the current work this will take the form  $\Delta \gamma \sqrt{\pi a}$  where  $\Delta \gamma = \Delta \tau / G$ . Here G is the shear modulus.

As the stress levels increase and attain the plain specimen fatigue limit, with which we are concerned in this paper, the total strain range can be readily converted into a plastic strain range via the cyclic stress strain curve. Note that Jono and co-workers (11) were the first to report that plastic strains can be recorded

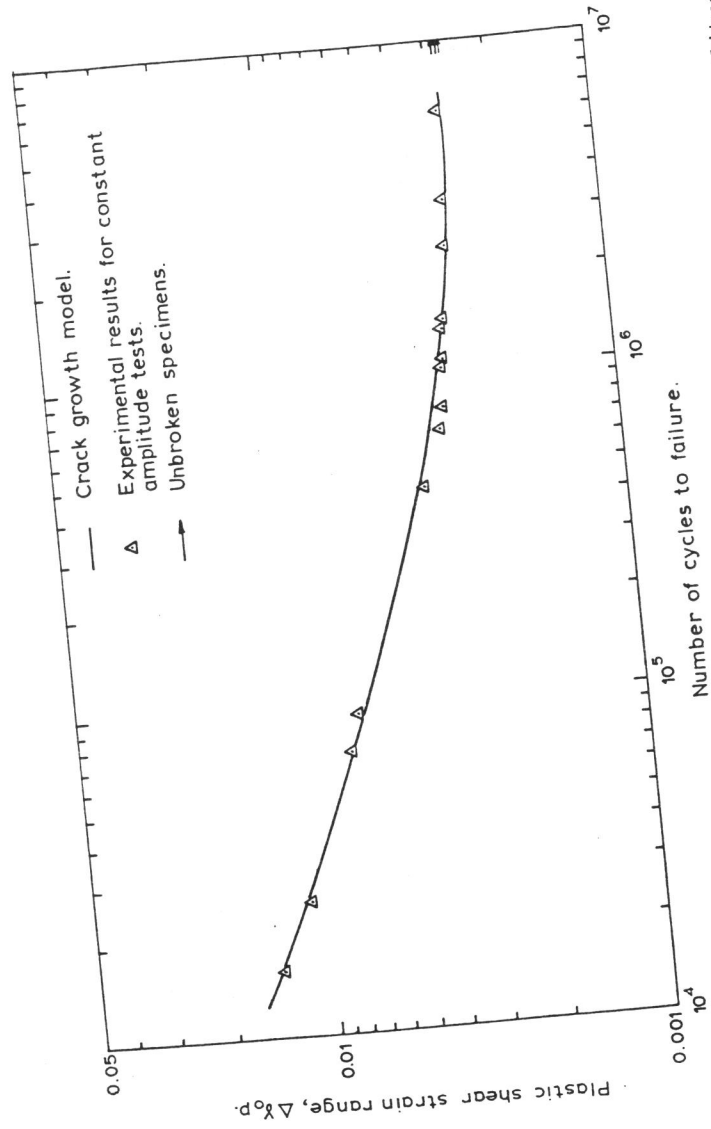


Fig 5. Experimental and predicted life of specimens tested at constant stress amplitude.

below the fatigue limit. Using this simple approach to link low stress to high strain fatigue several workers (12) (13) (14) (15) (16) have shown that small crack growth behaviour, i.e. beyond the microstructurally short crack phase, can be best described by equations of the simple form

$$da/dN = B \Delta \gamma_p^\beta a - C \text{ (}\mu\text{m/cycle)} \quad [3]$$

The advantages of this approach are obvious. For small cracks it is only required to know the strain range and actual crack lengths and a natural bridge is found between high stress microstructurally short cracks and low stress physically long cracks.

For the work referred to previously concerning the microstructure shown in Fig 2, the constants of equation [3] are given by:-

$$B = 17.4, \quad \beta = 2.68 \quad \text{and} \quad C = 8.26 \times 10^{-4} \text{ } \mu\text{m/cycle}$$

This latter term accounts for the threshold condition and permits some cyclic plasticity at the fatigue limit.

The equations [1] and [3] together with the material constants quoted will be used to predict fatigue lifetime and hence illustrate the usefulness of this approach to engineering design.

#### FATIGUE LIFETIME PREDICTIONS

Two different types of tests will be presented and the importance of short, small and long cracks in fatigue life prediction discussed.

##### 1. Conventional Fatigue

Fig 5 presents the results of constant shear stress range tests (16) obtained on a simple reversed torsion fatigue machine (17). Failure is defined as a surface crack length of 1mm. Crack growth beyond 1mm is rapid and the remaining lifetime so consumed is of little consequence to engineers. The experimental points show excellent agreement with the fatigue life prediction based on summing the lifetime spent in the short crack regime with that spent in the small crack growth regime, i.e. integrating equation [1] between the limits  $a = 0$  to  $a = d$  and similarly integrating equation [3] between the limits  $a = d$  to  $a = 1\text{mm}$ . However despite the fact that the prediction appears to be excellent as witnessed by Fig 5, it will be apparent in later discussion that beyond 1 million cycles to failure the method is not satisfactory, a fact that is again disguised by the logarithmic axis of number of cycles to failure. Beyond a million cycles to failure the summation of the integrated forms of equations [1] and [3] is conservative.

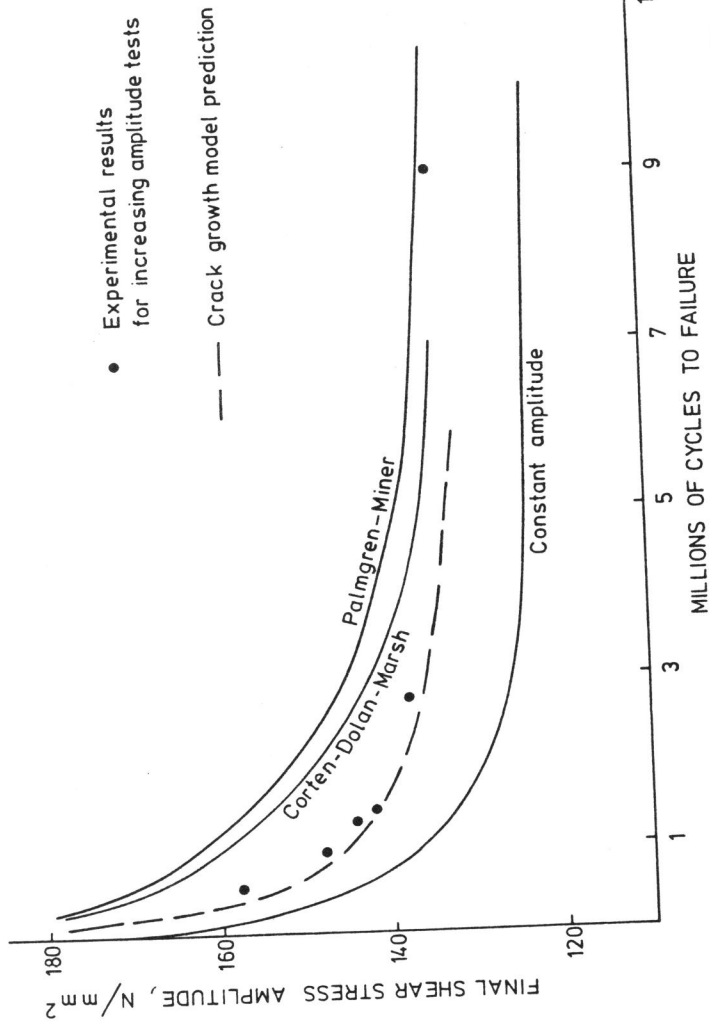


Fig 6. A comparison between classical cumulative damage laws, the crack growth model (equation [1] and equation [3]) and experimental data from increasing stress range tests.  
 A constant amplitude S-N curve is also shown for comparative purposes.

2. Increasing Amplitude Tests

In order to validate the approach outlined above a series of tests were performed in which  $\Delta\gamma_p$  was continuously increased throughout a test from a level below the fatigue limit (approx  $0.7 \Delta\tau_{FL}$ ) to a level above the fatigue limit. Such tests ensure that failure must inevitably occur and that no specimens will be wasted. A report on these tests is given in reference (1).

This kind of test programme is one form of a "cumulative damage" experiment without the complication of overload effects. Furthermore such a test programme permits an evaluation of the fatigue damage which occurs below the fatigue limit. Finally and most important it permits equations [1] and [3] to be developed for variable stress or strain range situations in order to investigate their usefulness in a more general situation.

Fig 5 shows the results of these tests and comparison is made with two other classical cumulative damage theories. The Palmgren-Miner theory (18)(19) provides a dangerous overestimation of lifetime since it can not allow for damage below the fatigue limit. The Marsh (20) modification to the Corten-Dolan theory (21) is slightly better but still provides an underestimation of damage accumulation and so overestimates lifetime; see (1). The crack growth prediction based on equations [1] and [3] is very good and gives a slight conservative judgement on lifetime prediction.

DISCUSSION

Analysis of the theoretical predictions and a comparison with the experimental results produces several important conclusions.

Fig 7 shows that for fatigue lifetimes below one million cycles to failure the small crack growth period dominates. However the microstructural short crack growth period consumes about 7% to 17% of the lifetime as judged from experimental data of the form of Fig 1. The implication is that crack growth occurs immediately from the very first cycle but then quickly slows down and arrests. This is in sympathy with the views expressed in reference (12). It is the period of crack arrest that can not at present be estimated. At 8 million cycles to failure the microstructurally short crack will be temporarily arrested for about 70% of life and in all probability it is this phase which creates the high degree of scatter associated with long life fatigue. During this period the crack will nominally be arrested but will be slowly advancing in isolated parts of the crack front at the weakest parts of the pearlite barrier should the stress level be above the fatigue limit.

No part of the lifetime can be attributed to the LEFM regime

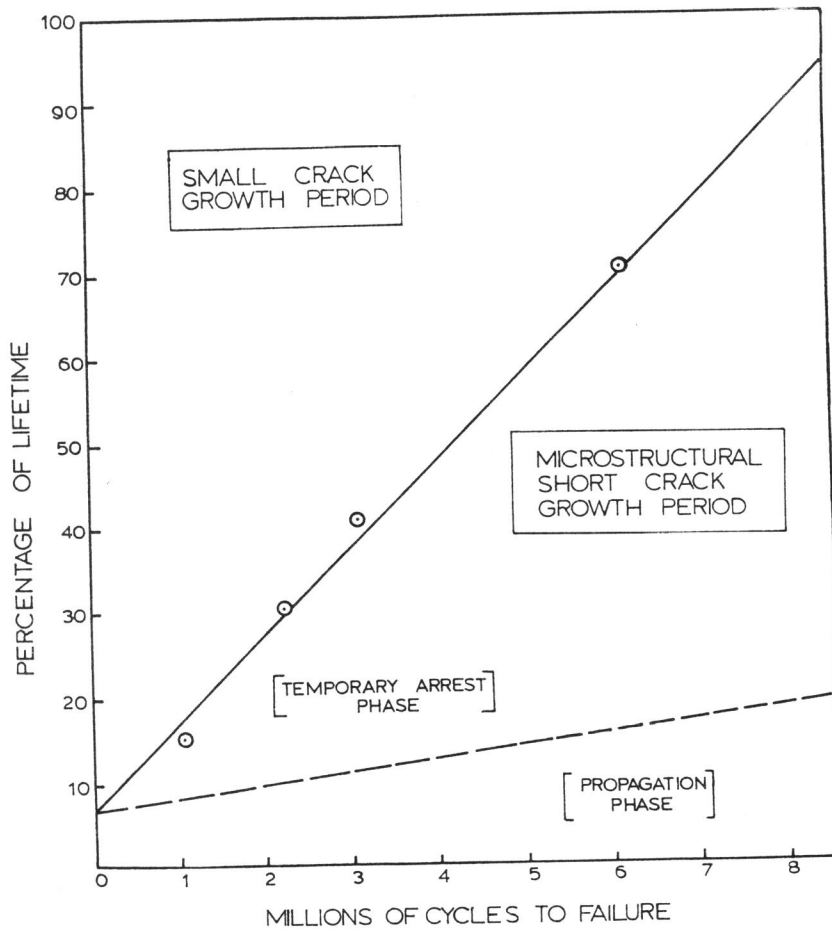


Fig 7. Division of lifetime between microstructurally short and physically small fatigue crack growth as determined by experiment on a plain (0.4%) carbon steel.

since the stress levels are too high.

The dashed line of Fig 7 is the theoretical limit of the short crack growth phase as determined via equation [1] and which is illustrated in Fig 1 by the inflexion on the damage curve. In the case of increasing stress amplitude experiments the inflexion is sharp and a zero temporary arrest period exists. As the rate of increase in stress amplitude decreases (or in the limit when the rate is zero as in constant amplitude tests) then the temporary arrest period is prolonged and the possibility of a non-propagating crack increased.

The success of the model on predicting lifetime in the cumulative damage type test series is due to two factors, the first is that the model, via equation [1] permits a damage assessment to be made in terms of crack length, for stress levels below the fatigue limit and secondly the temporary arrest period is minimised. This is because equation [1], in stress range terms, becomes

$$\frac{da}{dN} = 2.72 \times 10^{-31} \Delta\tau.a^{10.55} - 8.26 \times 10^{-4} \mu\text{m/cycle} \quad [4]$$

since the cyclic stress-strain curve is given by

$$\Delta\tau = 1034\Delta\gamma_p^{0.254} \quad [5]$$

The high stress exponent of 10.55 permits a rapid breakdown of the barrier and so the temporary arrest period is reduced.

From all of these remarks it is clear that the small crack propagation phase, as described by the elastic-plastic fracture mechanics equation [3], is most important in determining fatigue life and that to increase the fatigue resistance of a material, barriers to the continued growth of microstructurally short cracks are required. Finally in limited life fatigue the period of crack initiation can be neglected and that cracks can be assumed to grow from the beginning of cyclic loading at stress levels above the fatigue limit.

Fig 8 shows a typical Kitagawa plot but, like Fig 4c, it is a plot of zero crack growth rate. The three zones of microstructurally short cracks, physically small cracks and low stress (LEFM) cracks are clearly shown. The zone in which LEFM operates satisfactorily is purposely shaded because even a so-called "long crack" can become a small crack if the stress range level is raised, i.e. it can be described by EPFM. Perhaps therefore we should not refer to "small" and "long" cracks but be more precise and refer to these cracks as low stress cracks and high stress cracks respectively.

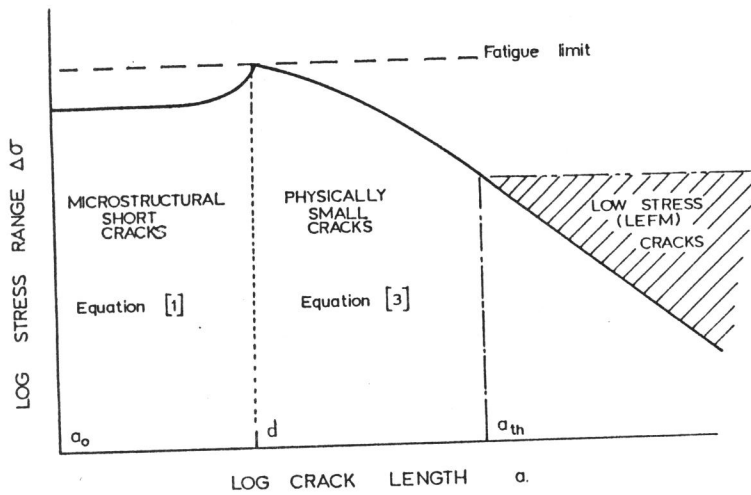


Fig 8. Three zones of crack growth behaviour at stress levels just above threshold conditions.

CONCLUSIONS

From much recent work on microstructurally short and physically small fatigue cracks the following conclusions can be made.

1. The growth rate of microstructurally short cracks can be described by an equation of the form

$$\frac{da}{dN} = A \Delta\gamma_p^\alpha (d-a)$$

where d represents a microstructural barrier to further growth.

2. The period of growth of microstructurally short cracks is short.
3. The growth of physically small fatigue cracks can dominate the limited lifetial regime, e.g. up to a million cycles to failure.
4. The growth rate of physically small cracks, i.e. cracks longer than the spacing of barriers to growth of microstructurally short cracks, can be described by an equation of the form

$$\frac{da}{dN} = B \Delta\gamma_p^\beta - C$$



where C represents a threshold condition.

5. Crack initiation in limited life fatigue is negligible and fatigue cracks can be assumed to grow immediately.
6. The LEFM approach, even in modified form using the  $\Delta K_{eff}$  method, is not recommended for studying small cracks at stress levels close to the fatigue limit of un-notched materials.
7. Shear strain range can be a unifying parameter linking three basic laws of fatigue crack growth covering, respectively, the microstructurally short crack, the physically small crack and the low stress (LEFM) crack.

#### ACKNOWLEDGEMENTS

This paper has called upon much recent research conducted by many workers at the University of Sheffield, Mechanical Engineering Department. I thank them all for their efforts and contributions but in particular I would name M.W. Brown, E.R. de los Rios and H.J. Mohamed.

Grateful acknowledgement is also given to the Science and Engineering Research Council of the UK and Rio Tinto Zinc for funding parts of this study.

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