THE EXPECTED POSITION OF FATIGUE FRACTURE PLANES ACCORDING TO VARIANCE OF THE REDUCED STRESS

W. Bedkowski, E. Macha*

In the paper the fatigue criterion for multi axial loadings is presented. It concerns the random state of stress, which components have zero mean values. It was assumed that the plane in which the maximum of the variance of the reduced stress appears is critical for the material. It was demonstrated that for each stationary random stress state there is one or more critical planes where the fatigue fracture plane can be expected.

INTRODUCTION

Fatigue fracture of machine parts and structures occurring as a result of multi-axial random loads takes place in case of random states of stress. It means that components of the stress state are stochastic processes and that principal stresses change at random their magnitudes and directions at selected points of the material.

The criterion of the maximum strain in the direction perpendicular to a fracture plane, proposed in the earlier papers (1), (2), is one of the fatigue criteria for the random triaxial stress state. In case of this criterion it is assumed that fatigue fracture is influenced by a strain in the direction perpendicular to the expected fracture plane and the plane is perpendicular to the mean direction of the maximum principal strain $\varepsilon_1(t)$. In a particular case the criterion agrees with the maximum principal strain criterion concerning sinusoidal loadings. If the presented criterion is used for evaluation of fatigue life of materials, the fatigue fracture plane should be known, which is determined in macroscopic scale by the unit normal vector:

* Technical University of Opole, Poland

$$\bar{\eta} = \hat{\ell}_1 \, \bar{i} + \hat{m}_1 \, \bar{j} + \hat{n}_1 \, \bar{k} \tag{1}$$

Assuming that the material is isotropic, the strain $\varepsilon_\eta(t)$ in the $\bar{\eta}$ direction can be expressed in the following way (1), (2):

$$\varepsilon_\eta(t) = \frac{1}{E} \{[\hat{\ell}_1^2 \, (1+\nu) - \nu] \, \sigma_{xx}(t) + [\hat{m}_1^2 \, (1+\nu) - \nu] \, \sigma_{yy}(t) + [\hat{n}_1^2 \, (1+\nu) - \nu] \, \sigma_{zz}(t) + 2 \, (1+\nu) \, \hat{\ell}_1 \, \hat{m}_1 \, \sigma_{xy}(t) + 2 \, (1+\nu) \, \hat{\ell}_1 \, \hat{m}_1 \, \sigma_{xz}(t) + 2 \, (1+\nu) \, \hat{m}_1 \, \hat{n}_1 \, \sigma_{yz}(t)\} \tag{2}$$

According to the discussed criterion $E \cdot \varepsilon_\eta(t) = \sigma_{red}(t)$.
The value of $\sigma_{red}(t)$ determines failure cumulating and fracture, cf. figure 1.
Fatigue life calculated on the base of the presented criterion under a given random loading state depends on the assumed values $\hat{\ell}_1$, $\hat{m}_1$ and $\hat{n}_1$. It is known that the variance of the acting stress is one of very important parameters influencing the fatigue life. It is more important when the stress has the normal probability distribution. The higher its value is, the fatigue life lower is. In the next considerations we assume that the cases where the variance of the reduced stress reaches its maximum are the most dangerous for the material.
Analysing formula (2) we can notice that the reduced stress depends on components $\sigma_{ij}(t)$ in a linear way and on the mean direction cosines in a non-linear way.
Formula (2) can be written in the following way:

$$\sigma_{red}(t) = \sum_{k=1}^{6} \alpha_k \, X_k(t) \tag{3}$$

Then the variance of the reduced stress can be calculated from the following relationship:

$$\mu_{\sigma_{red}}(\tau) = \sum_{s=1}^{6} \sum_{t=1}^{6} \alpha_s \, \alpha_t \, \mu_{xst}(\tau) \tag{4}$$

According to the assumptions estimation of the expected fracture plane consists in determining the conditional maximum of the variance of the reduced stress.

The results of fatigue tests under multiaxial random loadings are not available, so for generating the random triaxial stress state the digital simulation was used (3), (6), (7).
The random stress state was generated as 6 random sequences of numbers and each of them had the normal probability distribution and wide-band spectrum of frequency. Triaxial state of stress was replaced by uniaxial state by determining a time course of the reduced stress according to the discussed fatigue criterion. The matrix of covariance of components of the simulated stress state is equal to:

$$\mu_{xij} = \begin{bmatrix} 902.77 & 455.41 & 274.49 & -274.56 & -455.48 & 51.97 \\ 455.41 & 864.90 & 133.23 & -133.21 & -864.88 & 36.01 \\ 274.49 & 133.23 & 853.20 & -853.26 & -133.29 & 171.31 \\ -274.56 & -133.21 & -853.26 & 853.32 & 133.27 & -171.33 \\ -455.48 & -864.88 & -133.29 & 133.27 & 864.87 & -36.03 \\ 51.87 & 36.01 & 171.31 & -171.33 & -36.03 & 892.76 \end{bmatrix} \quad [MPa^2] \qquad (5)$$

On the basis of calculations the following function maximum of the variance of the reduced stress according to equation (4) considering the values of the covariance matrix according to equation (5) was obtained:

$$(\mu_{\sigma_{red}})_{MAX} = 2448.1614 \ [MPa^2]$$

for:

$$\hat{\ell}_1 = -0.5939$$
$$\hat{m}_1 = -0.7369$$
$$\hat{n}_1 = 0.3229 \qquad (6)$$

The graph of the variance function and the position of the expected fatigue fracture plane simulated state of stress are presented in figure 2. From the graph it results that also local maxima occur, but - in consideration of the function value in these points - fracture on these planes is not too probable.

In the paper the expected fatigue fracture planes for some particular cases of stationary ergodic random state of stress were also determined with the assumption that the components are not mutually correlated and their own variances are equal:

$$\mu_{xij} = \begin{cases} \mu_x & \text{when } i=j \\ 0 & \text{when } i \neq j \end{cases} \qquad (i,\ j=1,\ \ldots.,6) \qquad (7)$$

The exemplary graph for triaxial tension-compression ($\sigma_{ii} \neq 0$, $i = x,y\ z$, the remaining components = 0) is presented in figure 3. In this case $\mu_{\sigma_{red}}$, according to equation (4), is equal to:

$$\mu_{\sigma_{red}} = [\hat{\ell}_1^2\ (1+\nu) - \nu]^2\ \mu_x + [\hat{m}_1^2\ (1+\nu) - \nu]^2\ \mu_x + [\hat{n}_1^2\ (1+\nu) - \nu]^2\ \mu_x \tag{8}$$

and the maximum value is equal to:

$$\left(\frac{\mu_{\sigma_{red}}}{\mu_x}\right)_{MAX} = 1 + 2\nu^2$$

for:

$$\begin{aligned} \hat{\ell}_1 &= \pm 1 & \hat{m}_1 &= 0 & \hat{n}_1 &= 0 \\ \hat{\ell}_1 &= 0 & \hat{m}_1 &= \pm 1 & \hat{n}_1 &= 0 \\ \hat{\ell}_1 &= 0 & \hat{m}_1 &= 0 & \hat{n}_1 &= \pm 1 \end{aligned} \tag{9}$$

Three fracture planes, probable to the same degree, were obtained. The final evaluation of the correctness of the described method of determining the position of the expected fatigue fracture plane requires fatigue tests of materials for various random state of stress. It should be noticed that application of the proposed method for biaxial simusoidal loadings leads to the expected agreement with the experiments. The expected directions of the fatigue fracture for stress state used in tests carried out by Nishihara and Kawamoto (tests of cylindrical specimens under bending and torsion) (cf. reference (4)) amd by Rotvel (biaxial tension-compression of the cylindrical specimens (cf. reference (5)) agree with the directions obtained during this analysis.

CONCLUSIONS

* The fatigue life of the material under the given state of stress, calculated on the basis of reduced stress according to the presented criterion, depends strongly on the assumed fatigue fracture plane orientation.
* For each stationary random state of stress there is one or more planes which are critical as far as fatigue life of the material is concerned. On these planes the reduced stress variance reaches its maximum value and the fatigue fracture can be expected there.

* In some special states of stress it is possible to observe an analogy between the critical planes location under static loads and those under random fatigue loads, if normal fatigue stresses are treated as principal stresses. It refers to the planes of the maximum principal stresses, the planes of the maximum shear stresses and the octahedral planes.
* An analysis of selected cases of random loads shows that - generally speaking - location of the critical planes do not depend on values of Poisson's ratio for a given material.

SYMBOLS USED

$\bar{\eta}$ = normal vector describing location of fracture plane

$\hat{\ell}_1, \hat{m}_1, \hat{n}_1$ = mean values of direction cosines

$\bar{i}, \bar{j}, \bar{k}$ = versors of axes x,y,z, respectively

$\varepsilon_1(t)$ = maximum principal strain

E = Young's modulus

ν = Poisson's ratio

$\sigma_{ij}(t)$ = components of stress state (i,j = x,y,z)

$\varepsilon_\eta(t)$ = strain in direction $\bar{\eta}$ erpendicular to fracture plane

σ_{af} = fatigue strength limit under sinusoidal tension- compression

$\sigma_{red}(t)$ = reduced stress

X_k = one-dimensional stochastic processes equal to components of stress tensor (k=1,...,6)

a_k = coefficients depending on $\hat{\ell}_1, \hat{m}_1, \hat{n}_1$

$\mu_{\sigma_{red}}(\tau)$ = variance of reduced stress

a_s, a_t = coefficients depending on $\hat{\ell}_1, \hat{m}_1, \hat{n}_1$, in non-linear way

$\mu_{xst}(\tau)$ = variances (for s=t) or covariances (for s≠t) of random variables X_k

REFERENCES

(1) Macha E., "Fatigue Failure Criteria for Materials under Random Triaxial State of Stress". Advances in Fracture Research 1984, Proceedings of the 6th International Conference on Fracture ICF6, New Delhi 1984, Pergamon Press, Vol. 3, pp. 1896-1902.

(2) Macha, E., "Mathematical Models of the Life to Fracture for Materials Subjected to Random Complex Stress Systems", Scientific Papers of the Inst. of Material Science and Applied Mech. of Wroclaw Tech. Univ. No. 41 1979 (in Polish) p. 99.

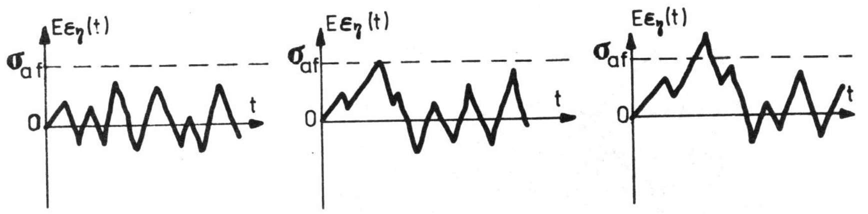
(3) Macha E., "Statistical Characteristics of Strains, Principal Stresses and Direction Cosines in Random Stress State", Studia Geotechnica et Mechanica, Vol. VI, No. 2, 1984, pp. 21-54.

(4) Nishihara, T., Kawamoto, M., "The Strength of Metals under Combined Alternating Bending and Torsion with Phase Difference", Memoirs of the College of Engineering, Kyoto Imperial Univ. Vol. XI, No. 4, 1945.

(5) Rotvel, F., "Biaxial Fatigue Tests with Zero Mean Stresses using Tubular Specimens", Int. J. Mech. Sci., Vol. 12, 7, 1970, pp. 597-613.

(6) Macha, E., "Simulation of Material Life-Time under Random Triaxial Stress State", Simulation Systems 79. Editors: L. Dekker, G. Savastano, G.C. Vansteenkiste, North Holland Publishing Company, 1980, pp. 425-435.

(7) Macha, E., "Simulation of Fatigue Process in Material Subjected to Random Complex State of Stress", Simulation Systems. Editor: L. Dekker, North Holland Publishing Company, 1976, pp. 1033-1041



$\max_t \{E\,\varepsilon_\eta(t)\} < \sigma_{af}$ | $\max_t \{E\,\varepsilon_\eta(t)\} = \sigma_{af}$ | $\max_t \{E\,\varepsilon_\eta(t)\} > \sigma_{af}$

no fatigue fracture | limit state | fatigue fracture can occur

Figure 1 Results of applying stress $E\,\varepsilon_\eta(t) = \sigma_{red}(t)$ with various maximum values

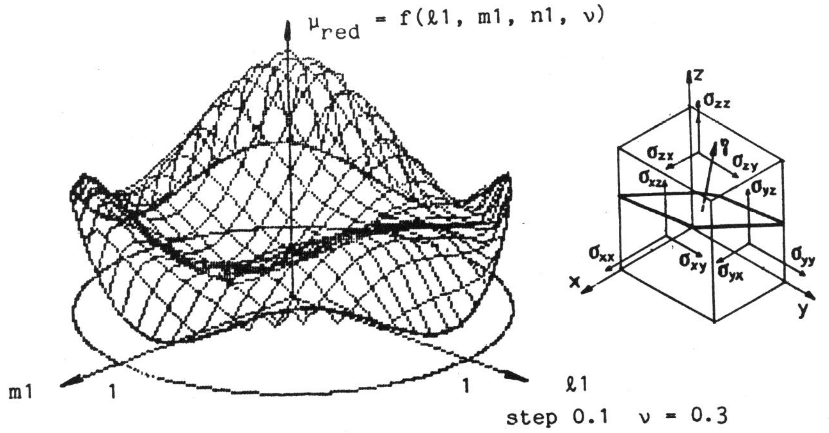


Figure 2 Graph of μ_{red}, situation of expected fracture plane for simultaneous state stress $\hat{n}_1 = (1 - \hat{l}_1^2 - \hat{m}_1^2)^{0.5}$

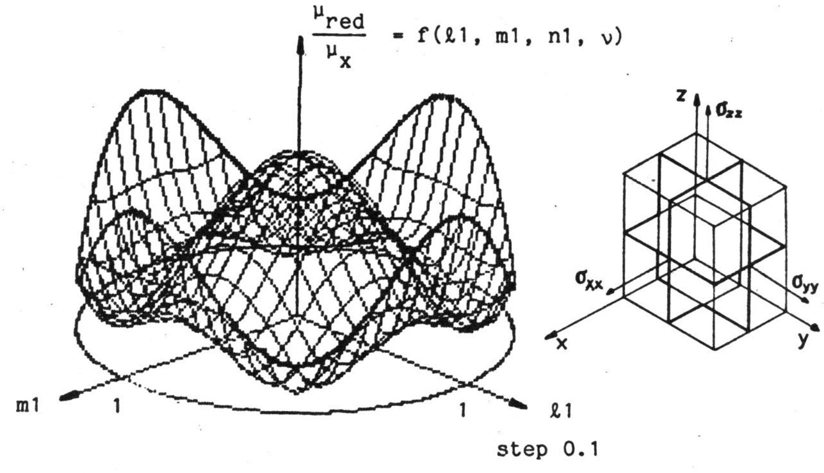


Figure 3 Graph of μ_{red}/μ_x, situation of expected fracture plane for triaxial tension-compression, $\nu = 0.3$