

## COMPARISON OF R-RESISTANCE CURVES

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R-resistance curves may be obtained using multiple or single specimen testing techniques, and various specimen configurations. Furthermore, various methods for measuring crack length can be utilised, including dc potential difference; unloading elastic compliance; and physical measurements of the crack front after fracture. The results obtained for tests on compact tension and three-point bend specimens, for two different thicknesses are presented, and the R-resistance curves ( $J-\Delta a$ ) are compared.

INTRODUCTION

For elastic materials, the study of the behaviour of cracks and defects can be undertaken using conventional linear elastic fracture mechanics (LEFM). The crack tip conditions can be characterised by the stress intensity factor ( $K_I$ ), and the plane strain fracture toughness by the critical stress intensity factor ( $K_{IC}$ ). Under situations where LEFM conditions are contravened, due to excessive plasticity at the crack tip,  $K_I$  is no longer a valid parameter and recourse must be made to the application of elastic-plastic fracture mechanics (EPFM).

In order to characterise the crack tip stress fields in ductile materials, the concept of a path independent integral was proposed by Eshelby (1) and significant advances in the development and application of the J-integral have been made by Hutchinson (2) and Rice and Rosengren (3). Subsequently, the use of the J-integral as a means of characterising static cracking in ductile materials has been further developed by Rice (4). Now in order to assess the performance of a cracked structure under elastic-plastic conditions, it is necessary to define the fracture toughness, and to ensure that the J-integral does not exceed an appropriate limiting value. This is achieved by experimentally determining the relationship between the J-integral and the crack extension ( $\Delta a$ ), i.e. by the use of a so-called R-resistance curve. From such a curve, the onset of crack extension and the subsequent stable crack growth can be determined and, hopefully, used to assess the acceptability of materials and the risk of fracture. The value used from an R-resistance curve to make such assessments is a matter of philosophy, and may correspond to the onset of crack extension (the blunting line) or the J-integral corresponding to the peak load reached in an R-resistance test.

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TEST METHODS

Two methods are commonly used to obtain R-resistance curves, namely the multiple and single specimen techniques.

The multiple specimen technique requires testing 4 to 6 pre-cracked test pieces loaded to different specific values of load-line displacement, and obtaining an accurate record of corresponding measurements of load and COD. Final crack length are then measured by fracturing each test piece, physically measuring the crack front at several positions, and averaging the results. For some materials simply fracturing at low temperature may be sufficient to identify the final crack front, whilst in other instances some other technique, such as heat tinting, might be necessary. In this latter case, the possibility of secondary cracking due to the heating and cooling should not be overlooked. Integrating the load-COD curves and applying appropriate geometric correction factors enables the J-integral to be calculated, and a plot of the resulting J- $\Delta a$  points (one point for each specimen) defines the R-resistance curve.

The single specimen technique involves testing one pre-cracked specimen by loading incrementally throughout a load increasing test, and at various stations determining the load, COD, crack length, and the area under the appropriate section of the load-COD curve. Usually, at each station, the load is held constant to allow the crack to stabilise, some unloading is introduced (usually about 10% but no more than 20%) in order to measure the unloading elastic compliance and hence determine the crack length. Typically some 10 to 50 stations are employed, giving a corresponding number of J- $\Delta a$  points on the R-resistance curve. Both multiple and single specimen testing techniques are described in ASTM E-813 (5).

Now not only are multiple and single specimen techniques used to determine R-resistance curves, but different specimen types are frequently employed. In this paper, experimental results obtained by different techniques for the compact tension and three point bend specimens are considered.

TESTING AND ANALYSIS

The material studied is a high strength age-hardening wrought copper nickel alloy (6) having the following mechanical properties:

Tensile strength,	Su = 744 MPa
0.2% Roof strength,	Sp = 526 MPa
% Reduction in Area,	RA = 16.3
Young's modulus,	E = 143 000 MPa

Compact tension and three point bend specimens were manufactured to conform to standard dimensions. Single specimen and multiple specimen tests were conducted, using an Instron 8032 microprocessor controlled testing system for the compact tension and a Mayes servo-hydraulic testing machine for the three-point bend specimens. Pre-cracking was carried out to obtain appropriate (a/W) ratios for the cracked test pieces (typically about 0.5) prior to obtaining the R-resistance data. Crack length, which is probably the most important measurement to be made, was determined using various techniques, including direct current potential difference (7); unloading elastic compliance (8); where appropriate, using fractomat surface crack length gauges (9); and by direct physical measurements from the fracture surfaces. In this latter case, heat tinting was conducted to final fracture in order to identify the various stages of cracking, but some secondary cracking was encountered as illustrated in

Figure 1.

Calculations for the J-integral were made using the result (10)

$$J = \frac{2A}{B(W-a)} \left[ \frac{1+\alpha}{1+\alpha^2} \right] (\text{MN/m}) \quad \dots (1)$$

where A is the area under the appropriate section of the load displacement curve (Nm); B is the specimen width (mm); W the specimen depth (mm); a the crack length at the appropriate point on the load-displacement curve (mm); and  $\alpha$  is a dimensionless parameter equal to unity for three-point bend and for the compact tension calculated from

$$\alpha = \left[ \left( \frac{2a_0}{W-a} \right)^2 + \left( \frac{4a_0}{W-a} \right) + 2 \right]^{\frac{1}{2}} - \left[ \frac{2a_0}{W-a} + 1 \right] \quad \dots (2)$$

where  $a_0$  is the original crack length, including the fatigue pre-crack. The relationship between crack length and the normalised elastic unloading compliance enables the crack length to be determined, i.e.

$$\frac{a}{W} = f(\text{CEB}) \quad \dots (3)$$

where

$$C = \text{unloading elastic compliance} \\ = \frac{\text{load-line displacement } (\delta)}{\text{load } (P)}$$

$$E = \text{elastic modulus;}$$

Appropriate relationships are found in the literature (11) (12) (13). Typically, a polynomial relationship is used, of the form

$$\frac{a}{W} = C_0 + C_1 U + C_2 U^2 + C_3 U^3 + C_4 U^4 + C_5 U^5 \quad \dots (4)$$

$$U = \frac{1}{(\text{CEB})^{\frac{1}{2}} + 1} \quad \dots (5)$$

The coefficients for the compact tension specimen (12), based on load-line COD measurements, and for the three-point bend specimen (13) based on (a) COD measurements at the surface and (b) load line displacements measured using a linear displacement transducer, are indicated in Table 1. The crack length can also be calculated from measurements of dc potential difference, either using appropriate calibration procedures (14), or from an equation due to Johnson (15) and verified by Schwalbe (16). Thus for a centre crack panel of width  $2W$ , with an initial crack length  $a_0$  exhibiting a potential drop  $U_0$ , the crack length corresponding to potential drop U can be calculated from

$$a = \frac{2W}{\pi} \arccos \frac{\cosh(\pi y/2W)}{\cosh \frac{U}{U_0} \operatorname{arcosh} \frac{\cosh(\pi y/2W)}{\cos(\pi a_0/2W)}} \quad \dots (6)$$

where  $2y$  is the span of the potentiometric points.

TABLE 1 - Coefficients for Polynomial Equation Relation (a/W) with U

		$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
Compact Tension		1.0002	-4.06319	11.242	-106.043	464.335	-650.677
3-Point Bend	(a)	0.9945	-3.6925	1.70627	-36.472	106.443	-125.51
	(b)	1.013	-4.498	19.708	-361.706	2692.973	-7941.81

the advantages of using the normalised potential drop ( $U/U_0$ ) with equation (6) to determine crack length are discussed by Schwalbe (16).

Using the testing techniques and methods of analysis discussed, R-resistance curves, in the form of  $J-\Delta a$  plots, have been obtained for both three-point bend (Figure 2) and compact tension (Figure 3) specimens. The data points have been reduced using a linear regression technique in accordance with the ASTM standard (5), i.e.

$$J = m \cdot \Delta a + C \text{ (MN/m)} \quad \dots (7)$$

Data points outside the range  $0.15 \text{ mm} < \Delta a < 1.5 \text{ mm}$  were eliminated, but this restriction made little difference compared with regressing all the data points for the three-point bend tests.

In the case of the tests on compact tension specimens, non-linear relationships between  $J$  and  $\Delta a$  were observed, with the possibility of dividing the data points into two regions, i.e. those above and those below the knee in the curve. When applying the linear regression to these results, the early data points below the knee were eliminated. Figure 4 compares the linearly regressed data for three-point bend and compact tension tests.

In order to define the onset of stable crack extension,  $J_i$ , the intersection of the linear regression with the blunting line has been used, the blunting line being defined by the equation

$$J = 2 \sigma_f \Delta a \text{ (MN/m)} \quad \dots (8)$$

where

$$\sigma_f = \frac{1}{2} (S_u + S_p) \quad \dots (9)$$

Thus, corresponding to  $J_i$

$$\Delta a_i = \frac{C}{2 \sigma_f - m} \quad \dots (10)$$

The resulting  $J_i$  values have been converted to equivalent  $K_i$  values through the relationship

$$K_i = (E J_i)^{\frac{1}{2}} \quad \dots (11)$$

Table 2 summarises the results, the values for m and C being average values from several tests.

TABLE 2 - Summary of Test Data

	Three Point Bend		Compact Tension
	30	50	13
W(mm)			
m (MN)	435	384	86.8
J <sub>1</sub> (MN/m) Single	0.084	0.093	0.042
K <sub>1</sub> (MPa√m) Single	110	115	78
J <sub>2</sub> (MN/m) Multiple	0.210	0.164	-
K <sub>2</sub> (MP√m) Multiple	176	153	-
C (MN/m)	0.0557	0.0655	0.040

#### DISCUSSION OF RESULTS

The most critical measurement required in obtaining R-resistance curves is that of crack length. With the multiple specimen technique, physical measurements of the initial fatigue crack length, and the crack length at the terminal point are frequently relied upon. In the case of the material of this study, these physical measurements are extremely difficult to make. The crack growth mechanism under fatigue is predominantly intergranular (6), and the distinction between the fatigue pre-cracking and the crack extension during a load increasing test is difficult to determine. Further, heat tinting to distinguish the crack extension from final fracture introduces problems. In particular, secondary cracking ahead of the crack front may occur (Figure 1), making accurate physical measurements unreliable.

During fatigue pre-cracking, surface crack length measurements were made using fractomat gauges (9). In some instances, before carrying out the R-resistance test, the fractomat gauges were removed, the specimen surfaces polished, and direct measurements of the surface crack lengths were made optically and also using replica techniques. These confirmed the accuracy of the fractomat gauges and the direct current method of measuring the crack length. Further surface measurements were made, both directly and with replicas, at the end of an R-resistance test. Following heat tinting (various temperatures were used), the surfaces were re-polished and surface measurements again made before fracturing the specimens. This confirmed that the secondary cracking observed was associated with the heat tinting process.

The use of the direct current potential drop technique for determining crack length was found to be generally the most reliable method. The application of equation (6) was found to give good agreement with experimentally determined calibration curves for the three-point bend specimen, but was not applicable to the compact tension specimens.

Unloading elastic compliance methods for the determination of crack extension proved to be unreliable, unless great care was taken in the experimentation. This is particularly true for initial crack extension, where frictional effects and grip arrangements become important. In some instances, for example, indications that the crack was actually reducing in length with increase in load was indicated, although this is, of course, physically impossible. The unloading elastic compliance technique gave good correlation with dc potential drop (discarding some of the initial readings) for the compact tension specimens. In the case of three-point bend, measuring the compliance using a displacement transducer below the specimen gave poor results, which could not be relied upon. However, when the displacements were measured utilising a comparator bar, the predicted crack lengths were compatible with those obtained using dc potential drop, but the R-resistance curve was different from that obtained without the comparator bar.

The  $J-\Delta a$  plots for three-point bend and compact tension are compared in Figure 4, from which it is observed that for equivalent  $\Delta a$  values, the  $J$ -integral is higher for the three point bend tests. This is almost certainly due to the differences in constraint between the two types of specimen. Furthermore, the point identified as that corresponding to the onset of stable crack extension is lower for the case of compact tension than it is for three-point bend, although the scatter and variances in the compact tension tests are significantly greater than obtained in the three point tests.

The  $J_i$  values obtained in these studies correspond to valid  $J_{IC}$  values, since in all cases the thickness criterion is satisfied, i.e.

$$B > 15 \left( \frac{J_i}{S_p} \right) \quad \dots \quad (12)$$

#### CONCLUSIONS

1. R-resistance curves obtained for the three-point bend specimens are significantly different from those obtained for compact tension specimens.
2. The onset of crack extension for the compact tension tests occurs at a lower value of  $J_i(J_{IC})$  than for the three-point bend tests.
3. Data obtained using multiple-specimen testing is not very reliable for material studied, due to the difficulty in physically measuring the crack lengths.
4.  $J_i$  values obtained using single specimen testing were generally lower than obtained from multiple specimen testing.
5. Unloading elastic compliance was not always a reliable method for determining crack length.
6. Consistently reliable results for crack length measurements were obtained with the dc potential drop technique.
7. Johnson's equation relating normalised crack extension with normalised voltage was found to apply to the three-point bend test specimens, but not to

the compact tension specimens.

8. It is suggested that the three-point bend specimen is probably more suitable than compact tension for the determination of R-resistance curves, due to greater degree of repeatability and reduced scatter.

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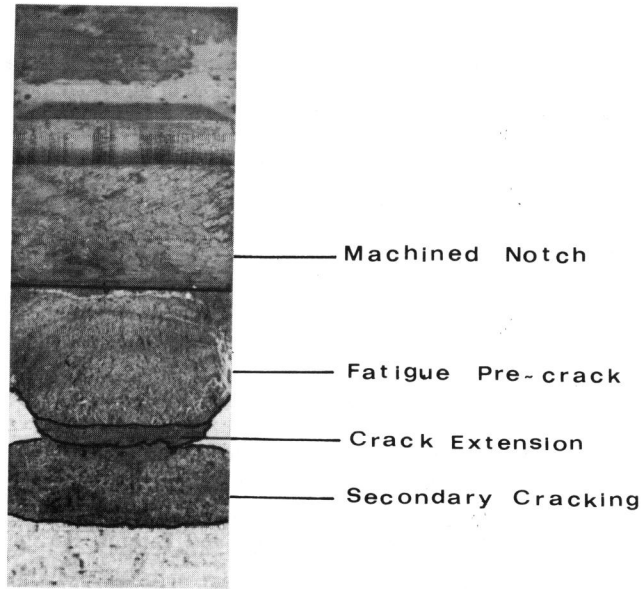


Figure 1 Typical Fracture Surface

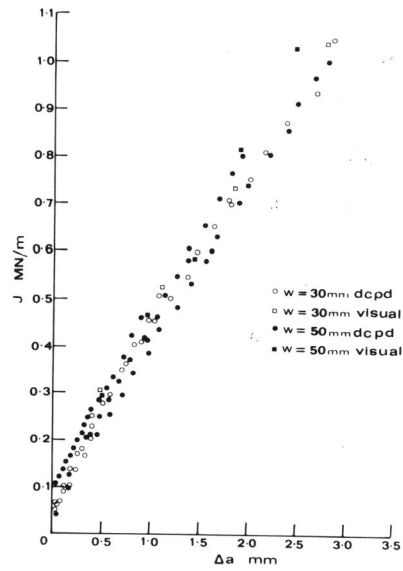


Figure 2 R-Resistance Data for Three Point Bend Tests



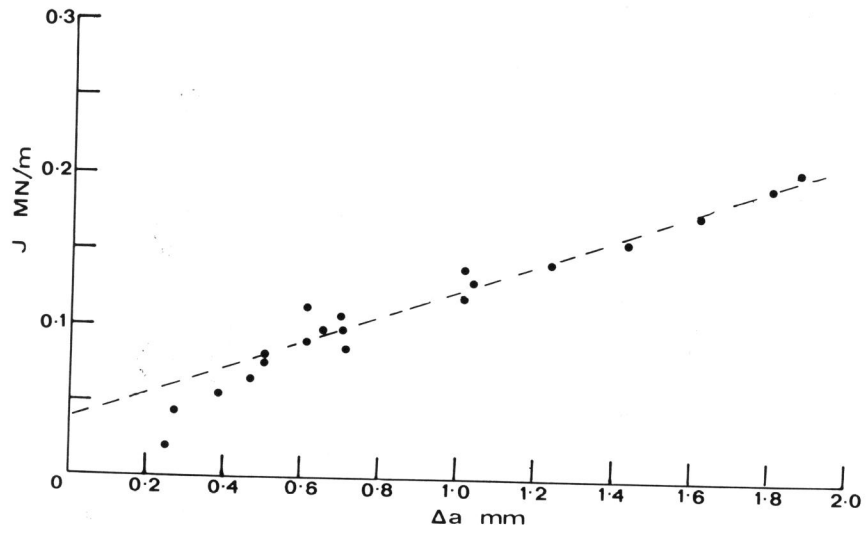


Figure 3 R-Resistance Data for Compact Tension Tests

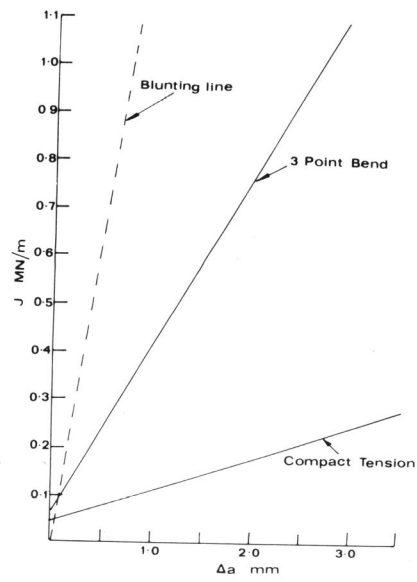


Figure 4 Regression Lines for Three Point Bend and Compact Tension Data