USE OF THE R-CURVE FOR DESIGN WITH CONTAINED YIELD

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ABSTRACT

The well-known use of G-R curves in terms of linear elastic fracture mechanics is re-examined in terms of recent developments of J-R curves in plasticity using the simple argument that the energy release rate available, I, mustbe equal to or greater than the total work dissipation rate, dw/Bda. The circumstances when this reduces to or differs from the well-known tangency condition between G or J and the R-curve are stated, and the relationship between this condition and an effective toughness of component that decreases with crack growth is discussed. Some estimates of I in contained yield are given. It is found that I≃G for bending configurations but I≃J for tension cases. The use of G corrected for plastic zone size as an approximation to J in contained yield is examined in the light of some existing residual strength data for wing panels.

KEY WORDS

Residual strength; R-curves; stable crack growth; unstable crack growth; fracture; fracture mechanics.

INTRODUCTION

The use of a rising curve of crack growth resistance versus crack growth, the so-called R-curve, came into prominence following several ASTM bulletins reporting on the work of a committee on the "Fracture Testing of High-strength Sheet Material" in the late 1950's, culminating in the well-known paper by Krafft et al, (1961). The concept has since been explored and firmly established in many papers, of which the works of Broek (1966), Hayer and McCabe (1972), Creager & Liu (1974) and Novak (1976) are representative of studies in both light alloys and thin section steel where the failure mode is by ductile tearing that is initially stable and amenable to description by lefm.

Recently, considerable interest has been shown in the R-curve concept expressed in terms of either COD (Tanaka & Harrison, 1978) or J (Garwood et al, 1978) for describing the ductile tearing in the presence of extensive plasticity. Particular interest has attached to the prediction of unstable ductile tearing (Paris et al, 1979), (Turner, 1979) in the absence of a change in micro-mode of behaviour. In the present paper the earlier use of R-curves expressed in terms of either G or K via lefm for problems where the net section stress is below yield is re-examined in the light of the viewpoint engendered by the studies of R-curves derived in conditions of extensive yield and expressed in terms of J.

USE OF J-R CURVES OR G-R CURVES

Two rather separate cases exist for the use of J in the analysis of stable crack growth via material resistance R-curves. One is the development of R-curves expressed in terms of J from test pieces that experience extensive yielding with the intention of predicting unstable crack growth in components that experience a significant degree of yield. Such work has been explored recently by Paris et al, (1979), Garwood et al, (1979), and others and was summarised by Turner (1979b). Yet more recent work follows this line of attack, for example, Shih et al (1979), Garwood (1979), Paris et al (1979), Turner (1979c) and a number of other studies. This use of J for R-curves in extensive plasticity is not further discussed here, except in so far as the viewpoints generated throw light on the use of R-curves in contained yield. The second use of J is indeed in contained yield. As noted above, this type of problem has been attacked in terms of lefm by Krafft et al (1961) and many later workers, as summarised for example, in ASTM (1973) and more recently by Turner (1979d). In a recent sequence of papers, Wilhem et al (1977), Ratwani & Wilhem (1978), Ratwani & Wilhem (1979), suggested that a J treatment was desirable even for certain problems where nominal stress σ and net section stress σ_n were less than σ_Y. It should be remarked that these papers deal with the residual strength of thin skin structures, with rivetted or bonded stringers, representative of aircraft wing panels. These require several complex steps of analysis, but it is only the plasticity aspect of the R-curve treatment that is discussed here.

It is generally accepted that G corrected for plasticity by use of an apparent crack length based on either the Irwin plastic zone correction factor or perhaps an equivalent compliance is an adequate representation of the degree of plasticity for perhaps 20% or even more in excess of the lefm value. In so far as J has emerged as an acceptable single parameter model of the intensity of crack tip deformation beyond the lefm range, then denoting the corrected value of G by G_p, it is supposed that $G_p \simeq J$, in contained yield, Fig. 1 B, C.

There is considerable experimental evidence that this is reasonably so using the Irwin term $r_p = (K/\sigma_Y)^2/2\pi$ (for plane stress) for intermediate levels of stress. Clearly, for stresses approaching yield $G_p < J$, greatly so once net section yield is reached, as for example, Fig.1. (A)

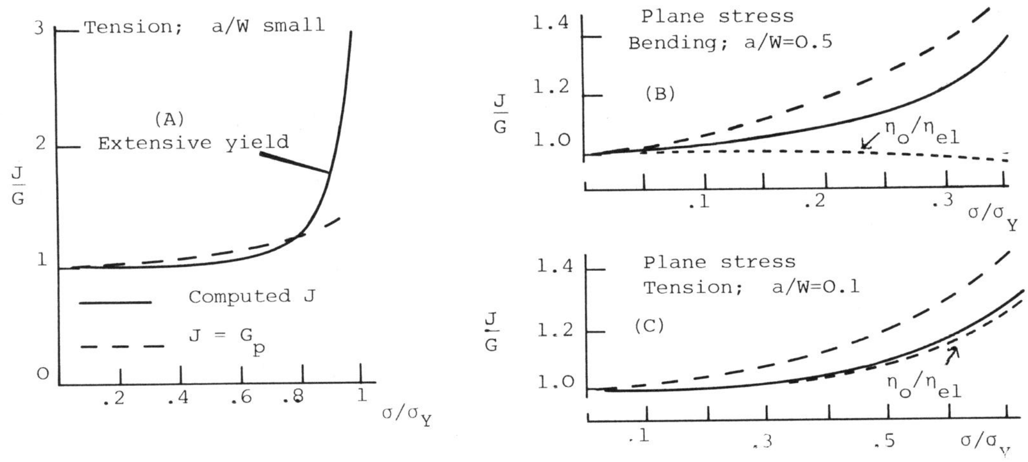

Fig.1. Variation of J with extent of yielding.

In early support for the COD model Burdekin & Stone (1966) also pointed out the similarity of the Irwin correction to the expansion of the Dugdale ln secant term. For an infinite plate, the differences between K, K_p (corrected by the Irwin factor) and K_D (inferred from the Dugdale model by taking $K^2 = EG$ and $G = \sigma_Y \delta$ where δ is the Dugdale term $\delta = (8\sigma_Y a/\pi E)\ \ln \sec (\pi\sigma/2\sigma_Y)$ are shown Table 1. For this configuration there is an error of 6% if plasticity is neglected for a stress level of $\sigma/\sigma_Y = 0.5$ and an error (relative to the Dugdale solution) of 6% if the Irwin term is accepted for a stress level of $\sigma/\sigma_Y = 0.8$.

TABLE 1. Effect of plasticity according to the Irwin correction to lefm for the Dugdale model.

σ/σ_Y	K_p/K	K_D/K	$(K_D-K_p)/K_D$
0.5	1.06	1.06	1%
0.6	1.09	1.09	1%
0.7	1.12	1.14	2%
0.8	1.15	1.22	6%
0.9	1.19	1.36	12%

In the writer's view there thus seems little need for the use of J for stress levels below yield such that the crack tip plasticity is still contained by an outer elastic region, although it is admittedly difficult to give explicit limits for that statement other than as a value for J/G or the like.

In the studies just mentioned by Wilhem and others, the R-curve was indeed derived by elastic methods corrected for plastic zone size, generally according to the procedure recommended in ASTM (1973b) (now 1978) using a crack line wedge loaded (CLWL) test piece.

Two separate elastic finite element analyses of the CLWL piece were made to give K and J (elastic). In deriving R-curves by both methods an allowance was made for plasticity by determining the effective crack length but both analyses are elastic and thus inherently related by $K^2 = EG = EJ(\text{elastic})$. The computed calibration for K was several per cent. lower than that quoted in ASTM (1973b) here denoted K_A. The J value, when translated to K as $\sqrt{EJ}$, here called K_J, was several per cent lower again. These computational differences cloud the issue and make certain definition of the R-curves impossible. Two main analyses were made for the treatment of the crack in the test panels, each conducted for two crack lengths, of which only the shorter is followed here. One was an elastic finite element solution to find K, including a treatment judged best suited to the rivetted reinforcing stringers. The second was a Dugdale type elastic-plastic computation,based on the Hayes & Williams (1972) procedure, with a few preliminary Prandtl-Reuss computations in support, again including (the same) treatment of the rivetted stringer joints. A typical Dugdale computation over-estimates the Prandtl-Reuss result by 4% at $\sigma/\sigma_Y = 0.5$ and under-estimates it by almost the same amount at $\sigma/\sigma_Y = 0.82$, the maximum stress level considered. This very reasonable agreement is taken to justify use of the Dugdale model in the light of the saving of computations.

Predictions of unstable crack growth are made in two ways, one appearing to be satisfactory, the other not. The way that appeared satisfactory was the use of the R-curve defined by $\sqrt{J}$ (elastic) with plastic zone correction and with the driving force computed in terms of $\sqrt{J}$ from the Dugdale computations. For one particular case this analysis showed arrest at the reinforcing stringer was feasible up to $\sigma/\sigma_Y < 0.75$.

The second prediction was made in terms of the R-curve defined by K_A with the driving force computed in terms of K not corrected for effect of plastic zone size. This prediction showed behaviour always stable (up to $\sigma/\sigma_Y = 0.82$) by a large margin. It is not here disputed (beyond the discrepancies in the various computed values already noted) that the J procedure is adequate. Indeed, the agreement with experiment at about 6% difference for several test panels is remarkably close. However, it is argued here that the K procedure can also be adequate for this problem if one datum R-curve is used (i.e. the discrepancies between the various calibrations eliminated whichever may in fact be correct) and the computed value of the applied K corrected for size of plastic zone to become K_p. The predictions of instability and arrest are then quite similar to the author's J method. As an arbitrary choice the R-curve defined by J (elastic) is used but expressed in terms of K as $R(K_J)$. As shown Fig.2, this falls some 20% below the $R(K_A)$ curve for reasons that are primarily attributable to the differences in elastic calibration since plastic methods of analysis were not used for the test piece. The applied value of K given in the data is here corrected for the size of plastic zone quite approximately by use of the correcting factor for a wide plate since a value more appropriate to a skin and stringer configuration is not known.

As seen, Fig.2, arrest might occur at the reinforcing stringer for a stress level up to about 0.78 σ_Y. This is clearly comparable to the prediction by the authors of $0.75\sigma_Y$ using the Dugdale J computations, although agreement between some data in the calculations even at elastic stress levels is still affected by a number of uncertainties that cannot be resolved in the absence of an agreed reference solution.

Resolution of the quite major differences between the original predictions in terms of K and J raises the question of whether there should be any difference between an elastic and elasto-plastic prediction in contained yield if the former is corrected for size of plastic zone. This question is discussed in the remainder of the paper in the light of the recent development of J-R curves.

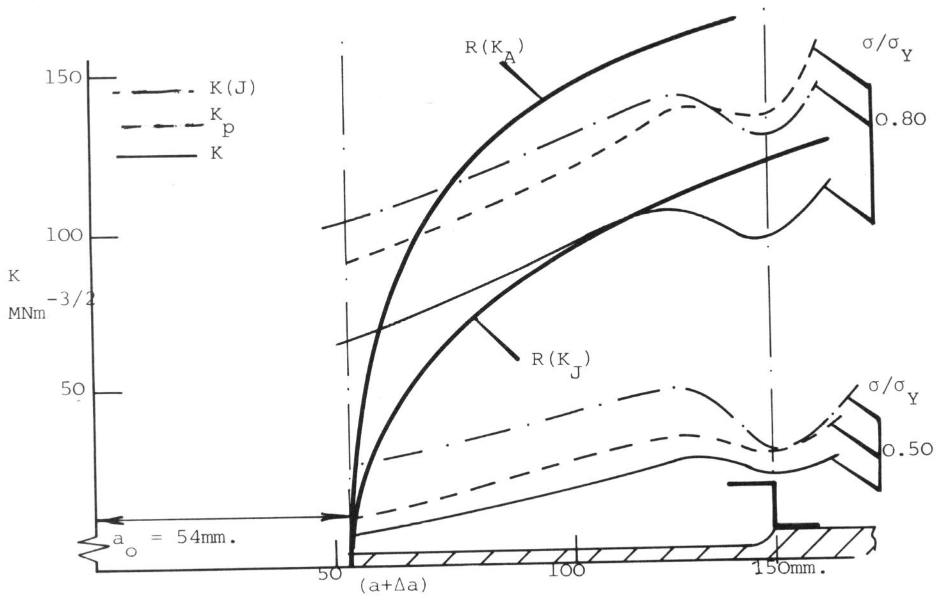

Fig.2. Comparison of driving force and R-curves derived by different methods in a ⌐L stiffened panel (after Wilhem & co-workers).

R-CURVES AS A MEASURE OF WORK DISSIPATION

It is now generally agreed that as discussed at some length in Turner (1979b) plasticity is a measure of the total work dissipation rate, some of which can be described, conceptually, as surface energy and the remainder of which is general plastic work, not necessarily or even primarily, closely adjacent to the fracture plane. It is arguable whether the two terms can be disentangled. When the plasticity is extensive, it is clearly dependent on the configuration of the test piece. One interpretation of the J-R curve is that, when suitably expressed, it provides a measure of plastic dissipation that has been normalised with respect to both size and shape of the component and this defines a material property, the toughness against tearing. The terms that in principle allow the normalisation are those that relate J or in lefm, G, to the work done, w,

$$G = \eta_{el}\, w/Bb \qquad \text{Eqn.1}$$

$$J = \eta_o\, w/Bb \qquad \text{Eqn.2}$$

where B is thickness, b is the ligament (W-a) and η a shape dependent term that in elasticity can be calculated from lefm shape factor and in gross plasticity can be estimated from the variation of limit load with crack growth (Turner, 1979b). From the conventional definitions of J (or G) in terms of work rate, it follows that amongst several possible definitions of η one is

$$\eta = -\frac{b}{w}\left.\frac{\partial w}{\partial a}\right|_q \qquad \text{Eqn.3}$$

where $|_q$ implies constant displacement. The extent to which these concepts are sufficient to describe actual tearing behaviour for a variety of configurations, stress states and degrees of inhomogeneity is uncertain, since it is still fiercely argued where any one parameter, be it J, COD or other, can provide an adequate description, even of the initiation of fracture. Nevertheless, the picture of J-R curves as a normalised work rate term provides a conceptual model that appears to contain all the correct features in so far as any one parameter model can do so and should be of most relevance as $J \to G$ and lefm remains adequate. When in lefm R-curves are expressed in terms of K it is implied that fracture is being described by a characterising parameter for which the single term K is sufficient to define the severity or magnitude. Because in strict lefm there is the well-known relationships between characterising parameter K and energy rate G

$$K^2 = E'G \qquad \text{Eqn.4}$$

where E' is the effective modulus ($E' \equiv E$ in plane stress; $E' = E/(1-\nu^2)$ in plane strain where ν is Poisson's Ratio) then a balance of energy rate is implied. However, in the non-linear case the identity between characterising parameter and energy rate is lost unless a strict interpretation is put on the J model which is thereby restricted to non-linear elastic (nle) behaviour. For realistic behaviour, the question arises whether, as plasticity is no longer negligible, the characterising or energetic argument should be followed and how different they may be. As a datum value against which to compare other estimates in plasticity, J is taken despite the admitted uncertainty over its use as a definitive criterion for fracture in plasticity. A corrected value of G, here written G_p,

$$G_p = G(Y,\sigma,(a + r_p)) \qquad \text{Eqn.5}$$

is used in terms of the Irwin plastic zone correction, where $Y = f(a + r_p)/W$ and (in plane stress)

$$r_p = EG/2\pi\sigma_Y^2 \qquad \text{Eqn.6}$$

If the correction is negligible, then the distinction between characterising and energetic arguments is irrelevant. If net section yield occurs $J \gg G_p$ so that a plasticity treatment as referenced above is necessary. The present discussion centres on the region of significant but contained yield where $J \simeq G_p$. Precise definition cannot be given to the stress level or extent of yielding where plasticity ceases to be "contained" and becomes "uncontained" since the effect depends upon configuration and the degree of difference between J and G_p that is tolerable for the purpose but broadly it is the "elbow" of the load-deflection diagram where the extent of plastic zone may be large in relation to the crack length and comparable to the ligament whilst not yet completely crossing it.

In discussing characterising or energetic pictures of fracture the distinction must be made between the initiation of fracture and subsequent slow crack growth or unstable behaviour. Since fracture from a pre-existing sharp defect can start without unstable growth, even in conditions of rigorous lefm, e.g. in a wedge loaded situation, then reaching a certain severity of crack tip stress field or deformation must be a sufficient criterion for onset of separation. There is no requirement that the energy for separation be supplied from internal sources of strain energy since further external work will be done to extend the fracture and indeed, increase the stored strain energy still further.

In rigorous lefm the rate of work absorption with crack growth is equal to the rate of energy release rate with crack growth, i.e. BGda. With non-linear material this equality is no longer so, except for non-linear elastic behaviour where both terms are J. Thus, with plasticity as the source of the non-linearity the work absorption rate that defines the R-curves is closely J (in so far as any single term can describe it) whereas the energy release rate that drives an instability is not J.

There are thus two quite separate questions. When is J (or other term) a suitable measure of work dissipation rate, characterising parameter, or both, and what is the measure of elastic energy release rate in an elastic-plastic-elastic (i.e. elastic plastic loading, linear elastic unloading) material?

The relevance of J (and thus G_p as approximation thereto for contained yield) to work absorption rate is exact for 'total' or 'deformation' theory plasticity. In such behaviour the ratio of the stress components (i.e. the triaxiality) is independent of the degree of plasticity for proportional loading and fixed boundary conditions (Turner et al, 1980) and it is generally supposed, with some computational evidence in support but no absolute proof, that the crack tip stress ratios remain sufficiently constant even for incremental material whilst yield is contained. In extensive yield the converse, that the triaxiality is a function of configuration and degree of yielding, is well-known in plane strain with minimal hardening (McClintock, 1965) but in plane stress the variation of the stress ratios must be limited since σ_z is maintained zero and only the ratio $\sigma_x : \sigma_y$ (transverse to axial) can alter. However, extensive yield is outside the regime of treatment by G_p so that the present argument is simply that in contained yield where $J \simeq G_p$, either term is an adequate measure of absorption rate. Even with no unloading J is strictly a characterising parameter only for 'total' theory plasticity and proportional volumetric strain ratio (as in the nle HRR model) since otherwise at least two terms, intensity and triaxiality, are required to describe the stress state. The adequacy of J as these conditions are lost is contentious, and must be determined experimentally for the material and circumstances in question.

Even if J is accepted as the correct or at least an adequate measure of work dissipation rate, it must still be asked whether the R-curve found with extensive plasticity can be related to the elastic R-curve. Studies by Garwood et al (1978) show conclusively that the R-curve has two components, one due to shear lip formation and one due to general plasticity that remains even when shear lip is eliminated by deep side grooves. This plasticity effect clearly disappears as lefm is approached and in K_c testing it is the shear lip component that causes the rising R-curve. It is not at all clear under what conditions the shear lip work rate will be the same in elastic or plastic tests since the size of shear lip is expected to be both thickness and configuration dependent. Thus, acceptance of a J-R curve for use in contained yield seems highly questionable unless the R-curve was obtained in contained yield (as indeed was the case by Wilhem et al) in which case any distinction between a G-R curve (with plastic zone correction) and a J-R curve is within the uncertainties of present analyses and understanding.

The energy release rate in the presence of plasticity was defined (Turner, 1979a) as I, Fig.3, where

$$I = G - q_{el} (\eta_o - \eta_{el}) \left.\frac{\partial Q}{B \partial a}\right|_q \qquad \text{Eqn.7a}$$

$$= G \{(2\eta_o/\eta_{el}) - 1\} \qquad \text{Eqn.7b}$$

BGda = ABC
BIda = ABDE
BC = dQ_{el}
BD = dQ_o

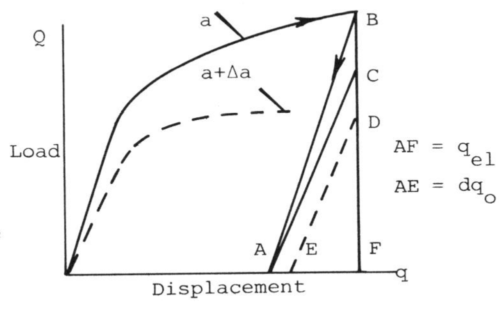

Fig.3. The elastic energy release rate in the presence of plasticity.

If deformation theory plasticity is used, then, within the restrictions already mentioned of proportional loading and fixed boundary conditions, during loading the behaviour is identical to non-linear elastic and η_o is independent of the extent of deformation and in particular $\eta_o = \eta_{el}$. In that case I = G. If deformation theory plasticity followed by linear elastic unloading (called ideal elastic-plastic-elastic (ideal epe) behaviour were followed, then the release rate would still be G even with gross plasticity. In reality, with incremental behaviour, in some configurations triaxality is maintained (e.g. deep notch bending in plane strain) in which case $\eta_o \simeq \eta_{el}$ and $I \simeq G$, whereas in others (e.g. centre cracked tension) triaxality is lost as yield becomes uncontained, $\eta_o > \eta_{el}$ and I > G. The upper limit of I is complete recovery of energy, i.e. $I \leqslant J$. As a guide the statement of $G \leqslant I \leqslant J$ is made but some qualifications to that statement may be necessary. For most purposes the lower bound is $I \geqslant G$, although there may be certain cases where I < G, for example rigid plastic or perhaps velocity damping type materials.

In so far as the loose approximation $J \simeq G$ is adequate in contained yield, then the distinction between G and J would only be significant in gross yield, where J >> G. However, if the difference between G and G_p is deemed significant, then the release rate I is not presently known within the limits $G \leqslant I \leqslant G_p$. The limit I = G is exact for so-called ideal epe behaviour. The limit I = J is exact for nle behaviour which is not physically relevant to metals. The condition $I \rightarrow J$ is met where crack tip triaxiality (or indeed, biaxiality, for plane stress) is lost as plasticity spreads. It is well-known that the shape of plastic zone differs from one test configuration to another (Larsson & Carlsson, 1973) but there are no solutions for the stress state in the contained yield regime known to the writer other than computational data. If the plastic zone is very well contained by the outer elastic field, it may be though that biaxiality is maintained. Once a ssy model ceases to be adequate and the effect of lateral boundaries is felt, no doubt the result depends on type of loading and degree of hardening.

Applying these arguments to G-R curves viewed as a balance of energy rate, it follows that the ordinate of the R-curve should be G_p, since that is the best measure of the energy dissipation; the abscissa should be Δa (actual) rather than $\Delta a + r_p$ since J (to which G_p approximates) is a function of the actual crack length. The ordinate of the energy release rate, excess of which causes instability, should strictly be I, for which in well-contained yield, no explicit formula is known and for which G might be a better approximation than G_p, perhaps according to configuration. If, on the other hand, the G-R curves are viewed as a balance of characterising severities, albeit expressed in the G notation rather than K, then G_p is the better estimate of intensity for both the material behaviour and the applied loading. It is not self-evident what the abscissa should be. Although $\Delta a + r_p$ seems commonly favoured, the writer inclines to the view that G_p is still being used as an estimate of J which is a function of the actual crack length so that Δa (actual) again is the more logical abscissa.

CONDITIONS FOR INSTABILITY

The instability in lefm is usually expressed as the tangency condition

$$\left.\frac{\partial G}{\partial a}\right|_Q \text{(applied)} > \frac{dR}{da} \text{(material)} \qquad \text{Eqn.8}$$

where, here, the applied condition is taken at constant load Q. Analysis at fixed displacement, is also possible using $(\partial G/\partial a)|_q$. By differentiating Eqn.1 the applied term can be re-expressed as

$$\left.\frac{\partial G}{\partial a}\right|_Q = \frac{G}{b}\,(f_1(\eta) + \eta) \qquad \text{Eqn.9a}$$

or $$\left.\frac{\partial G}{\partial a}\right|_q = \frac{G}{b}\,(f_1(\eta) - \eta)$$ Eqn.9b

where $f_1(\eta) = 1 + (b/\eta)\,(d\eta/da)$. Obviously the expression for $(\partial G/\partial a)_Q$ is the same as that found by differentiating $EG = Y^2\sigma^2 a$; i.e.

$$\left.\frac{\partial G}{\partial a}\right|_Q = \frac{2G}{Y}\frac{dY}{da} + \frac{G}{a}$$ Eqn.9c

The η notation is preferred for ease of relating $\partial/\partial a|_Q$ to $\partial/\partial a|_q$ and for relating ∂G to ∂J as plasticity spreads. The instability statement is

$$I > dw/Bda$$ Eqn.10a

which by differentiating Eqn. 2 gives

$$I > \frac{b}{\eta_o}\left\{\frac{dJ}{da} - \frac{J}{b}(f_1(\eta))\right\}$$ Eqn.10b

For lefm Eqn.1 is used and then $J \equiv G$ and η_{el} is implied instead of η_o. For rigorous lefm this is the same as

$\partial G/\partial a$ (applied) $>$ dR/da (mat) where R is G(mat) Eqn.11

For truly nle material I = J and Eqn.10 is the same as

$\partial J/\partial a$ (applied) $>$ dR/da(mat) where R is J(mat) Eqn.12

For ideal epe material I > G(mat), Eqn.10 is the same as

$\partial J/\partial a$ (applied) $-$ $(\eta/b)(J-G) >$ dR/da(mat)

where R is J(mat) Eqn.13

With real (incremental) epe material

$\partial J/\partial a$ (applied) $-$ $(\eta/b)(J-I) >$ dR/da(mat)
where R is J(mat) Eqn.14

For $I \to G$ this case reduces to Eqn.13 and for $I \to J$ it reduces to Eqn.12.

An alternative viewpoint is to treat the right hand side of Eqn.10b as an effective toughness R_{eff} (Turner, 1979c).

$$R_{eff} = \frac{b'}{\eta}\left\{\frac{dR}{da} - \frac{R}{b}f_1(\eta)\right\}$$ Eqn.15

where in general R implies J(mat) and η implies η_o. For lefm $R = G_{mat}$ and $\eta = \eta_{el}$. The term R_{eff} obviously <u>reduces</u> with crack growth corresponding to the reduction in slope of the R-curve. The term can be treated as an effective toughness of the <u>component</u>, influenced both by the material R-curve, which at least conceptually is a material property, and the size and shape factors, b and η that depend on the component. This concept corresponds to the reducing R-curve discussed in the pre-Krafft, Sullivan & Boyle (1961) literature but generally thereafter dropped in favour of the "material only" R-curve effect. Written in this form, for lefm, (instead of Eqn.11)

$$G > R_{eff} \text{ (with } R = G_{(mat)})$$ Eqn.16a

and for true nle material instead of Eqn.12

$$J > R_{eff} \text{ (with } R = J_{(mat)})$$ Eqn.16b

For ideal epe material (instead of Eqn.13) and real (incremental) material (instead of Eqn.14)

$$I > R_{eff} \text{ (with } R = J(\text{mat})) \qquad \text{Eqn.16c}$$

The value of I will differ in the last two cases according to the value of η_o/η_{el} in Eqn.7, since $\eta_o/\eta_{el} = 1$ for ideal epe. Numerical evaluation of Eqns.16,a,b,c, will of course give the same result for instability as Eqns.11-14.

The differences between Eqns.12 & 13a,b,become large as dJ/da becomes small, i.e. after substantial crack growth.

In the use of J-R curves Hutchinson & Paris (1979) denote small crack growth by $\omega >> 1$ where ω is defined or $\omega = (b/J)(dJ/da)$ in order that a characterising J field is retained at the crack tip. In strict lefm or nle material the restriction is not required, since only the derivative term appears. In contained yield treated by lefm, the difference between Eqn.14 and Eqn.11 or 12 might be significant, since the absolute amount of crack growth prior to instability tends to be larger in relation to the thickness and the slope of the R-curve much reduced from its initial value.

Treating R as J(mat), a more general statement of small amounts of growth is seen from Eqn.15 to be

$$\omega >> f_1(\eta) \qquad \text{Eqn.17}$$

but for a number of deep notch configurations $f_1(\eta) \simeq 1$ so that the Hutchinson & Paris (1979) conditions is recovered. If Eqn.17 is satisfied and all terms in Eqn.10 multiplied by E/σ_y^2 then, of course, all the cases can be expressed in terms of the T or tearing modulus notation of Paris et al (1979). It must be realised that as crack growth continues, so T reduces with dR/da. The value of T (applied) would be $\eta I/b$, the precise value of which for any configuration depends upon the stress strain law (linear elastic, nle, ideal epe, incremental) chosen.

SOME ESTIMATES OF I IN CONTAINED YIELD

If the difference between G and J is not important, then there is no further argument to be made over the value of I. If it is judged significant then the value of I is also important. There is no rigorous way known to the writer of evaluating I for real elastic-plastic materials, so that an uncertainty exists over the value of the driving force curve, and hence of its tangency point with the material resistance curve. As gross yield is approached there is some evidence that $I \simeq G$ (or at best, I is closer to G than to J) for configuration with high constraint, since in Eqn.7, $\eta_o \simeq \eta_{el}$. On the other hand for a low constraint configuration, such as centre cracked tension, $\eta_o > \eta_{el}$ by an amount such that $I \to J$, which in extensive yield may be much greater than G. The extent to which these trends can be traced back into contained yield or to which contained yield is well modelled by the high constraint Prandtl field (for non-hardening material in plane strain) for all boundary configurations is not clear. Some computed data for η_o/η_{el} in the contained yield region are shown, Fig.1, and Fig.4 the value I/G inferred from Eqn.7b, and the estimate of G_p/G. It appears from this rather scanty evidence that the ratios η_o/η_{el} and hence I/G, depend, as expected, on the configuration as well as the extent of plasticity.

In particular, for deep notch three-point bending, I is closely G, whereas in centre-cracked tension, I > G by subtantial amounts for a deformation about that for yield of the uncracked body. For non-hardening behaviour or deep notches, where the deformation at yield is concentrated at the notch, this effect is quite marked. With shallower notches and some work hardening sufficient to spread the yield away from the notch, the increase of I over G is much less marked.

With some work hardening or shallower notches to cause yield to the front face in-bending, the near constancy of I/G to unity might be expected to be less closely maintained. In the present data it is noted that G_p is not as close an estimate of J as might have been expected. For example, with J/G = 1.076 (Fig.1,B), $G_p/G = 1.180$ at $r_p/a = .053$ in bending. There is thus a difference of 8% of G or 50% of $(G_p-G)/G$. In short, G_p appears to over-estimate J for moderate stress levels, but, of course, under-estimate it for high stress levels as gross yield occurs. On the present data there seems little point in preferring G_p rather than G, since J is near the mean value of G and G_p. It cannot be demonstrated here whether or not this discrepancy between G_p and J is because of the obvious approximate nature of the Irwin correction factor, or because of inaccuracy in the computation of J. The discrepancy netween G_p and J is comparable for both tension and bending data here and data from an entirely separate source would be required to resolve that question. It is generally known that in gross plasticity $\eta \simeq \eta_{el} \simeq 2$ for deep notch bending, whereas $\eta \neq \eta_{el}$ in tension. Thus, the different trends in η_o/η_{el} and in I/G are supported by analytical (non-computed) data, albeit in the gross rather than contained yield situation. It is possible to estimate I/G in contained yield from the plastic zone correction and other data (see Appendix). This estimate follows the trend noted that I/G increases for tension but not for bending. However, it appears to over-estimate both effects (i.e. I/G too large in tension; I/G too small in bending) in relation to the values of Fig.4 estimated from the computations, corresponding to the difference between G_p and J already noted. It is perhaps unreasonable to expect the relatively small differences in question to be predicted with close accuracy, but conversely, some estimate of the accuracy of both the plastic zone correction procedure and of the computational methods seems essential if differences from lefm are to be taken account of at all.

CONCLUSION

It is concluded here that the elastic energy release rate in the presence of plasticity, I, is strictly neither G nor J but depends upon the configuration for real plasticity materials. The value I/G, tends to rise above unity for tension cases by an amount comparable to J/G, whereas for bending cases it does not. The effect is attributed to the partial loss of constraint for incremental plasticity material in the former but maintenance of it in the latter. If the constraint (triaxiality ratios) were constant with degree of deformation, then I = J, for perfect nle behaviour or $I = G_p$, for ideal epe behaviour, and the effects of configuration under discussion would not occur. In so far as the computations of J/G and lefm value corrected for the size of plastic zone, G_p/G, differ in contained yield by some 5% - 10%, it is not possible to say with assurance which value is the more nearly correct within contained yield and a corresponding uncertainty must remain over the numerical prediction of unstable crack growth when viewed as a balance of energy rates.

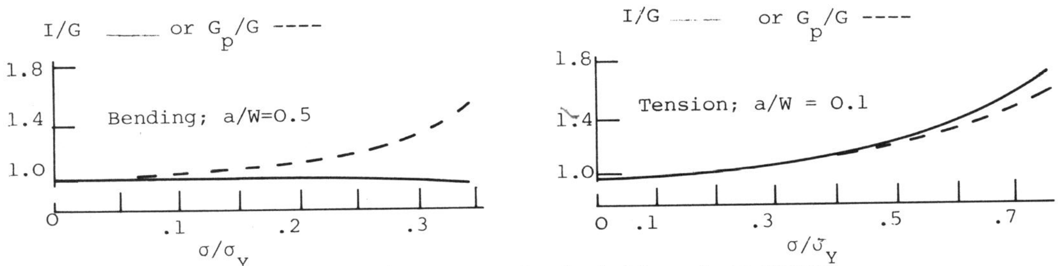

Fig.4. Estimates of I/G in contained yield; plane stress

APPENDIX

If with a small degree of yielding but no actual crack growth a slightly non-linear deflection diagram is obtained, Fig.5, then the following relationships can be written :

Non-linear	ORDG	=	w ;	$J = \eta_{el}\, w/Bb$
Triangle	OAE	=	w_{el};	$G = \eta_{el}\, w_{el}/Bb$
Triangle	OBF	=	w_{eq};	$J = G_{eq} = \eta_{el}\, w_{eq}/Bb$
Triangle	ODG	=	w_p ;	$G_p = \eta_p\, w_p/Bb$ (=J)

where ODG refers to an effective crack length $a + r_p$ (with r_p defined by Eqn.6) for which it is supposed $G_p = J$ at least in intention) and η_p is η_{el} for crack length $a + r_p$. The actual load at A or D is denoted Q_A. The end of the substantially linear region is denoted Q_R. The ratio Q_R/Q_A (<1) is denoted α and can be estimated for any load deflection record. Clearly, the true value of w is slightly greater than the linear approximation to ORDG (i.e. RD linear instead of curved). Thus,

w > triangle ORH + trapezium HRDG

$$\therefore \quad w/B \simeq [G_p b_p (\alpha+1)/\eta_p] - Gb\, \alpha/\eta_{el} \qquad \text{Eqn.18}$$

where $b_p = a - r_p$

$$\therefore \quad \eta_o/\eta_{el} = 1/\left[(\alpha + 1)\ (\eta_{el}\, b_p/\eta_p b) - \alpha G/G_p\right] \qquad \text{Eqn.19}$$

If $a + r_p$ is a slightly longer crack $a + \delta a$, and η_p is η_{el} at $(a + \delta a)$

then $$(\eta_{el}/b)_a\, (b/\eta_{el})_{a+\delta a} = 1 - (f_1(\eta))\,\delta a/b \qquad \text{Eqn.20}$$

and η_o (i.e. the value of η in contained yield treated by the lefm plastic zone correction) is η_{el} at $(a + r_p)$

$$\therefore \text{ contained yield } \eta_o/\eta_{el} \lesssim 1 - \alpha(1-(G/G_p)) + f_1(\eta)(\alpha+1)\, r_p/b \qquad \text{Eqn.21}$$

Also $I/G = (2\eta_o/\eta_{el}) - 1$ (Eqn.7b)

$$\therefore \text{ In contained yield } I/G \lesssim 1 - 2\alpha\ (1 - (G/G_p)) + 2f_1(\eta)(\alpha+1)\, r_p/b \qquad \text{Eqn.22}$$

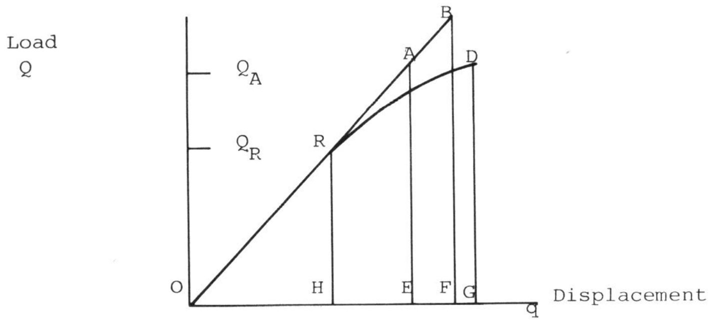

Fig.5. A slightly non-linear load-deflection diagram for contained yield

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