THE STRESS INTENSITY FACTORS FOR GRIFFITH CRACK(S) IN AN ORTHOTROPIC STRIP IN THE PRESENCE OF ASYMMETRICAL BODY FORCES

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ABSTRACT

The closed form expressions of stress intensity factors for Griffith crack(s) opened by asymmetrical body forces in a rigidly lubricated orthotropic strip have been obtained by using finite Fourier transform. The determination of plastic-zone length and the conditions of partial closing of the crack are also given for asymmetrical forces. A special case of point body force is considered for wood of Oak.

INTRODUCTION

The modern advance in application of composite materials necessitates the stress analysis of the structures. The region bounded by ribs, spars, and stringers in a wing of an aeroplane can be reduced to the problems of infinite strip with edges rigidly lubricated in plane strain conditions. The problem of two symmetrically placed Griffith cracks in the isotropic strip with symmetrical body forces by Parihar and Kushwaha [1]. The stress intensity factor due to a Griffith crack in an orthotropic infinite medium has been obtained by Kushwaha [2].

Only recently Satpathi and Parhi [3] have solved the problem of stresses in an orthotropic strip containing a Griffith crack opened by uniform pressure at crack faces using potential function method.

The title problem is the extention of [1,2] and the references thereof.

The convention for notations of stress and displacement components and super-script is followed from [4]. Physically the problem is with rigidly lubricated edges having stress free crack(s) with the continuity conditions in off-crack region. We assume that the axes of material symmetry coincide with the axes of co-ordinate. All physical quantities vanish as $|y| \rightarrow \infty$ where y is the axis normal to crack axis. Mathematically, we are to solve the following boundary value problem with the boundary and continuity conditions, namely;

$$O_{xy}(\pm a, y) = 0, \quad u_{x}(\pm a, y) = 0, \quad 0 \le |y| < \infty,$$
 (1.1)

and

$$\sigma_{yy}(x,0^{\pm}) = \sigma_{xy}(x,0^{\pm}) = 0, \quad 0 \le |x| \le c,$$
 (1.2)

$$u_{y}(x,0^{+}) = u_{y}(x,0^{-}), u_{x}(x_{1}0^{+}) = u_{x}(x,0^{-}), c \leq |x| \leq a,$$
 (1.3)

$$\sigma_{yy}(x,0^{+}) = \sigma_{yy}(x,0^{-}), \ \sigma_{xy}(x,0^{+}) = \sigma_{xy}(x,0), \ c \leq |x| \leq a,$$
 (1.4)

for a Griffith crack occupying the region (y = 0) o $\leq |x| < C$. The superscript (\pm) gives the value of physical components from y > 0 and y < 0. 2a is the width of the strip.

Similarly for two symmetrically placed Griffitherack occupying the region (y=0) $b \le |x| \le c$ and edge condition (1.1) and the following,

$$\sigma_{yy}(x,0^{\pm}) = \sigma_{xy}(x,0^{\pm}) = 0, \quad b < |x| < c,$$

$$(1.5)$$

$$\sigma_{xy}(x,0^{+}) = \sigma_{xy}(x,0^{-}), \ \sigma_{yy}(x,0) = \sigma_{yy}(x,0^{-}), \ o \leqslant |x| \leqslant b, \ c \leqslant |x| \leqslant a, (1.6)$$

$$u_{\mathbf{y}}(\mathbf{x},0^{+}) = u_{\mathbf{y}}(\mathbf{x},0^{-}), \ u_{\mathbf{x}}(\mathbf{x},0^{+}) = u_{\mathbf{x}}(\mathbf{x},0^{-}), \ o \leq |\mathbf{x}| \leq b, \ c \leq |\mathbf{x}| \leq a.$$
 (1.7)

For the above mentioned problems, we divide these into two parts, namely, Body Force Problem and Elasticity Problem. The solution of body force problems will be independent of number cracks in the medium. Individual problem is further sub divided into two namely, symmetrical and anti-symmetrical. The symmetry of the geometry will reduce the domain of solution to the domain $[o,a] \times [o,\infty]$

We have used the following definitions of Fourier transform.

$$f_{cs}^{s,a}\begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix}, \zeta = \int_{0}^{a} \int_{0}^{\infty} f^{s,a}(x,y) \cos \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix} x \sin \zeta y dxdy,$$

$$\alpha_n = n\pi/a, \quad \beta_n = (n - 1/2)\pi/a \quad , \qquad (1.8)$$

with the usual definition of inversion. The plan of the paper is as follows. Section 2 solves the problem of body force. Section 3 solves the elasticity of a Griffith crack and section 4 deals with plastic zone length at the crack tip. The elasticity problem of two Griffith crack is solved in section 5. Section 6 deals with partial closing with two models. To illustrate we consider one special case of point body force in section 7.

2. BODY FORCE PROBLEM

We solve the equations of equilibrium in the presence of body force [X,Y] along with the boundary conditions.

$$\sigma_{xy}^{s(k)}(a,y) = \sigma_{xy}^{a(k)} = 0, k = 1,2, 0 \le |y| < \infty,$$
 (2.1)

$$u_y^{s(k)}(x,0) = u_y^{a(k)}(x,0) = 0, k = 1,2, o \leq |x| \leq a,$$
 (2.2)

$$u_{x}^{s(k)}(a,y) = u_{x}^{a(k)}(a,y) = 0, k = 1,2, os|y|<\infty,$$
 (2.3)

where super script (s.a) refer to symmetrical and anti-symmetrical parts of the

problem super script (1 & 2) refer to physical quantities for y > 0, y < 0 corresponding to body force problem.

Symmetrical: We assume that the body force components [X,Y] are odd and even; even and odd with reference to variables x and y which satisfy (2.1) - (2.3). Taking appropriate Fourier transforms of equations of equilibrium and of stress-strain relations and then inverting, we get

$$u_{x}^{s(k)}(x,y) = \underbrace{4}_{n=1}^{\infty} \sum_{n=1}^{\infty} \sin_{n}^{\alpha} x \int_{0}^{\infty} \left[w_{1} x_{sc}^{k} + (-1)^{k} w_{2} x_{cs}^{k} \right] \cos_{3}y \, ds, \ k = 1.2$$
(2.4)

$$u_{y}^{s(k)}(x,y) = \frac{1}{5}u_{yc}^{s(k)}(o,y) + \sum_{n=1}^{\infty} u_{yc}^{s(k)}(a,y) \cos a x, k = 1,2$$
 (2.5)

$$u_{yc}^{s(k)}(x_{n}^{k},y) = \frac{-4}{\pi a_{0}} \left[w_{2} x_{sc}^{k} + (-1)^{k} w_{3} x_{cs}^{k} \right] \sin y dy,$$
 (2.6)

where w_1 , w_2 , w_3 are taken from [2] after replacing ξ by \prec_n . It is being done to save the space.

Anti-symmetrical: As for symmetrical case we shall follow the same analysis except in this case the components [X,Y] are even and odd functions of x and y, respectively. We get the displacement components as

$$u_{x}^{a(k)}(x,y) = \underbrace{4\sum_{n=1}^{\infty} \cos(\beta_{n}x) \int_{0}^{\infty} \left[w_{1} X_{cc}^{k} + (-1)^{k+1} w_{2} Y_{ss}^{k} \right] \cos(y) d\zeta, \quad (2.7)$$

$$u_{y}^{a(k)}(x,y) = 4 \sum_{N=1}^{\infty} \sin(\beta_{n}x) \int_{0}^{\infty} [w_{2} X_{cc}^{k} + (-1)^{k+1} w_{3}Y_{ss}^{k}] \sin(yd\tau), \qquad (2.8)$$

where w_1 , w_2 , w_3 are to be obtained from [2] given by equations (2.6) after replacing ξ by β_n .

3. AN INTERIOR GRIFFITH CRACK

We solve the equations of equilibrium, in the absence of body forces. Following the method of Kushwaha [2] and taking the components of displacement for symmetrical part of the problem as

$$u_{x}^{s(j)}(x,y) = \sum_{n=1}^{\infty} \sin \alpha_{n} x \alpha_{n}^{-1} \left[a_{11} H, \frac{j}{yy} - \alpha_{n}^{2} a_{12} H^{j} \right], j = 3,4$$
 (3.1)

$$u_{y}^{s(j)}(x,y) = \frac{1}{2}A_{0} + \sum_{n=1}^{\infty} \propto_{n}^{-2} \cos \propto_{n}^{\infty} x \left[a_{11}^{H}, j_{yyy}^{j} - \alpha_{n}^{2}(a_{12}^{+} a_{66}^{-})H_{yy}^{j} \right] (3.2)$$

with H^j given as

$$(\mathbf{r}_{1} - \mathbf{r}_{2}) \ \mathbf{H}^{\dot{J}}(\overset{\wedge}{\mathbf{n}}, \mathbf{y}) = \left[(\mathbf{r}_{1} - \mathbf{r}_{2}) \ \mathbf{A}_{\mathbf{n}}^{(\dot{J})} - \mathbf{B}_{\mathbf{n}}^{(\dot{J})} \right] e^{(-1)^{\dot{J}}} \overset{\wedge}{\mathbf{n}} \mathbf{y} \mathbf{r}_{1} + \mathbf{B}_{\mathbf{n}} e^{(-1)^{\dot{J}}} \mathbf{r}_{2} \overset{\wedge}{\mathbf{n}} \mathbf{y} \mathbf{r}_{1} + \mathbf{B}_{\mathbf{n}}^{\dot{J}} \mathbf{r}_{2} \overset{\wedge}{\mathbf{n}} \mathbf{y} \mathbf{r}_{1}$$

where r_1 & r_2 are two roots of equation given by (2.10) of [2] and $a_{11}^{-}a_{66}$ are constants of the medium. Similarly we solve for anti symmetrical system of body forces as

$$u_{x}^{a(j)}(x,y) = \sum_{n=1}^{\infty} \cos(\beta_{n}x) \beta_{n}^{-1} \left[a_{11} G_{yy}^{j} - \beta_{n}^{2} a_{12} G^{j} \right], j = 3,4 (3.4)$$

$$u_y^{a(j)}(x,y) = \sum_{n=1}^{\infty} \beta_n^{-2} \sin \beta_n x \left[a_{11} G_{,yyy}^{j} - \beta_n^{2} (a_{12} + a_{66}) G_{,y}^{j} \right], \quad (3.5)$$

with G^j given as

$$(\mathbf{r}_{1} - \mathbf{r}_{2}) G^{j}(\beta_{n}, \mathbf{y}) = \left[(\mathbf{r}_{1} - \mathbf{r}_{2}) G^{(j)}_{n} - D^{(j)}_{n} \right] e^{(-1)^{j} \mathbf{r}_{1} \beta_{n} \mathbf{y}}_{+D_{n} e}^{(-1)^{j} \mathbf{r}_{2} \beta_{n} \mathbf{y}}$$
(3.6)

We can easily evaluate the components of stress for two cases, from the equations (3.1) - (3.6) and the stress-strain relations. Having satisfied the boundary conditions and continuity conditions (1.1) - (1.4), the problems are reduced to system of dual trigonometrical series relations whose solutions are obtained by the method of Parihar [5].

Then we calculate the components of stress and of displacement in the vicinity of crack tip $(\pm c, \circ)$ and use the definition of stress-intensity factor [1],

$$K_{c}^{S} = \frac{2a^{-1}\sin(\alpha c/2)}{\sqrt{\alpha \sin \alpha c}} \int_{0}^{c} \left[\cos(\alpha y/2) \mathcal{F}_{1}(y) / \sqrt{G(y,c)}\right] dy, \qquad (3.7)$$

$$\mathbb{E}_{\mathbf{c}}^{\mathbf{a}} = \left(2\mathbf{a}^{-1}/\sqrt{\mathbf{qsinqc}}\right) \int_{0}^{\mathbf{c}} \left[\sin\left(\mathbf{q}\mathbf{y}\right) G_{1}(\mathbf{y})/\sqrt{G(\mathbf{y},\mathbf{c})}\right] d\mathbf{y} , \qquad (3.8)$$

with
$$G(y,c) = \cos qy - \cos qc, q = \pi/a$$
, (3.9)

$$F_{1}(x) = \sigma_{yy}^{s(1)}(x,0) + \sigma_{yy}^{s(2)}(x,0), G_{1}(x) = \sigma_{yy}^{a(1)}(x,0) + \sigma_{yy}^{a(2)}(x,0), G_{1}(x) = \sigma_{yy}^{a(2)}(x,0) + \sigma_{yy}^{a(2)}(x,0), G_{1}(x) = \sigma_{yy}^{a(2)}(x,0) + \sigma_{yy}^{a(2)}(x,0), G_{1}(x) = \sigma_{yy}^{a(2)}(x,0) + \sigma_{yy}^{a(2)}(x,0)$$

$$N_{c}^{s} = (2a^{-1}\cos(qc/2)/\sqrt{q\sin qc}) \int_{0}^{c} \left[F_{2}(y) + \frac{1}{(r_{1} - r_{2})} \int_{c}^{a} F_{3}'(y)\right] \left[\sin(qy/2)/(r_{1} - r_{2})\right] \int_{0}^{a} \left[F_{3}(y)\right] \left[\sin(qy/2)/(r_{1} - r_{2})\right] \int_{0}^{a} F_{3}'(y) \left[\sin(qy/2)/(r_{1}$$

$$N_c^a = (2a^{-1}\sqrt{\sin qc/q}) \int_0^c G_4(y) \frac{dy}{\sqrt{G(y,c)}},$$

$$(3.11)$$

$$(3.12)$$

$$G_4(y) = \sin y G_2(y) + a^{-1} \frac{\cos(qy/2)}{r_1 - r_2} \int_{c}^{a} (\sin(qt/2)/G(y,t))G_3'(t)dt,$$
 (3.13)

$$G_{2}(t) = \sum_{n=1}^{\infty} \left[\sigma_{yy}^{a(1)}(\beta_{n}, 0) - \sigma_{yy}^{a(2)}(\beta_{n}, 0) \right] \cos \beta_{n} t = \sum_{n=1}^{\infty} p_{1}(n) \cos (\beta_{n}t),$$
(3.14)

$$G_3(t) = \sum_{n=1}^{\infty} p_1(n) \cos \beta_n t + u_x^{a(1)}(t,0) - u_x^{a(2)}(t,0)$$
 (3.15)

$$F_{2}(t) = \sum_{n=1}^{\infty} p_{0}(n) \sin \beta_{n}t, p_{0}(n) = \sigma_{yyc}^{s(1)}(\beta_{n}, 0) - \sigma_{yyc}^{s(2)}(\beta_{n}, 0) /(3.16)$$

$$F_{3}(t) = (a_{11} - a_{12}) \left[(r_{1}^{2} - r_{2}^{2}) \right]^{1/2} a_{11}^{-1} \left[F_{2}(t) + u_{x}^{s(1)}(t, 0) - u_{x}^{s(2)}(t, 0) \right]$$
(3.17)

It is not difficult to evaluate the expressions for crack shape. But however, due to lack of space I am deleting those. The problem of two exterior Griffith cracks is important from experimental point of view, but, however, mathematically it is similar to that of one of section 3.

4. PLASTIC ZONE LENGTH

Dugdal [5] had proposed an elastic model for plane extension problem of a straight crack. The same has been extended to Orthotropic medium by Kushwaha [2]. The material was assumed to flow after yielding with constant tensile stress, T. We used Tsai-Wu criterion for yielding. In the present paper we extend the analysis of [6] to asymmetrical loading, causing yielding due to shearing stress too.

Thus, the boundary value problems to solve is with boundary conditions (1.1) and the boundary conditions (1.2) and (1.4) replaced by

$$\sigma_{yy}(x,0^{\pm}) = \sigma_{xy}(x,0^{\pm}) = 0, \quad 0 \le |x| < d,$$
 (4.1)

$$\sigma_{yy}(x,0^{\pm}) = -P_1, \quad \sigma_{xy}(x,0^{\pm}) = -Q_1, \quad d \in |x| < 0,$$
 (4.2)

for interior Griffith crack and

$$\sigma_{yy}(x,0^{\pm}) = \sigma_{xy}(x,0^{\pm}) = 0, \quad d \leq |x| \leq a$$
, (4.3)

$$O_{yy}(x, O^{\pm}) = -P_1, O_{xy}(x, O^{\pm}) = -Q_1, C < |x| \le d$$
 (4.4)

for exterior Griffith cracks whose results are not reported here. Where \textbf{P}_1 and \textbf{Q}_1 are evaluated from the Tsai-Wu yield criterion for plan-strain conditions and given as

$$2P_1T_3 = -T_2 \pm \sqrt{T_2^2 + 4T_3(1 - \alpha)} , \qquad (4.5)$$

$$20_1 F_{66} = -F_6 \pm \sqrt{F_6^2 + 4F_{66} \alpha}, \qquad (4.6)$$

where T_1 , T_2 , T_3 ; F_6 , F_{66} etc. are given in appendix [2], d, α are arbitrary

constants to be determined. The finiteness of normal and of shearing stress at $(\pm c,0)$ will determine d as well as α . The solution of boundary value problems given by equations (4.1) - (4.4) can easily be obtained through the analysis of sections 2 - 4.

Thus using the equations (3.7), (3.8), (3.11), (3.12); we get for the determination of d which inturn gives plastic zone length as (c-d) for single crack. We get from

$$\int_{0}^{d} \frac{F_{1}(y)\cos(qy/2)dy}{\sqrt{G(y,c)}} - \frac{\sqrt{2}}{q}P_{1}\cos^{-1}\frac{\sin(\alpha d/2)}{\sin(\alpha c/2)} +$$
(4.7)

$$\int_{0}^{d} \frac{G_{1}(y) \sin(qy)dy}{\sqrt{G(y,c)}} = \frac{2}{q} P_{1} \left[\sqrt{G(d,c)} - \sqrt{G(o,c)} \right], \qquad (4.8)$$

and

$$\int_{0}^{d} \frac{F_{2}(y) \sin(y/2)dy}{\sqrt{G(y,c)}} + \int_{c}^{a} \frac{F_{3}'(y)\sin(qy/2)dy}{(r_{1}-r_{2})\sqrt{G(y,c)}} + \int_{0}^{d} \frac{G_{2}(y)dy}{\sqrt{G(y,c)}} + \frac{1}{2} \int_{0}^{d} \frac{G_{2}(y)dy}{\sqrt{G(y,c)}} dx$$

$$+ a^{-1} \int_{0}^{d} \frac{\cos(qy/2)dy}{(r_{1}-r_{2})\sqrt{G(y,c)}} \int_{c}^{a} \frac{\sin(\alpha t/2) G_{3}'(t)dt}{G(y,t)}$$

$$= \frac{\sqrt{2}}{q} Q_{1} \cos^{-1} \left[\frac{\pi \sec(qd/2)}{2} \right] + \frac{2}{q} Q_{1} \left[\sqrt{G(d,c)} - \sqrt{G(0,c)} \right]. (4.9)$$

5. TWO GRIFFITH CRACKS

 $\delta(y) = [1G(b,y) G(y,c) 1]^{\frac{1}{2}}$

To solve the problem two symmetrically placed Griffith cracks with boundary conditions (1.1), (1.5) - (1.7) we follow same analysis and section 2-3. The problem is reduced to triple trigonometrical series relations whose solutions are obtained by the method of Parihar [5]. Just to save the space we shall report only stress intensity factors of symmetrical case only.

The solutions of triple series relations, involving the constants, are given as

$$g_{1}(t) = \frac{a^{-2}}{\delta(t)} \left[\int_{b}^{c} \frac{\delta \omega \sin(\alpha x) F_{1}(y) dx}{G(x,t)} + L_{1} \right] = \frac{a^{-2}}{\delta(t)} \Delta_{1}(t) \quad (5.1)$$

$$g_{2}(t) = \frac{a^{2} \sin(\alpha t)}{\delta(t)} \left[\int_{b}^{c} \frac{\delta(x) F_{4}(x) dx}{G(x,t)} + L_{2} \right] = \frac{a^{-2}}{\delta(t)} \Delta_{2}(t), \quad (5.2)$$
with $\int_{b}^{c} g_{1}(t) dt = 0, \quad \int_{b}^{c} g_{2}(t) dt = F_{3}(b) - F_{3}(c), \quad (5.3)$

Using the definitions of stress-intensity factors we get the expressions as

$$K_{xi}^{s} = (-1)^{i} M(x_{i}) \Delta_{1}(c), N_{xi}^{s} = (-1)^{i} M(x_{i}) \Delta_{2}(c), i = 1,2,$$

$$M(y) = [9 \sin \varphi G(b,c)]^{i} N_{xi}^{s}, x_{1} = b, x_{2} = c \qquad (5.5)$$

where \triangle_1 (y) and \triangle_2 (y) are defined in (5.1) - (5.2) and F₄ as F₄ (y) = F₂ (y) + a⁻¹ [$\sin (q \mathcal{M}_2)/(\mathcal{Y}_1 - \mathcal{Y}_2)$] $\int_c^a [F_3'(t) \cos (c_V t/2)/(y,t)] dt$ (5.6)

6. PARTIAL CLOSING OF THE CRACK

Burniston's Model (BM): As pointed out by Burniston [7] that the crack faces may meet other than the crack tips. Parihar and Kushwaha [1] had extended to isotropic strip with symmetrical body forces. In the present section we are extending the analysis of [1,2] to orthotropic strip under asymmetrical system of body forces. We know from practical experience that when crack faces one separated cannot be brought to continuum concept of closure. However, we can stop propagation by providing the structures and such conditions that the crack faces may have tendency to move closure. The concept of partial closing of Burniston rests upon the idea that if the displacements $\mathbf{u}_{\mathbf{y}}(\mathbf{x},0)$ caused by constant pressure, T, is cancelled by body force then crack would be called partially closed.

Thus the problem under consideration is that the crack $0 \le |x| \le c$ (y=0) is opened by constant internal pressure, T, at crack faces and closed by asymmetrical system of body forces. The problem is characterised by the equations (1.1), (1.6) - (1.7) while (1.5) is changed to

$$\sigma_{xy}(x, 0^{\pm}) = 0, \sigma_{yy}(x, 0^{\pm}) = -T, b < |x| < c$$
, (6.1)

where b is unknown in this case which will be determined from the finiteness condition of resultant stress at (b,0) i.e. vanishing of stress-intensity factors (with vector sum).

Following the analysis of section 5 we can easily solve the boundary value problem. Thus we get

$$\left(\mathbf{1}+\mathbf{r}_{1}\mathbf{r}_{2}\right)\left(\overrightarrow{\mathbf{k}_{b}^{s}+\mathbf{k}_{b}^{a}}\right)+\overrightarrow{\mathbf{N}_{b}^{s}+\mathbf{N}_{b}^{a}}=0$$
(6.2)

If body forces become symmetrical (with respect to both the axes) $K_b^a = N_b^s = N_b^a = 0$

Kushwaha Model (KM):

As emphasised in previous model that after opening, the crack faces may not close upto the state as before the opening. However, application of stresses may destroy the elastic nature of meeting surfaces and develop plasticity in the region. Mathematically, the boundary value problem is as follows. The boundary conditions (1.1), (1.3) - (1.4) while (1.2) changes to

$$\sigma_{xy}(x,0^{\pm}) = 0_1, \, \sigma_{yy}(x,0^{\pm}) = P_1, \, 0 \le |x| < b$$
, (6.3)

$$\sigma_{yy}(x,0^{\pm})=p, \sigma_{xy}(x,0^{\pm})=0, b \leq |x| < c$$
, (6.4)

with crack opening with constant pressure, P. at crack faces. P_1 and Q_1 are given by equations (4.5) - (4.6) with unknown. Thus there are two unknowns, namely b and α , which will be determined through the equations given below.

$$-P_{1}\frac{\sqrt{2}}{q}\sin^{-1}(\frac{\sin qb/2}{\sin qc/2}) + \frac{a}{2}\sqrt{\frac{2}{q}}P_{\cos^{-1}}(\frac{\sin qb/2}{\sin qc/2}) - \frac{P_{1}}{q}\sqrt{G(b,c)} - \sqrt{G(0,c)}$$

$$+ \int_{b}^{c} \left[\cos(qy/2) F_{1}(y) + \sin(qy) G_{1}(y) \right] / G(y,c) dy = 0 \quad (6.5)$$

$$- Q_{1} \frac{\sqrt{2}}{q} \left[\cos h^{-1} \left\{ \frac{x}{2} \sec(qc/2) \right\} - \cos h^{-1} \left(\frac{\cos qb/2}{\cos qc/2} \right) \right] - \frac{Q_{1}}{q} \left[\sqrt{G(b,c)} - \sqrt{G(o,c)} \right]$$

$$+ \int_{b}^{c} \frac{\sin(qy/2) F_{2}(y) + G_{2}(y)dy}{\sqrt{G(y,c)}} + \frac{1}{(r_{1} - r_{2})}$$

$$= \int_{c}^{a} \frac{\sin(qt/2) \left\{ G'_{3}(t) + F'_{3}(t) \right\} dt}{\sqrt{G(c,t)}} = 0 \quad (6.6)$$

where F_1 , F_2 , F_3 ; G_1 , G_2 and G_3 are given by equations (3.10), (3.14) - (3.17). In the next section we shall consider an example of point body force.

Thus we clearly see that in the length of closed crack the normal displacement is zero for B-Model for K-Model it is not so. Secondly, in BM the resultant is finite at one point i.e. (b, 0) while in KM it is constant throughout the closed length of the crack.

7. AN EXAMPLE

Since revets or stiffeners can be simulated by point body forces in mathematical analysis, therefore, the following example is of practical importance. $X_{0} = Y_{e} = 0, \quad X_{e} = 0, \quad Y_{0} = P\delta(x) \left\{ \delta(y - h) - \delta(y + h) \right\} (7.1)$

$$X_0 = Y_0 = 0$$
, $X_0 = 0$, $Y_0 = P\delta(x) \{\delta(y - h) - \delta(y + h)\}$ (7.1)

which means a constant force of magnitude P is acting at $(0, \pm h)$ in positive and negative directions of y, respectively. Since loading is symmetrical with respect to crack faces, we shall get anti-symmetrical part of the problem to be zero identically and $F_2(x) = F_3(x) = 0$. Thus obtaining the transform of (7.1) and substitute in equations (2.3) -(6.5) and get use of the stress-strain relations with first of (3.10) and evaluate the integrals in (3.7), we get

$$K_{c}^{s} = \frac{P}{\sqrt{2\pi a \cot(qe/2)}} \left[r_{3} (r_{1}) V_{o}(r_{1}h) - r_{3}(r_{2}) V_{o}(r_{2}h) \right] (7.2)$$

$$v_0(y) = \cosh(\alpha y/2) / \sqrt{R(y,c)}, r_3(y) = a_{11}^{-1} \left\{ a_{12} + a_{66} - a_{11} y^2 \right\} / (r_1^2 - r_2^2)$$
 (7.3)

The values of $(\pi/2c/P)$ % for different values of h/c and of c/a obtained from equation (7.2) of wood of oak are given below.

TABLE - 1

h/c	Grains parallel to x-axis		Grains parallel to y-axis	
	c/a = 0.9	c/a = 0.5	c/a = 0.9	c/a 0 0.5
0.0	4.512	1.772	4.522	1.782
0.5	4.504	1.753	4.512	1.769
1.00	4.480	1.572	4.509	1.766
2.00	4.467	1.377	4.484	1.613
4.00	4.466	1.258	4.468.	1.365

Partial Closing: (BM)

The solution of equation (6.2) with $F_4(x)=0$ and $F_1(x)$ is obtained as in preceeding lines with the change of P by Q. The graph is plotted for (Q/cT) against b/c for different value of h/c and c/a in Figure 1. Though expression are lengthy ones, yet they are closed form.

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