THE SIGNIFICANCE OF MODE 1 BRANCH CRACKS FOR COMBINED MODE FAILURE

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ABSTRACT

Cracks in isotropic materials are observed to grow perpendicular to the maximum principal applied tensile stress, or, put more precisely into fracture mechanics terms, in Mode I. For cracks not initially oriented in the preferred direction, prediction of failure involves finding criteria for the formation and propagation of Mode I branch cracks. No satisfactory criterion for the formation of Mode I branch cracks appears to exist, but, through an examination of stress intensity factors for branch cracks, an approximate, lower bound failure envelope for the general case of a crack inclined to the applied stress has been constructed. Comparison with experiment is hampered by lack of general agreement on what constitutes a valid test for a specimen containing an inclined crack.

KEYWORDS

Fatigue crack growth; branch cracks; stress intensity factors; failure criteria; failure envelope.

NOTATION

a	Crack length
δa	Branch crack length
K	With subscripts I, II, III to denote mode, stress intensity factor for main crack
K_c	Critical value of K_I for crack growth
K_{Ic}	Plane strain value of K_c
k	With subscripts I, II, III to denote mode, stress intensity factor for branch crack
k_I^*	Maximum value of k_I
Δ	As prefix, range in fatigue cycle

θ Angle of branch crack

ν Poisson's ratio

ϕ Inclination of crack element (Fig. 3)

INTRODUCTION

For situations where the crack tip stress field can be characterized by stress intensity factors it is a matter of observation that, for both static and fatigue loadings, cracks in isotropic materials tend to grow perpendicular to the maximum principal applied tensile stress, or, put more precisely into fracture mechanics terms, in the opening mode (Mode I) (Frost, Marsh and Pook, 1974; Knauss, 1970; Cotterell, 1966). In fatigue the statement applies only to a macrocrack. The qualification 'tend' is included to allow for those cases where geometrical constraints prevent pure Mode I growth. A Mode I crack is not necessarily straight and crack trajectories are not readily determined (Pook, 1980).

Conventional specimens used to determine crack growth properties of materials are designed so that only Mode I displacements are present. However, in the general case of a crack-like flaw from which a service failure originates, Mode II or Mode III displacements or both may be present. Under such combined loadings, initial crack growth will in general not be in the plane of the initial flaw, and stress intensity factors will change radically as soon as crack growth starts. The direction of crack growth will normally be such that K_I, the opening mode stress intensity factor, has its maximum value and K_{II} and K_{III}, the edge sliding and shear mode stress intensity factors, are zero.

For a Mode I crack under static loading, crack growth occurs when a critical value of K_I, called K_c, is exceeded. Under plane strain conditions this becomes a geometry independent property, K_{Ic} (British Standards Institution, 1977). Similarly for fatigue loading, crack growth occurs when a critical (usually called threshold) value of ΔK_I, the range of K_I in the fatigue cycle, is exceeded (Frost, Marsh and Pook, 1974). As compressive stresses simply close a Mode I crack, that is K_I cannot be negative, ΔK_I is calculated from the positive part of the fatigue cycle only. In this paper it is assumed that the critical values are sharply defined although this is not necessarily the case in practice (Pook, 1980). In the remainder of the paper the prefix Δ is only added where it is needed for clarity, and except where noted it is assumed that K_{II} or K_{III}, which can be of either sign, do not pass through zero during the fatigue cycle. Other assumptions are:

a Ratios between K_I, K_{II} and K_{III} do not vary during loading. (An example of non-proportional fatigue loading is discussed in the Appendix.)

b The crack is initially stress free; in particular there are no residual stresses due to a crack tip plastic zone developed by some type of prior loading.

c The main crack is straight and, for three-dimensional cracks, the main crack is flat and the crack front straight, or the appropriate radii of curvature are large compared with branch crack length.

As cracks tend to grow in Mode I it is pertinent to consider the behaviour of a small Mode I branch at an initial (main) crack. Short crack limitations (Lankford, 1980) mean that stress intensity factors are not a valid basis for the discussion of combined mode problems unless the initial (main) crack is of the order of a quarter millimetre long (Pook, 1980). The general case of a quasi-two-dimensional (main) crack, where only Mode I and II displacements can be present, is discussed, and the discussion extended to the general three-dimensional case, which may involve Mode III displacements.

THE ANGLED CRACK PROBLEM

An angled crack is usually shown as a central crack in a large sheet, with the crack inclined to the applied stress. Here, it is taken to mean the quasi-two-dimensional case where only Mode I or Mode II displacements, or a combination of the two can be present. For an angled crack, crack growth is at an angle θ to the initial (main) crack as shown schematically in Fig. 1; for θ to be positive K_{II} must be negative. Criteria are needed for the formation of a branch crack, its initial direction and whether or not the branch, once formed, will continue to grow. Crack growth from an angled crack follows a curved path. A complete solution of the problem would require determination of this path and corresponding stress intensity factors. Here it is generally assumed that once a branch starts to propagate, failure will follow. It is important to distinguish between criteria for the formation of a branch crack, and criteria for its propagation. Depending on circumstances either event may dominate behaviour, a point which does not appear to be generally appreciated (Pook, 1980).

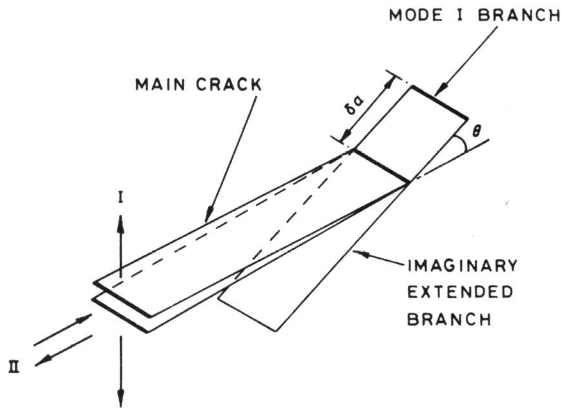

Fig. 1. Quasi-two-dimensional crack with Mode I branch.

Criteria for crack growth from an angled crack have been extensively discussed in the literature; Swedlow (1976) has surveyed some of them. In the literature, criteria are often assessed by comparing actual and predicted directions of crack growth. Results tend to be unconvincing both because it is difficult to measure initial directions accurately, and because predicted directions are not sharply defined (Pook, 1980). Minor deviations from isotropy are therefore likely to have a significant influence on initial crack direction. Various criteria have been used to construct failure envelopes for combined Mode I/II loading.

Two main approaches to the problem are used (Swedlow, 1976). The simpler is to assume that the crack tip stress field for the main crack, in terms of one parameter or another, is controlling so that the angular behaviour of the parameter selected should be examined for the vicinity of the crack tip. This implicitly assumes that branch crack formation is the critical event. In particular various aspects of the strain energy density have been extensively discussed, with a view to predicting the load to cause failure under a static load and the initial direction of crack growth (Swedlow, 1976).

The alternative is to examine the behaviour of a branch crack (Fig. 1). This is appropriate where the branch forms easily, or can be regarded as already present, perhaps due to a metallurgical discontinuity, so that branch crack propagation is the controlling event. The observation that cracks tend to grow in Mode I indicates that the values of θ should be selected such that K_{II} for the branch crack (k_{II}) is zero. For a straight branch crack, available data suggest that K_I for the branch crack (k_I) has its maximum value k_I^* for an angle θ at which k_{II} is zero, although this does not appear to have been formally proved (Pook, 1980). The value of k_I in the vicinity of its maximum is only weakly dependent on θ.

Approximate solutions to branched crack problems in the quasi-two-dimensional case can be obtained by solving the problem of an application of the prior traction on the branch due to the main crack stress field, but merely ensuring that the 'extended branch' (Fig. 1) is traction free (Howard, 1978). The method becomes increasingly inaccurate as θ increases. Application of the method, most easily accomplished by comparison of appropriate crack tip stress field components, given for example by Paris and Sih (1965), leads to

$$k_I = \cos\frac{\theta}{2}\left(K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right) \tag{1}$$

$$k_{II} = \tfrac{1}{2}\cos\frac{\theta}{2}\{K_I \sin\theta + K_{II}(3\cos\theta - 1)\}. \tag{2}$$

It follows that the direction for k_I to be a maximum and k_{II} zero is given by

$$K_I \sin\theta = K_{II}(3\cos\theta - 1) \quad (70.5^\circ\ldots \leqslant \theta \leqslant -70.5^\circ\ldots). \tag{3}$$

The failure envelope obtained from this approximate solution is shown in Fig. 2.

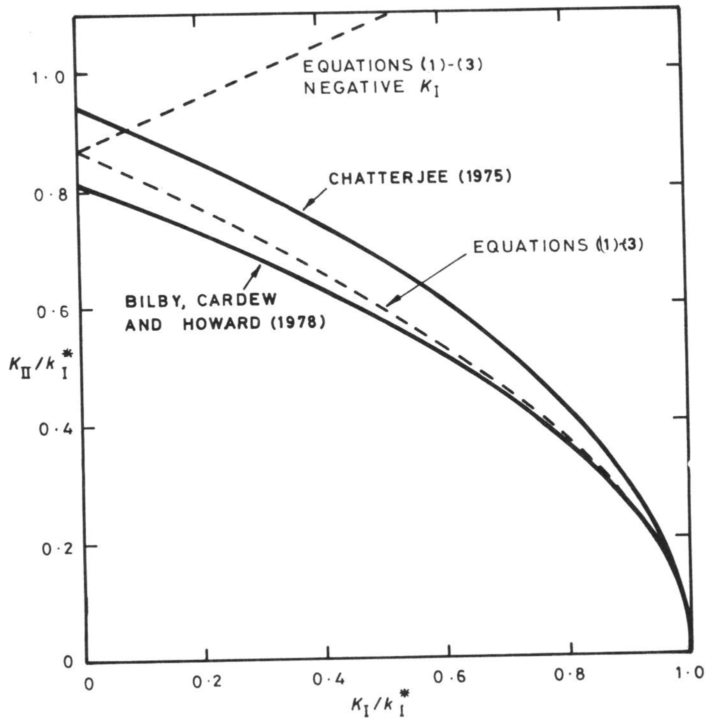

Fig. 2. Failure envelopes for combined Mode I/II loading.

As compressive stresses simply close a crack, only Mode II displacements will be present for an angled crack loaded in compression, and the value of K_{II} will control behaviour. However, friction between the crack surfaces acts to reduce K_{II} so calculating K_{II} for the frictionless state gives a lower bound (Swedlow, 1976). Stress intensity factors can be used to characterize stresses at the tips of narrow slits. Negative values of K_I then have meaning and act to reduce the value of k_I^*, as shown in Fig. 2; the corresponding value of θ is given by a root of equation (3) outside the indicated range. The sign of K_{II} (and K_{III}) is ignored in Fig. 2 and subsequent similar envelopes. When ΔK_{II} (or ΔK_{III}) is required it must be calculated separately for the positive and negative parts of the fatigue cycle where this passes through zero. This is because of the change in direction of a Mode I branch when K_{II} (or K_{III}) changes sign.

The first crack tip stress field parameter to be examined as a fracture criterion was the hoop stress (Erdogan and Sih, 1963). However, this is equivalent to the determination of k_I by the extended branch method.

Comparison of failure envelopes based on several criteria shows wide variation in predicted behaviour (Radaj and Heib, 1978), although the envelope based on strain energy density is similar to those in Fig. 2. It was concluded that, because different materials appeared to obey different criteria, the predictive utility of the various envelopes was limited. However, because failure cannot take place unless k_I^* exceeds the appropriate critical value, an envelope based on k_I^* provides a lower bound for failure, appropriate for design purposes. Examination of a range of published data for static and fatigue loading confirms this: data all fall on or above the envelope based on equations (1)-(3); some fatigue data fall well above it (Pook, 1980). Examination of numerical data shows that the envelope obtained depends on the precise boundary conditions (Pook, 1980). Data due to Chatterjee (1975) and Bilby, Cardew and Howard (1978) led to the highest and lowest envelopes found (Fig. 2). Despite its limitations, consideration of the stress intensity factors for a Mode I branch crack appears to be the most useful general approach to the understanding of the behaviour of angled cracks.

In practical angled-crack testing, variations from the ideal quasi-two-dimensional crack shape shown in Fig. 1 can present problems. In pure Mode I, variations from the ideal initial crack shape merely lead to some uncertainty in the value of K_I, but in combined mode testing such deviations can introduce unwanted modes. For example, in a specimen intended to give pure Mode II displacements, crack front curvature introduces unwanted Mode III displacements, and also means that the branch crack cannot be flat. In angled-crack testing in fatigue, extraneous Mode III displacements appear to facilitate branch crack formation (Pook, 1980); in the absence of Mode III displacements branch crack formation appears to be controlled by K_I for the main crack.

THE MODE III PROBLEM

The understanding of the behaviour of a Mode III crack requires the stress intensity factor to be determined for a Mode I branch at a Mode III crack. This is more difficult than the angled-crack problem because of its three-dimensional nature; an approximate solution is given below.

For a Mode III branch at a Mode III crack (Fig. 3) use of the extended branch method gives

$$k_{III} = K_{III} \tag{4}$$

a result which is independent of θ. For a flat element rotated through an angle ϕ,

$$k_I = K_{III} \sin 2\phi \quad (5)$$

$$k_{III} = K_{III} \cos 2\phi \quad (6)$$

which again is independent of θ. Thus for $\phi = 45^o$, k_I has its maximum value k_I^* and $k_{III} = 0$, that is

$$k_I^* = K_{III}. \quad (7)$$

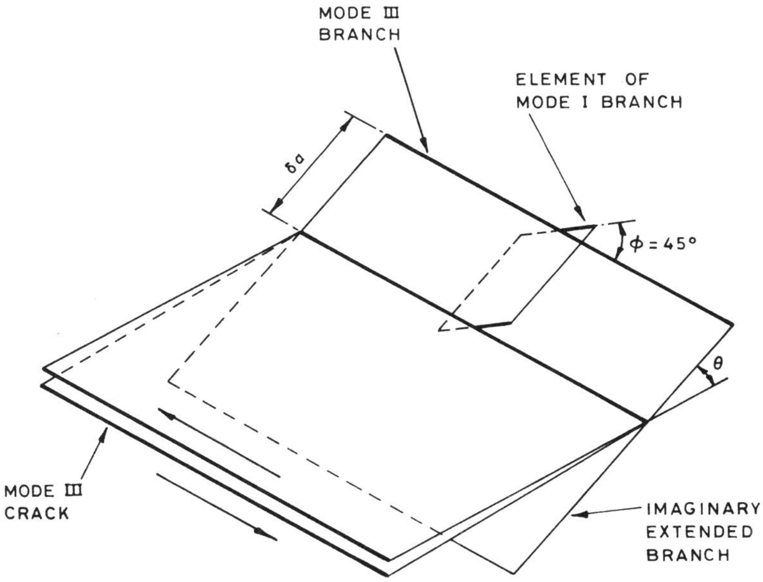

Fig. 3. Mode I and Mode III branches at a Mode III crack.

These flat Mode I elements only intersect the main crack front at one point, so they cannot be assembled to form a smooth-surfaced branch crack. However the element crack fronts can be assembled into a smooth helix, and a smooth ruled surface added to form a helical branch crack (Fig. 4). For this helical crack the value of ϕ for which K_I is a maximum and (presumably) $k_{III} = 0$ will not necessarily be exactly 45^o. Such a branch cannot grow without changing shape. A flat branch crack, such as the circular branch, can grow keeping the same shape, but there is no orientation for which only Mode I displacements are present around the whole branch crack front. Hence crack growth from a Mode III crack can be expected to be a complex process (Pook, 1980).

More generally, any branch crack consisting of a ruled surface with the straight lines passing through an origin can grow and remain smooth. It is not clear whether it is possible to find a configuration of this type such that only Mode I displacements are present along the whole crack front. The semi-circular branch crack shown in Fig. 4 has its front on the helical surface.

Branch cracks of similar configuration have been observed by Knauss (1970) in static tests on a glassy polymer; these presumably approach pure Mode I along the whole crack front. Comparison of stress intensity factors for elliptical and straight-fronted cracks shows that the effect of crack front curvature is to reduce the stress intensity factor to below that for the equivalent straight-fronted crack.

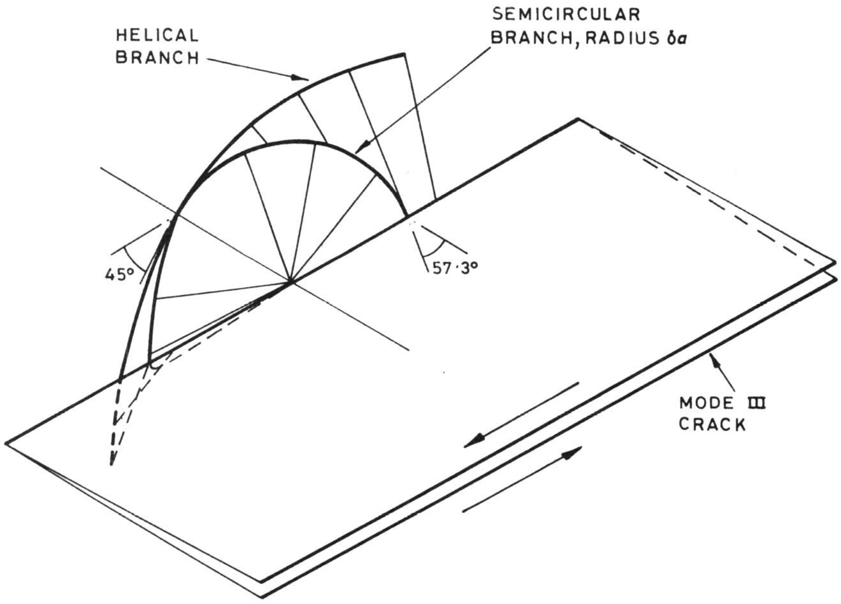

Fig. 4. Helical and semi-circular branches at a Mode III crack.

A helix possesses curvature, so even for a helical branch (Fig. 4) the stress intensity factors can be expected to be below those given by equations (5) and (6). A conservative indication of the effect of curvature on a Mode I branch crack can be obtained by considering a section at angle ϕ through the cylinder containing the helix; this section is an ellipse with minor axis 2a and major axis 2a cosec ϕ, and at the end of the minor axis has the same curvature as the helix. For this ellipse equation (7) becomes

$$k_I^* = 0.740 K_{III}. \tag{8}$$

This implies that the ratio of critical values of K_{III} and K_I is 1.35 compared with a ratio of $(1 - 2\nu)^{\frac{1}{2}}$, which is 0.577 for $\nu = \frac{1}{3}$, obtained from strain energy density. This may account for the observation (Section 2) that Mode III displacements encourage branch crack formation.

THE SLANT CRACK PROBLEM

A slant crack is most familiar at the growth on planes at 45^o through the specimen thickness often observed in thin sheets. This is associated with yielding on such 45^o planes (Pook, 1976) and is an example of a thin sheet problem where a quasi-two-dimensional approach is not really adequate (Frost, Marsh and Pook, 1974). Here it is taken to mean any crack where a combination of Mode I and III displacements are present, with the Mode III problem a limiting case.

Again considering an element of a Mode I branch as shown in Fig. 3, the extended branch method gives

$$k_I^* = \frac{K_I(1 + 2\nu) + \{K_I^2(1 - 2\nu)^2 + 4K_{III}^2\}^{\frac{1}{2}}}{2} \tag{9}$$

where ν is Poisson's ratio. This maximum value occurs when θ = zero and ϕ is given by

$$\tan 2\phi = 2K_{III}/K_I(1 - 2\nu) \quad (45^o \leqslant \phi \leqslant -45^o). \tag{10}$$

In developing these equations it is necessary to use the plane strain form of the Mode I crack tip stress field equations (Pook, 1971). The failure envelope obtained from equation (9) with $\nu = \frac{1}{3}$ is shown in Fig. 5 and is given by

$$\frac{K_{III}}{k_I^*} = 1 - \left\{\frac{(1 + 2\nu)K_I}{k_I^*} + 2\nu\left(\frac{K_I}{k_I^*}\right)^2\right\}^{\frac{1}{2}}. \tag{11}$$

As for the angled-crack problem, a narrow slit can have a negative value of K_I: this reduces k_I^* and ϕ falls outside the range shown at equation (10).

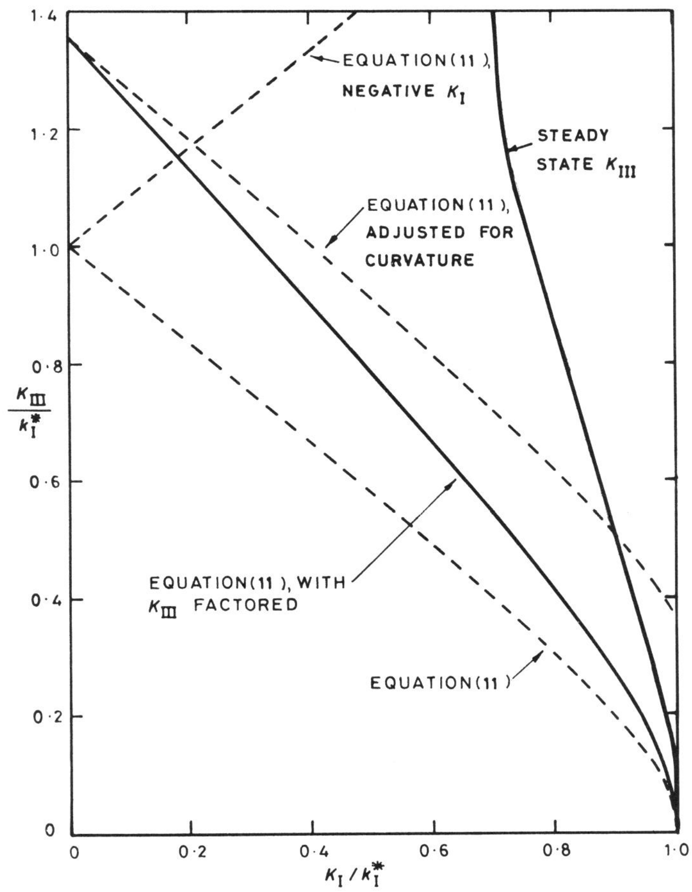

Fig. 5. Failure envelopes for combined Mode I/III loading.

As θ increases from zero the maximum possible value of k_I for a branch crack decreases and increasing values of k_{II} inevitably appear, so a pure Mode I helical crack cannot be formed. It is tempting to attempt a curvature adjustment for a Mode I branch by considering the situation at θ = zero, and using the same method as for the pure Mode III problem. This does extrapolate to the correct value for a pure Mode I main crack but gives a clearly unconservative envelope (Fig. 5). In the absence of a satisfactory solution it is suggested that interpolation between the pure Mode I and pure Mode III cases be achieved by replacing K_{III} in equation (9) by $K_{III}/0.740$, giving the envelope shown in Fig. 5. Limited available data for static and fatigue loadings mostly lie above the suggested envelope (Pook, 1980). An apparent exception is provided by two static tests under pure Mode III loading (Shah, 1974), however, these particular results must be regarded as suspect on the grounds of excessive plasticity. For pure Mode III the occurrence of branch cracks at the value of $\phi(45^{o})$ predicted by equation (10) is well established (Knauss, 1970; Pook, 1980; Shah, 1974). Three specimens preserved from the slant crack tests described by Pook (1971) were examined and showed small facets indicating branch crack formation at angles close to those predicted by equation (10).

THE GENERAL CASE

Extension of the extended branch method to the general case of combined Mode I, II and III loading is straightforward. Consideration of a Mode I element shows that the actual value of k_I^* and values of ϕ and θ, are obtained by first calculating an apparent value of k_I^* and actual value of θ from equations (1) and (3) using the values of K_I and K_{II}. The problem is then treated as if it were a slant crack using the apparent value of k_I^* in place of K_I together with the value of K_{III}. This gives the values of θ and k_I^*. The failure envelope shown in Fig. 6 was constructed using $K_{III}/0.740$ in place of K_{III} in order to correct for branch crack front curvature.

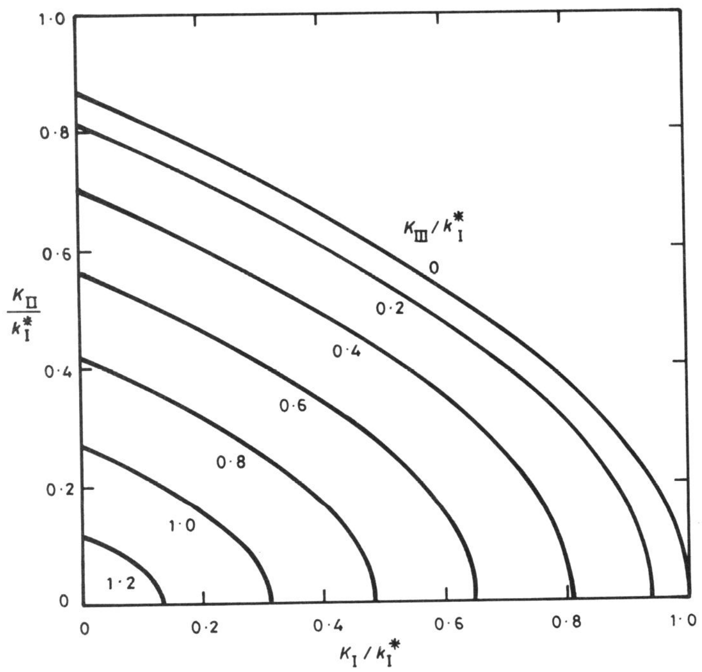

Fig. 6. Failure envelope for combined Mode I, II and III loading.

For rough calculations it is suggested that the simple expression

$$k_I^* = K_I + K_{II} + K_{III} \tag{12}$$

be used. It is conservative except in the vicinity of $k_I^* = K_{II}$. The expression

$$k_I^* = K_I + 1.22K_{II} + 0.74K_{III} \tag{13}$$

is more accurate and conservative everywhere.

DISCUSSION

The statement that cracks in an isotropic, elastic material tend to grow in Mode I can at best be regarded as a useful generalization based on observation. It appears in the literature in a number of forms (Pook, 1980), usually without justification, although the occurrence of Mode I crack growth could equally well be used as a test of isotropy, or assuming isotropy, as a test of essentially elastic behaviour.

It has been assumed that k_I has a maximum value when $k_{II} = k_{III} = 0$. This follows directly from the use of the extended branch approximate method, and for the quasi-two-dimensional case is true within the limits of accuracy of various sets of numerical data, but there appears to be no proof that it is generally true. Calculation of the maximum value of k_I, k_I^* is not easy. For the three-dimensional case, only approximate calculations are possible and the results are influenced by Poisson's ratio. The various uncertainties mean that the lower bound failure envelope for the general case shown in Fig. 6 can only be regarded as an approximation. As already pointed out the lack of generally agreed criteria for a valid combined mode test makes comparison with experimental data difficult, but various data sets examined all fell on or above this envelope. In particular the definition of plane strain, which was developed empirically for the Mode I case (Frost, Marsh and Pook, 1974), is unclear for the quasi-two-dimensional case, and poses considerable problems for slant cracks (Pook, 1971).

CONCLUSIONS

1 It is a matter of observation that from a macroscopic viewpoint cracks in an isotropic homogeneous material grow in Mode I under essentially elastic conditions.

2 No satisfactory criterion for the formation of a Mode I branch crack exists.

3 A lower bound failure envelope has been constructed for the general case of combined Mode I, II and III loading through examination of stress intensity factors for Mode I branch cracks, but computational difficulties leave some uncertainty.

4 Comparison with experiment is complicated by the lack of general agreement on what constitutes a valid combined loading test.

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APPENDIX

AN EXAMPLE OF NON-PROPORTIONAL FATIGUE LOADING

In some circumstances the ratios between K_I, K_{II} and K_{III} vary during the fatigue cycle so that it is not possible to find a direction for a branch crack such that k_{II} and k_{III} remain zero throughout the fatigue cycle. It would appear reasonable to assume that crack growth will take place in the direction for which Δk_I has its maximum value. However, if the case of a crack with zero to tension loading producing Mode I displacements, and a steady state Mode III loading is considered, the assumption implies that crack growth takes place in the plane of the main crack with K_{III} having no effect on behaviour.

The value of the fatigue crack growth threshold decreases as the mean value of K_I increases, which suggests that lower bound failure estimates should be based on the branch crack direction such that the combination of Δk_I and mean k_I produces the lowest failure loads. The failure envelope calculated by this method assuming (Frost, Marsh and Pook, 1974) that the threshold is proportional to (alternating K_I/mean $K_I)^{\frac{1}{3}}$ is shown in Fig. 5. Other relationships give slightly different envelopes. Treating the steady state K_{III} as if it were K_I simplifies calculations and is conservative.