METHODS FOR PREDICTING RESIDUAL STRENGTH OF THIN-WALLED COMPONENTS

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ABSTRACT

Residual strength tests were carried out on componentlike samples with different types of cracks. The results were used to check the accuracy of results obtained by different residual strength prediction methods. The test specimens were taken from 7475-T761 aluminium alloy sheets of 1 mm thickness.

The following residual strength prediction methods were investigated: R-curve, Newman's two-parameter method and the application of facture toughness obtained on centre crack specimens to componentlike specimens using Feddersen's method.

The R-curve method showed very good agreement between test result and prediction.

The R-curve used was obtained by the potential method.

KEYWORDS

Residual strength, elastoplastic, componentlike specimen, residual strength prediction methods, K_c-value, two-parameter concept of Newman, R-curve concept.

SYMBOLS USED

K_c	$(N/mm^{3/2})$	Fracture toughness
σ_c	(N/mm^2)	Failure stress relative to gross section area
a	(mm)	Crack length
Y		Correction function
F	(%)	Mean error $(= \frac{\sigma_{c,calc.} - \sigma_{c,test}}{\sigma_{c,test}}) \cdot 100\%$

$R_{p0 \cdot 2}$	(N/mm^2)	Yield stress
R_m	(N/mm^2)	Ultimate strength
E	(N/mm^2)	Modulus of elasticity
Z		Elongation at failure
W	(mm)	Width of specimen
G	(N/mm)	Energy release rate
R	(N/mm)	Crack resistance

INTRODUCTION

Failure of a cracked component usually occurs when the stress intensity at the crack tip reaches a critical value.

Within the plane strain condition the critical stress intensity may essentially be regarded as material constant.

Within the plane stress condition the critical stress intensity is a quantity dependent on the thickness of the specimen.

As the plastic zone increases and the crack propagates steadily until the specimen fails, the critical stress intensity also becomes dependent on crack length and sample width. Such dependency is found not only on ductile materials but also on thin-walled components, e.g. sheets of high strength metallic materials. Hence, the conventional K-method seems to have only limited applicability in the range of elastoplastic fracture processes.

The literature already contains different methods for predicting residual strength outside the linear elastic range.

The purpose of this study is to check different prediction methods against test results.

RESIDUAL STRENGTH DETERMINATION THROUGH TESTING

Tests on centre crack specimens and componentlike specimens had to be carried out to check the different residual strength prediction methods.

Centre crack specimens were tested in order to determine the material characteristics for the various prediction methods. The intention was to use the results of the residual strength tests on componentlike specimens to check the various prediction methods. The test programme and the results of the residual strength tests on the various types of specimens are shown in Figs. 1 to 4/1/. Fig. 5/1/ shows the R-curves obtained by different test methods. The test methods used were the compliance method and the potential method. The test results shown are mean values of two tests for each parameter.

The test specimens were taken in L-T direction from one batch of clad 7475T761 aluminium alloy. The sheet thickness was 1 mm. Its mechanical and chemical properties are shown in Figs. 6 and 7.

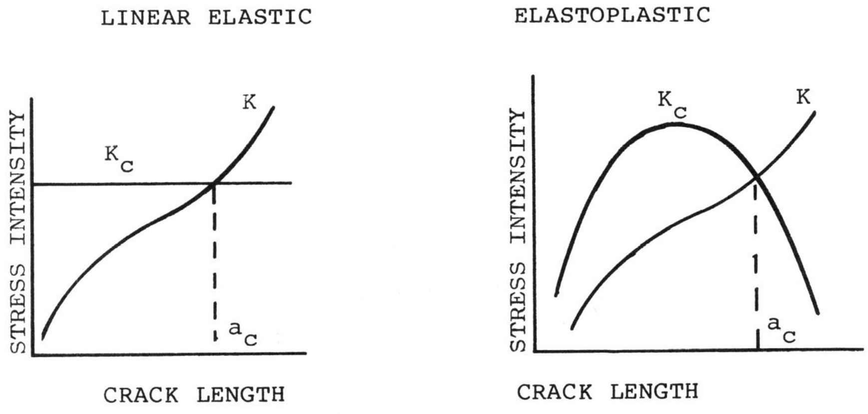
The crack starters for the residual strength tests were produced by saw cuts. The notch width was approx. 0.3 mm. It had been shown in previous tests that, for sheets in 7475T761 of 1 mm thickness, fatigue cracks and saw cuts have the same effect on residual strength. The R-curves, however, were determined according to reference /2/ using centre crack specimens with fatigue cracks.

APPLICATION OF CENTRE CRACK SPECIMEN FRACTURE TOUGHNESS TO COMPONENTLIKE SPECIMENS TO DETERMINE RESIDUAL STRENGTH

The underlying idea of this simple method is the assumption that a failure condition will also occur in the elastoplastic range whenever stress intensity reaches a critical value, i.e. fracture toughness.

As opposed to the linear elastic range, fracture toughness in the elastoplastic range is, in addition, dependent on crack length and specimen or component width.

Hence, it is necessary to know fracture toughness versus crack length for a specific specimen or component size (see schematic diagramme) in order to be able to predict the residual strength.



Schematic Representation of Procedure Used to Determine Critical Crack Length in the Linear Elastic and Elastoplastic Ranges.

According to the above procedure residual strength or critical crack length is determined as follows:

- Failure strength of centre crack specimens is determined for a specific crack length ratio, e.g. 2a/W = 1/3.

- Using Feddersen's method /3/ σ_c can then be determined over the entire crack length range.

- Fracture toughness can be calculated with the formula

$$K_c = \sigma_c \sqrt{\pi a} \; Y_{\text{centre crack}} \tag{1}$$

  for any crack length using the residual strength curve.

The above diagrammes show that the intersection of the K_c curve and the stress intensity line is the critical crack length of a component.

Failure stress is calculated as follows:

$$\sigma_c = \frac{K_{c,\ \text{centre crack specimen}}}{\sqrt{\pi a_c}\, Y_{\text{component}}} \tag{2}$$

Comparison of predicted mean residual strength with the test results (see Fig. 8) shows, for componentlike specimens with symmetric cracks, a mean error of approx. 13 %, whereas for specimens with a single edge crack the error is greater then 27 %.

On the assumption that the correction function $Y_{\text{component}}$ used in the above formula truly reflects the actual stress intensity exerted on the componentlike specimens during the test the result suggests the following presumption:

This method will only produce useful results when fracture toughness is obtained on a specimen which is identical to the component not only with respect to width and crack length ratio but also with respect to tension and bending and the type of crack, such as symmetric or non-symmetric. It would, therefore, appear that this method could not be recommended for general application.

As fracture toughness of thin-walled components is normally obtained on centre crack specimens under purely tensile stress it should be applied only to components of similar configuration.

RESIDUAL STRENGTH PREDICTION BY NEWMAN'S TWO-PARAMETER METHOD

The procedure developed by Newman to predict residual strength /4/ takes into account the elastoplastic behaviour of the material.

The failure criterion

$$S_n = \frac{K_F}{\sqrt{\pi a}\, Y + \frac{m K_F}{S_u}} \tag{3}$$

where $S_n > P_{p0 \cdot 2}$,

contains the two material parameters K_F and m.
S_n is the failure stress relative to the remaining cross section area. Stress S_u results from the load required to completely plastify the remaining cross section area.

In the case of centre crack specimen S_u is equal to R_m. For the three-point bending test specimen Newman quoted for S_u a value of 1.5 R_m and for the compact tensile test specimen 1.62 R_m, applicable to a crack length ratio of a/w = 0.5.

The parameters K_F and m are assumed to be constant for a defined material thickness. They are an indication of the susceptibility of the material to cracking.

Rupture behaviour of materials with m → 0 is called brittle, whereas materials with m → 1 are said to be extremely tough.

Parameters K_F and m are determined in residual strength tests on centre crack specimens. The test specimens shall cover a wide range of specimen width and crack length ratio.

The following equations are used to calculate the parameters:

$$m = \frac{\sum_{i=1}^{N} \frac{\sigma_{NC,i}}{R_m} \sum_{i=1}^{N} K_{C,i}^2 - \sum_{i=1}^{N} K_{C,i} \sum_{i=1}^{N} K_{C,i} \frac{\sigma_{NC,i}}{R_m}}{\sum_{i=1}^{N} \left(\frac{\sigma_{NC,i}}{R_m}\right)^2 \sum_{i=1}^{N} K_{C,i}^2 - \left(\sum_{i=1}^{N} K_{C,i} \frac{\sigma_{NC,i}}{R_m}\right)^2} \tag{4}$$

$$K_f = \frac{\sum_{i=1}^{N} K_{C,i}^2}{\sum_{i=1}^{N} K_{C,i} - m \sum_{i=1}^{N} K_{C,i} \frac{\sigma_{NC,i}}{R_m}} \tag{5}$$

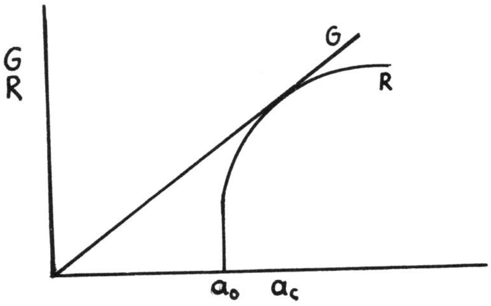
A comparison between test and prediction shows that the mean error in the prediction of the residual strength is not greater than 10.1 % for specimens with a single edge crack, whereas for specimens with a symmetric crack type i.e. with two cracks starting from the hole, the mean error is only 4 % and for specimens with double edge crack 8,5 % (see Fig. 8). Therefore, Newman's two-parameter method will render better results than the procedure described above.

A disadvantage of the prediction methods investigated so far is that they all require a relatively great number of residual strength tests.

DETERMINING RESIDUAL STRENGTH USING A R-CURVE.

The R-curve method is essentially based on the knowledge that a specimen or component will fail when the crack extension load exceeds the crack resistance of the material.

$$\frac{\partial G}{\partial a} \gg \frac{\partial R}{\partial a} \tag{6}$$



The R-curve, which is a material characteristic and said to be dependent on material thickness only, covers steady crack propagation and, depending on the test method used, also the influence of the plastic zone.

In this study the R-curve was derived using both the compliance method which, by the effective crack length, also covers the plastic zone, and the potential method which only considers physical crack length.

The R-curve obtained by the potential method was used to determine residual strength both with and without application of Irwin's correction which takes into account the plastic zone.

The mean errors (see Fig. 8) of the failure conditions or critical stresses derived from the R-curves shown in Fig. 5, as compared with the test results, were as follows: Relatively great differences occurred between the tests and the predictions based on R-curves obtained by the compliance method. The mean error was between 8 % and 18 %.

Residual strength predictions were somewhat better where an R-curve obtained by the potential method was used WITHOUT Irwin's correction. The mean error then was between 11.3 % and 14.8 %.

The best results were obtained with an R-curve derived by the potential method including Irwin's correction. The mean error did not exceed 8.1 %.

Thus, the R-curve derived by the potential method and corrected by Irwin's correction appears to be the most suitable of the previously used methods for residual strength prediction. Only few tests are required because the R-curve is largely independent of the crack length and component or specimen width /5/.

CONCLUSION

When comparing and assessing the investigated methods it should be kept in mind that the tests were performed on three componentlike specimens of one material only.

Strictly speaking, the statements are only valid for the parameter range covered by the tests concerning in particular quantitative results.

It may, however, be assumed that trends found in a qualitative assessment will also be applicable to other parameter ranges.

Taking a discriminating approach, it is very difficulty to state which prediction method should be preferred.

A comparison of the different methods should consider not only the accuracy of the prediction method but also criteria, such as easy handling, number of required tests, required test set-up and availability of material characteristics, e.g. residual strength, fracture toughness or R-curves.

Fig. 9 shows an assessment of the various prediction methods from the above-mentioned points of view.

When comparing the accuracy of the prediction methods, the use of R-curve derived by the potential method and corrected by Irwin's correction can be given the rating "excellent".

On the contrary Newman's method and the application of fracture toughness obtained with centre crack specimens to components use prediction methods that are very easy to handle.

The R-curve method is particularly good with respect to the number of tests required to derive the applicable characteristics.

The R-curve method, however, uses a relatively complicated test set-up. A practice for R-curve determination is still in progress /2/.

Regarding the availability of material characteristics it may be noted that a great number of residual strength tests have already been made for various materials of different thickness, whereas only few R-curves are currently available.

It may, however, be pointed out in general that the disadvantages of the method that applies the fracture toughness derived from centre crack specimens to components, and of Newman's method are essentially inherent in the method. The disadvantages of the method that uses R-curves obtained by the potential method and corrected by Irwin's correction are not inherent in the method. Thus elimination of these disadvantages seems to be a matter of time. This is valid in particular for test engineering and the availability of material characteristics.

Hence, the R-curve method has a good change of increasing application in future.

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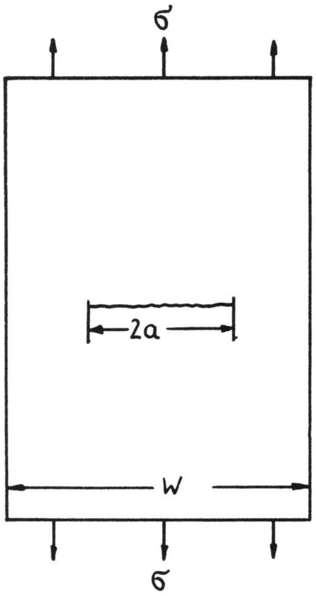


Stress intensity

$K = \sigma \sqrt{\pi a}\, Y$

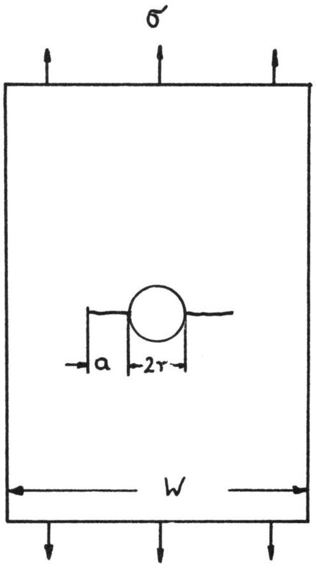
Correction function

$$Y = (1 - 0{,}025F^2 + 0{,}06F^4)\sqrt{\sec\frac{\pi a}{W}}$$

$F = 2a/W$

w (mm)	2 a/w (-)	a (mm)	σ_c (N/mm^2)
300.	0.10	15.05	352.8
300.	0.33	50.25	214.0
300.	0.50	75.00	155.3
100.	0.10	5.05	387.6
100.	0.33	16.65	280.4
100.	0.50	25.00	203.2

Fig. 1 Centre Crack Specimen Tests



Stress intensity

$$K = \sigma \sqrt{\pi a}\, Y$$

Correction Function

$$Y = Y_1 \; Y_2 \; Y_3$$

$$Y_1 = \sqrt{\sec\left(\frac{2r + a}{2(W-a)}\right)}$$

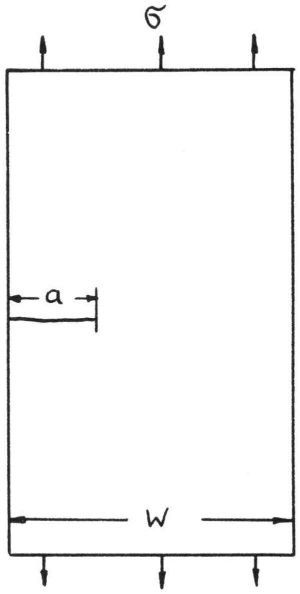
$$Y_2 = 1-0.15F+3.46F^2-4.47F^3+3.52F^4$$

$$F = (1 + a/r)^{-1}$$

$$Y_3 = \sqrt{\sec\frac{2r}{W}\pi}$$

w (mm)	$\frac{2\,(a+r)}{w}$ (-)	a (mm)	r (mm)	σ_C (N/mm^2)
300.	0.13	5.20	15.05	316.1
300.	0.17	10.40	15.10	298.7
300.	0.24	20.45	15.05	259.8
100.	0.26	3.15	10.00	305.6
100.	0.33	6.75	10.00	272.4
100.	0.40	10.25	10.00	244.4

Fig. 2 Test of Componentlike Specimens with Cracks Starting from a Hole



Stress intensity

$K = \sigma\sqrt{\pi a}\, Y$

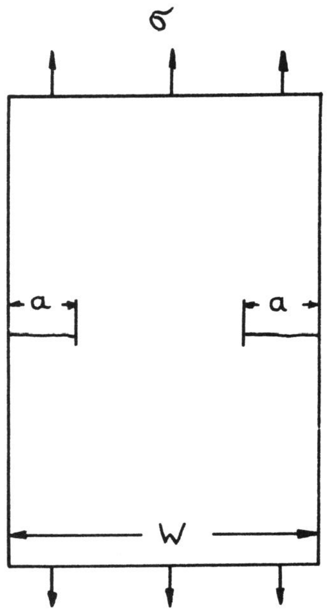
Correction function

$Y=5(20-13F-7F^2)^{-0.5}$

where $F = \frac{a}{W}$

w (mm)	a/w (-)	a (mm)	σ_C (N/mm^2)
150.	0.10	15.45	340.4
150.	0.33	50.50	235.6
150.	0.50	75.05	174.0
50.	0.10	5.25	379.8
50.	0.33	17.20	228.8
50.	0.50	25.25	162.5
100.	0.33	33.85	233.3
300.	0.33	100.20	164.6

Fig. 3 Test of Componentlike Specimens with Single Edge Crack



Stress intensity

$K = \sigma \sqrt{\pi a}\, Y$

Correction function

$$Y = \frac{1,122 - 0,561F - 0,205F^2 + 0,471F^3 - 0,19F^4}{\sqrt{1-F}}$$

$F = 2a/W$

w (mm)	2 a/w (-)	a (mm)	σ_c (N/mm^2)
100.	0.33	16.55	295.4
300.	0.33	50.20	245.6
100.	0.10	5.00	394.4
300.	0.10	15.15	344.5
100.	0.50	25.05	220.5
300.	0.50	75.10	185.7

Fig. 4 Tests of Componentlike Specimens with Double Edge Crack

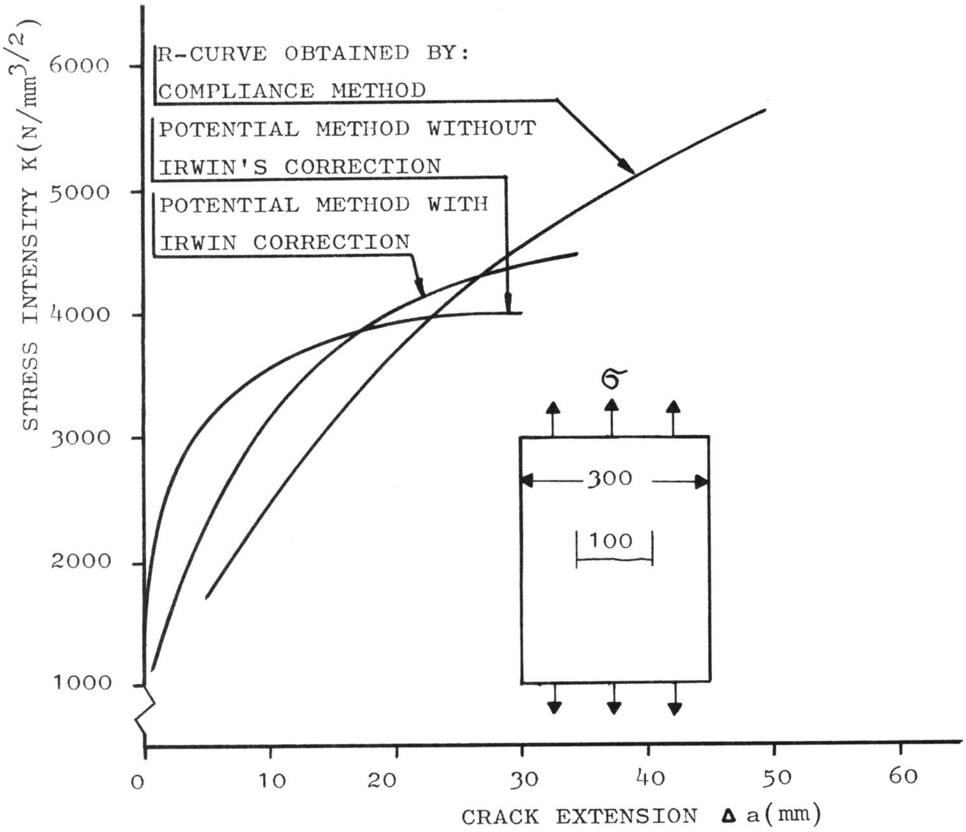


Fig. 5 R-Curves Obtained by Different Methods

$R_{p0 \cdot 2}$ (N/mm^2)	R_m (N/mm^2)	E (N/mm^2)	Z (%)
451	495	67600	10.5

Fig. 6 Mechanical Properties of Aluminim Alloy 7475 T761

Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti
0.1 max.	0.12 max.	1.2 to 1.9	0.06 max.	1.9 to 2.6	0.18 to 0.25	5.2 to 6.2	0.06 max.

Fig. 7 Chemical Properties of Aluminim Alloy 7475 T761
According to Material Specification

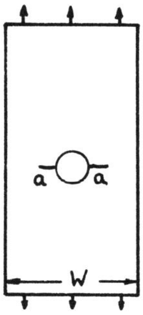
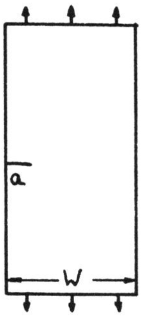
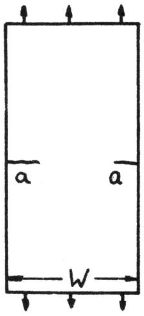
Test Paramters Material: AL 7475-T761 Material thickness: 1 mm Specimen width: from 50 mm to 300 mm Crack length Ratio: $\frac{a}{W}$; $\frac{2a}{W}$; $\frac{2(a+R)}{W}$ from 0.1 to 0.5			
R-Curve obtained by compliance method	± 14,2 %	± 18,3 %	- 8,3 %
R-Curve obtained by potential method without Irwin's correction	+ 11,3 %	± 14,8 %	± 12,0 %
R-Curve obtained by potential method with Irwin's correction applied	+ 5,5 %	- 8,1 %	± 4,0 %
Newman's two-parameter method	- 4,0 %	± 10,1 %	- 8,5 %
Use of K_c values obtained on centre crack specimens with same width and crack length ratio as componentlike specimen	- 11,0	- 27,0	- 12,0

Fig. 8 Mean Errors in the Prediction of Residual Strength of Componentlike Specimens Using Different Methods of Prediction.

Prediction Method	Prediction Method Accuracy	Prediction Method Implementation	Required Number of Tests	Test Set-Up	Availability of Material Characteristics
Application of fracture toughness determined on centre crack samples to components	Of limited adequacy only	Simple, no computer required	Rather high, a minimum of 2 tests for each specimen width	Simple	Relatively good
Newman's two-parameter method	Good	Relatively simple, no computer required	Very high, tests required with different crack lengths and specimen widths	Simple	Relatively good
R-curve obtained by compliance method	Adequate	Complicated, computer required	Very small, 2 to 3 tests only	Relatively complicated, no mandatory procedure	Inadequate
R-curve obtained by potential method, without Irwin's correction	Satisfactory	Complicated, computer required	Very small, 2 to 3 tests only	Relatively complicated, no mandatory procedure	Inadequate
R-curve obtained by potential method, with Irwin's correction	Excellent	Complicated, computer required	Very small, 2 to 3 tests only	Relatively complicated, no mandatory procedures	Inadequate

Fig. 9 Assessment of Residual Strength Prediction Methods