LOAD CYCLING OF CRACKED PLATES IN TENSION

C. M. Branco and J. Saldanha Peres

University of Minho, Largo do Paço, 4719 Braga Codex, Portugal

ABSTRACT

An analytical solution has been derived to compute the variation of total cyclic potential energy with crack length considering the elastic-plastic load cycling of cracked plates in tension. This solution is based on the energy interpretation of the J integral applied to fatigue crack growth in the elastic-plastic range and has been previously applied to the contoured DCB geometry. The solution is applicable to these geometries providing fully tensile loading and cyclic creep occurs. Crack growth rate da/dN is a function of the cyclic value of the J integral in the loading cycle ΔJ, in the form $da/dN = C\,\Delta J^{\beta}$ where C and β are two constants taken for mild steel BS15 at a loading frequency of 0.15 Hz.

Considering the above crack propagation law for mild steel BS15 ΔJ was obtained for diferent stress values as a function of crack length for load cycling in a single edge notched plate in tension. Also the predicted crack length against number of cycles plots were obtained.

Experimental crack propagation data is necessary to check the validity of this approach before it can be considered a valid parameter to describe elastic-plastic fatigue crack propagation in load cycling. This data should be obtained in a wide range of specimen geometries including cracked plates in tension.

KEYWORDS

Low-cycle fatigue, load cycling, general yielding fracture mechanics, plates.

INTRODUCTION

Linear elastic fracture mechanics (l.e.f.m.) has been successfully used to analyse the fatigue crack growth behaviour of high-strength materials where the extent of plasticity is confined to a small plastic zone at the crack tip. Consequently in the presence of a crack in a high-strength material failure will occur at nominally elastic stress levels. A considerable amount of research work has been carried out into the theoretical and experimental application of linear elastic fracture mechanics to fatigue crack propagation. Also a considerable number of crack growth "laws" has been developed to relate the fatigue crack growth rate, da/dN, to the range, ΔK of the stress intensity factor. Branco, Radon and Culver (1975, 1976) derived a crack growth law shown to correlate quite well with data obtained in aluminium alloys, mild steel BS15 and other low alloy steels.

When low and intermediate-strength alloys such as mild or other low alloy steels are considered fatigue crack growth may occur with relatively, large plastic deformations. Branco, Radon and Culver (1975, 1976) obtained crack propagation data at stress levels below the yield point. If large plastic deformations are developed during fatigue crack growth, l.e.f.m. can no longer characterize the local stress distribution at the crack tip. Rice (1968 a) has proposed the J contour integral to express the fracture toughness of these highly ductile materials. Mc Clintock (1968) has shown that for strain hardening materials following a Ramberg-Osgood relationship between effective stress σ and effective plastic strain $\bar{\varepsilon}_p$

$$\sigma = S\varepsilon_p^{\ n} \qquad (1)$$

where S is a constant and n the strain hardening exponent $(0 < n < 1)$, the stress and strain singularities at the crack tip may be expressed as a function of J. The equations presented in his work provide a physical interpretation of the J integral as representing the amplitude of stress and strain singularities under elastic-plastic conditions assuming a deformation theory of plasticity with strain hardening governed by equation (1). An important characteristic of the J integral is its path independence (Rice, 1968 a). Rice (1968 b) has shown that performing the integration along the contour coincident with the boundary of the body containing a crack of length a, the J integral, in terms of the potential energy difference, dU, between two identically loaded bodies having crack sizes differing infinitesimaly by da, is given by

$$J = -\frac{1}{B}\frac{dU}{da} \qquad (2)$$

where B is the specimen thickness.

Fatigue crack propagation in the plastic range has only recently been studied with some detail. Early work in this field using the J integral energy approach was carried out by Dowling (1976) and also Dowling and Begley (1976). In this work data were obtained on A533 B steel using centre cracked and compact tension specimens, and agreement was found in the results for both geometries when plotted as crack rate, da/dN, against ΔJ, the cyclic value of J.Cyclic J values were obtained in each cycle considering only the loading part and using the Rice, Paris and Merkle (1973) aproximate equation

$$\Delta J = \frac{2A}{Bb} \qquad (3)$$

where A is the area comprised by the loading curve of the load-deflection loop and the horizontal line passing through the assumed crack closure line, and b is the uncracked ligament length. However the method applies only to those two particular geometries and also cyclic J was not related to the material properties like the strain hardening and cyclic creep behaviour observed in elastic-plastic load cycling. In strain cycling Mowbray (1976) and also Kaisand and Mowbray (1979) related ΔJ with low cycle fatigue and fatigue crack growth rate properties. The derived relationship has the form of the well known Coffin-Manson equation, but the controlling variable for ΔJ is the applied strain energy density rather than plastic strain range. The fracture mechanics model derived by Mowbray (1976) was applied to the smooth bar low-cycle fatigue specimen with a semi-circular surface crack and da/dN was correlated with ΔJ using data obtained by Dowling (1976) for the A533-B steel.

In load cycling Branco, Radon and Culver (1977 a,b) correlated the fatigue crack growth rate with a cyclic operational value of J, ΔJ, obtained applying non linear beam theory with large deflections to the computation of the total cyclic potential energy in the contoured DCB geometry subjected to fully tensile loading above the yield point. This model takes into acconnt both strain hardening and cyclic creep behaviour occuring in load cycling above the yield stress. Correlation of data was obtained for mild steel BS15 in the form $da/dN = C\Delta J^{\beta}$ were C and β are experimental constants obtained at a loading frequency of 0.15Hz.

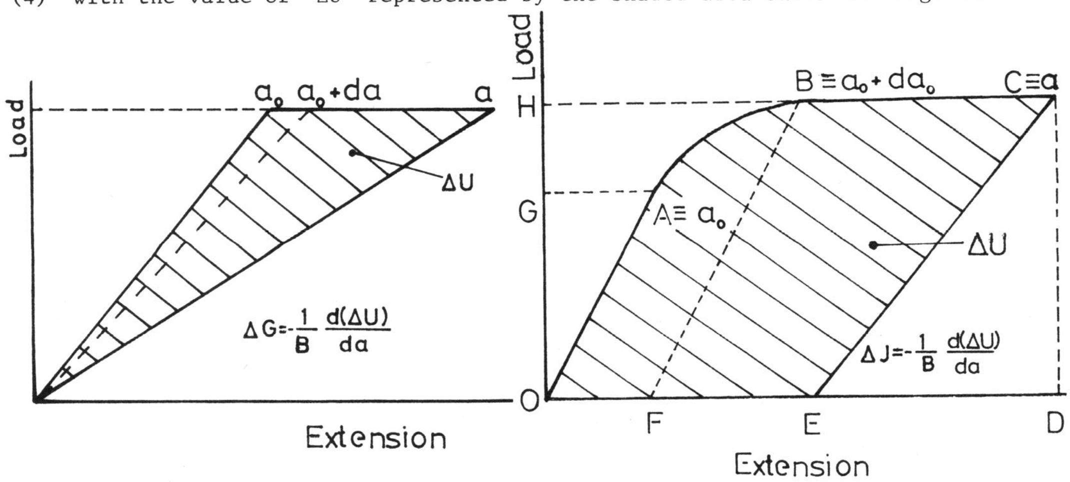
The present paper describes the development of the authors theoretical method for computing cyclic J values in load-cycling of cracked plates in tension. An application of this model is made for a single edge notched plate, using the data already obtained for the mild steel BS15.

THEORETICAL ANALYSIS

The load-displacement curves obtained in load-cycling for a specimen having an initial crack of length a_o are illustrated in Fig.1a for a linear elastic material and Fig.1b for cycling above the yield point. In both cases the crack has grown from a_o to a after N loading cycles. The area illustrated represents the decrease, ΔU, in total potential energy, after N cycles. For the linear elastic case a cyclic value ΔG of the monotonic crack extension force can be defined as

$$\Delta G = -\frac{1}{B}\frac{d(\Delta U)}{da} = \frac{\Delta P}{2B}\frac{d(\Delta\delta)}{da} \tag{4}$$

where ΔP is the loading range and $\Delta\delta$ is the corresponding range of extension for the assumed specimen gage length. For the elastic-plastic load cycling (Fig.1b) an operational value of ΔJ is given by an equation similar to equation (4) with the value of ΔU represented by the shaded area shown in Fig. 1b



a Linear elastic material. b Elastic-plastic material.

Fig. 1 Operational definitions of cyclic ΔG and ΔJ after N loading cycles.

However in elastic-plastic load cycling cyclic creep occurs and the corresponding cyclic creep extension accumulated after N loading cycles is represented by OE in Fig.1b. It should be pointed out that if crack growth had not occured the trace EC would have been parallel to BF. The elastic loading condition for EC is closely true since the yielding has been largely accomplished between points A and B.

The value of ΔU is obtained by the equation

$$\Delta U = \Delta U_N - \Delta U_1 \tag{5}$$

where ΔU_N is the total accumulated potential energy after N cycles with a crack length a (≡ OECBHGO) and ΔU_1 is the potential energy of the first cycle with a crack length a_o (≡ OABHGO). It can be seen that ΔU_N includes the cyclic creep component since crack growth will depend on the accumulation of strain caused by this phenomenon. The value of ΔU_1 is independent of the crack length a and therefore $d(\Delta U)/da = d(\Delta U_N)/da$. Hence ΔJ becomes

$$\Delta J = -\frac{1}{B}\frac{d}{da}\left(\Delta\delta_N\,\Delta P - \frac{\Delta P}{2}\,\Delta\delta_{el}\right) \tag{6}$$

where $\Delta\delta_N$ is the total extension for the specimen gage length (≡HC) and $\Delta\delta_{el}$ represents the unloading extension (≡ED). The displacement $\Delta\delta_N$ includes three factors

$$\Delta\delta_N = \Delta\delta_1 + \Delta\delta_{cc} + \Delta\delta_{el} \tag{7}$$

where $\Delta\delta_1$ is the maximum extension in the first cycle (≡HB), $\Delta\delta_{cc}$ is the cyclic creep extension accumulated after N loading cycles (≡OE) and $\Delta\delta_{el}$ is also the extension caused by crack growth (≡BC - FE). Thus equation (6) becomes

$$\Delta J = -\frac{1}{B}\left[\frac{d\,(\Delta\delta_1 + \Delta\delta_{cc})\Delta P}{da} + \frac{\Delta P}{2}\,\frac{d(\Delta\delta_{el})}{da}\right] \tag{8}$$

In an early work (Branco, Radon and Culver, 1977 b), fatigue crack growth rate da/dN correlated well with ΔJ calculated with a similar process for contoured DCB specimens of mild steel BS15. The fatigue crack growth data is shown in Fig.2 fitting a power law equation

$$\frac{da}{dN} = C\,(\Delta J)^{\beta} \tag{9}$$

with $C = 6.546 \times 10^{-5}$ and $\beta = 1.092$. These values were obtained applying a linear regression program to the experimental data points. The results presented in Fig. 2 indicate that the elastic-plastic data follows an extrapolation of the region II linear elastic data. In this region ΔJ is given by equation (4) equal to K^2/E from the crack propagation law derived by Branco, Radon and Culver (1976).

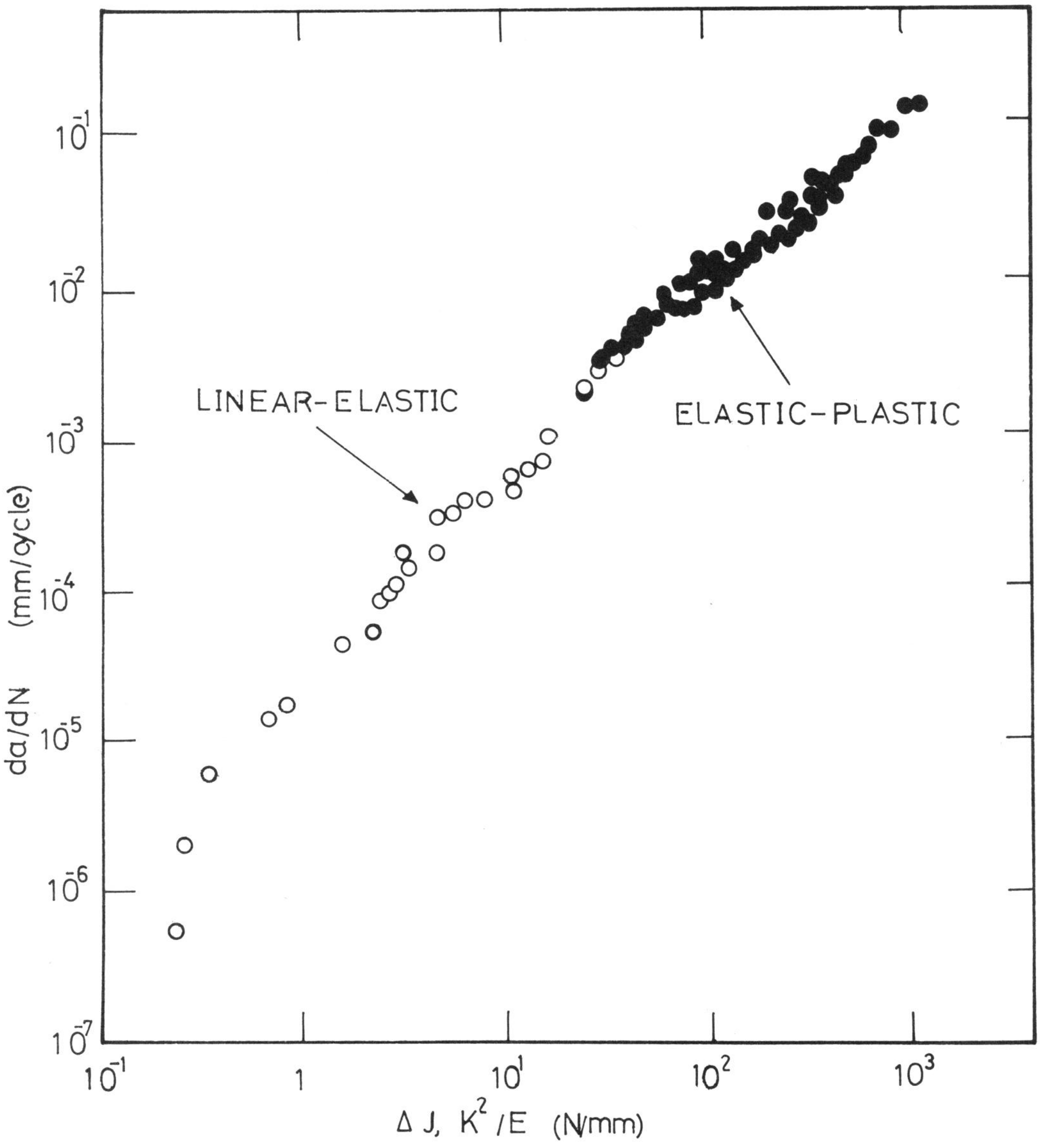


Fig. 2. Crack growth rate vs. ΔJ (or K^2/E). BS15. R = 0;0.15Hz

Calculation of ΔJ for Load Cycling of Cracked Plates in Tension (R = 0)

For these specimens the value of $\Delta\delta_N$ from equation (7) becomes

$$\Delta\delta_N = l_o\ (\varepsilon_{ys} + \varepsilon_{pl} + \varepsilon_{cc}) + \Delta\delta_{el} \qquad (10)$$

where ε_{ys} the total elastic strain, ε_{pl} the plastic strain, ε_{cc} the cyclic creep strain and lo the specimen gage length. The assumed stress-strain curve is an elastic-plastic one with exponential hardening. Therefore the first two terms in equation (9) are given by the equations

$$\varepsilon_{ys} = \frac{\sigma_{ys}}{E} \tag{11 a}$$

$$\varepsilon_{pl} = A(\sigma_{max} - \sigma_{ys})^n = A(\Delta\sigma)^n \tag{11 b}$$

where σ_{ys} is the yield stress, E the Young's modulus. A and n the strain hardening constant and exponent. The cyclic creep strain was also assumed to follow an exponential law

$$\varepsilon_{cc} = C_1 (\Delta\sigma)^m N \tag{12}$$

where C_1 is the cyclic creep constant and m the exponent. Equation (12) gives the accumulated cyclic creep strain as a function of the number of cycles and applied stress and was initially proposed by Benham and Ford (1962) to correlate load-cycling data obtained on plain steel specimens. Also a good correlation was obtained with equation (12) for data obtained in plain cylindrical specimens of mild steel BS15 (Branco, Radon and Culver, 1977 b). The second term in Equation (8) may be expressed as a function of the stress intensity factor, K

$$\frac{\Delta P}{2B}\,\frac{d(\Delta\delta_{el})}{da} = \frac{K^2}{E} = \frac{Y^2\sigma^2_{max}\pi a}{E} \tag{13}$$

where σ_{max} is the maximum nominal stress and Y is the dimensionless factor. Upon substitution of equations 11 a, 11 b, (12) and (13) in equation (8), ΔJ becomes

$$\Delta J = \ell_o\,\sigma_{max}\,W\,\frac{d}{da}\left(\frac{\sigma_{ys}}{E} + A\Delta\sigma^n + C_1\,\Delta\sigma^m N\right) + \frac{Y^2\sigma^2_{max}\pi a}{E} \tag{14}$$

Integrating comes

$$\int_0^{a_f} \Delta J da - \int_0^{a_f} \frac{Y^2\sigma^2_{max}\pi a}{E}\, da = \ell_o\,\sigma_{max}\,WC_1\,\Delta\sigma^m N_f \tag{15}$$

where W is the width for a plain specimen having a crack with an initial length 0 growing to a final length a_f in N_f cycles. For a single edge notched plate with $0 < a/w < 0.7$ the value of Y is given by

$$Y = 1.12 - 0.23\left(\frac{a}{w}\right) + 10.6\left(\frac{a}{w}\right)^2 - 21.7\left(\frac{a}{w}\right)^3 + 30.4\left(\frac{a}{w}\right)^4 \tag{16}$$

Therefore in this analysis a_f was considered to be equal to 0.7 w. From equation (9) the value of N_f becomes

$$N_f = \frac{1}{C}\int_0^{a_f} \frac{da}{\Delta J^\beta} \tag{17}$$

Substituting this equation in equation (15) comes

$$\int_0^{a_f} \Delta J da = \ell_o \sigma_{max} \frac{W C_1 \Delta\sigma^m}{C} \int_0^{a_f} \frac{da}{\Delta J^\beta} + \int_0^{a_f} \frac{Y^2 \sigma^2_{max} \pi a}{E} \qquad (18)$$

$$\therefore \quad \Delta J = \frac{\ell_o \sigma_{max} W C_1 \Delta\sigma^m}{C \Delta J^\beta} + \frac{Y^2 \sigma^2_{max} \pi a}{E} \qquad (19)$$

with the value of Y given by equation (16). Equation (19) was solved by an iterative process in a computer program with an accuracy of 10^{-2} for two consecutive values of ΔJ. The data used was obtained for mild steel BS15 and the corresponding values are indicated in Table 1. The program was run for six maximum nominal stress values of 550; 525; 500; 475; 450 and 425 MPa.

TABLE 1 Data used in Equations (19) and (11 b)-BS15 Steel

$\ell_o = 150$ mm	$C = 6.546 \times 10^{-5}$
$W = 25$ mm	$\beta = 1.092$
$C_1 = 2.0 \times 10^{-10}$ mm^2/N	$E = 2.07 \times 10^5$ MPa
$m = 3.06$	$n = 7.6$
$A = 1.9337 \times 10^{-18}$ mm^2/N	$\sigma_{ys} = 401$ MPa

RESULTS AND DISCUSSION

Figures 3 and 4 show the plots ΔJ against crack length for all the stress values. It is seen that ΔJ increases with the crack length and with the applied constant nominal stress in the specimen. The slopes of these curves are reasonably constant in the early stages of crack growth showing however a steep increase for crack lengths greater than 10 mm. The obtained J values are greater than 220 N/mm thus corresponding to the elastic-plastic range of the fatigue crack propagation curve for BS15 presented in Fig. 2. For the DCB specimen it was found that J decreases with increasing crack length (Branco, Radon and Culver 1977a). This result was also verified by the experimental results obtained for BS15.

In the side notched plate specimen subjected to tension the cyclic creep extension rate is constant with the applied load and the variation of compliance with crack length $(\partial C/\partial a)$ increases with the crack length. Therefore $d(\Delta J)/da$ is an increasing function (Figs. 3 and 4). In the DCB specimen $\partial C/\partial a$ is constant with the crack length and the cyclic creep extension rate decreases with increasing crack length due to the decrease in bending stress with the crack length. Therefore these two types of specimen geometries show different behaviour which is however consistent with the theoretical calculations made.

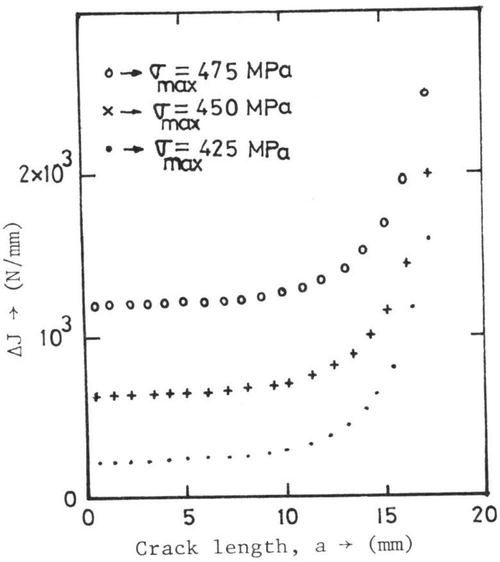


Fig. 3 Theoretical ΔJ VS. crack length. W = 25 mm. R = 0

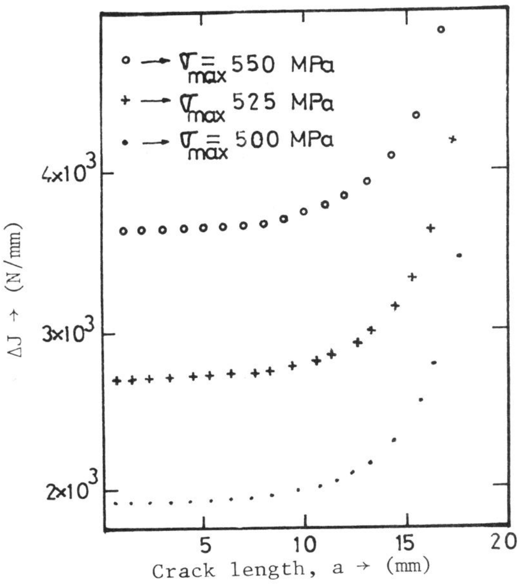


Fig. 4 Theoretical ΔJ VS. crack length W = 25 mm. R = 0.

Solution of equations (17) and (19) produced the plots crack length against number of cycles presented in Figs. 5 and 6. The results show a similar trend as for the corresponding ΔJ against crack length curves. With such high stress values (greater than the yield stress) crack growth rate is also very high and it is seen (Fig. 5) that the computed maximum number of cycles for the lowest stress value of 425 MPa is only 511 cycles to grow a crack from 0 to 0.7 W. An experimental testing programme is now being initiated to obtain crack growth data in these specimens.

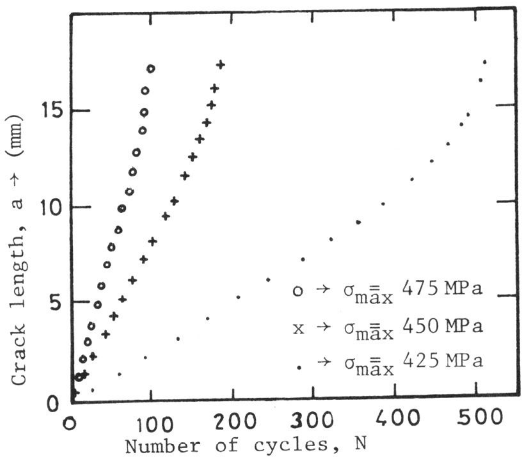


Fig. 5 Plot crack length VS. number of cycles. W = 25 mm. R = 0

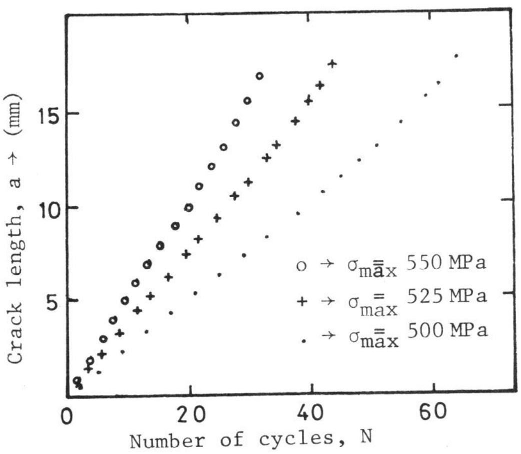


Fig. 6 Plot crack length VS. number of cycles. W = 25 mm. R = 0.

The computed load extension curves for the stress of 500 MPa are traced in Fig. 7. The cyclic creep extension is measured in the horizontal axis and the loading lines show an increase of slope caused by the increase of compliance due to crack growth. The curves shown in Fig. 7 can then be compared with the experimental load-extension curves. The ΔJ values can also be obtained from the experimental load-extension curves measuring the cyclic potencial energy ΔU (Fig. 1) and obtain its derivative in order to a.

Equation (19) was derived for nonimal stress values in agreement with the strain hardening and cyclic creep equations. Since plastic deformation is very large it would be probably more appropriate to use the true stress. Work is now in progress to solve this equation using the true stress, and obtaining its values from a constancy of volume relationship which may be obtained in the next load-cycling tests. Thus it is important first to assess which of the variables, thickness or width, will be reduced in the load cycling tests before one can stablish the relationship to be substituted in equation (19).

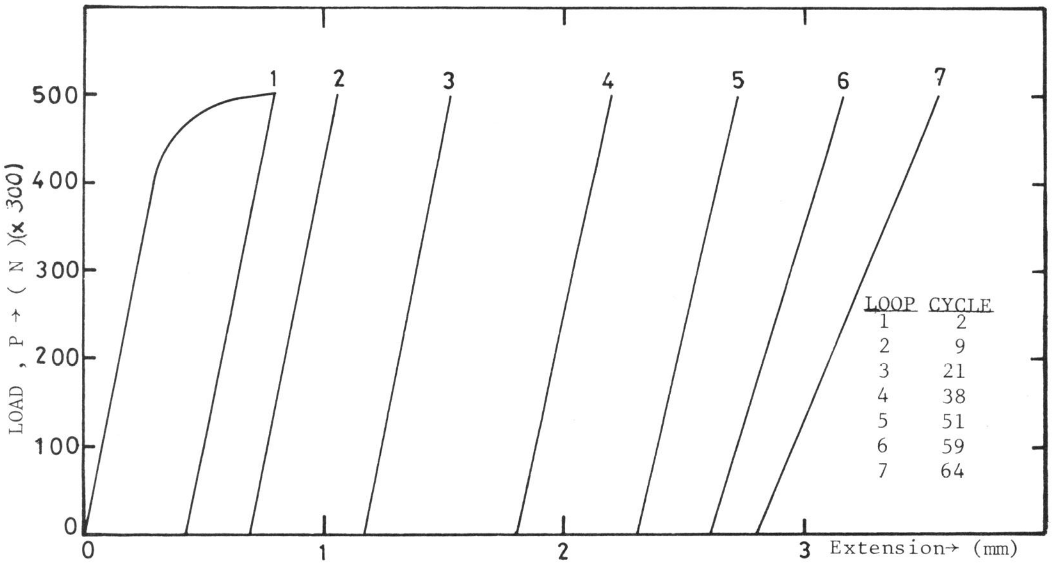


Fig. 7 Computed load-extension curves. W = 25 mm. R = 0. σ_{max} = 500 MPa. BS15

CONCLUSIONS

Based on the energy interpretation of the J contour integral, the variation of cyclic total potential energy with crack length was considered to define ΔJ, the cyclic value of J. An analytical method has been described to compute ΔJ values in plate specimens when cyclic creep occurs in tensile load cycling at stress levels above the yield point. It was found that ΔJ is given by an equation function among other variables of material properties like Young's modulus, and the strain hardening and cyclic creep parameters.

An application of this method was carried out for a single edge notched plate assuming a crack propagation law $da/dN = C\ (\Delta J)^{\beta}$ previonsly obtained by one of the authors in a similar elastic-plastic study of fatigue crack propagation in contoured DCB specimens of mild steel BS15, where ΔJ was defined by the same process refered to here.

In a single edge notched plate of mild steel BS15 ΔJ was obtained as a function of crack length for different stress values. A computation was made for the corresponding crack length against number of cycles curves and also the theoretical load-extension curves were obtained as a function of crack length.

Experimental data for other materials and in these specimen geometries is necessary to confirm the validity of this method to correlate fatigue crack growth rate in elastic-plastic load cycling.

REFERENCES

Benham, P.P. and H. Ford (1961). Low endurance fatigue of a mild steel and an Al-alloy. J. mech. Engng. Sci., 3, 2, 119-132

Branco, C.M., J.C. Radon, and L.E. Culver (1975). Influence of mean stress intensity on fatigue crack growth in an Al-alloy. J. mech. Engng. Sci., 17, 4, 199-205

Branco, C.M., J.C. Radon and L.E. Culver (1976). Growth of fatigue cracks in steels. Met. Sci., 10, 5, 149-155

Branco, C.M., J.C. Radon and L.E. Culver (1977 a). Elastic-plastic fatigue crack growth under load cycling. J. Strain Analysis Eng. Des., 12, 2, 71-80

Dowling, N.E. (1976). Geometry effects and the J - integral approach to elastic-plastic fatigue crack growth. In ASTM STP 601, Cracks and Fracture, American Society for Testing and Materials, pp. 19-32

Dowling, N.E. and J.A. Begley (1976). Fatigue crack growth during gross plasticity and the J - integral. In ASTM STP 590, Mechanics of Crack Growth, American Society for Testing and Materials, pp. 82-102

Kaisand, L.R. and D.F. Mowbray (1979). Relationships between low-cycle fatigue and fatigue crack growth rate properties. JTEVA, 7, 5, 245-253

Mc Clintock, F.A. (1968). In H. Liebowitz (Ed.), Fracture, Vol. III. Academic Press, New York. Chap. 2, pp 47-218

Mowbray, D.F. (1976). Derivation of a low-cycle fatigue relationship employing the J - integral approach to crack growth. In ASTM STP 601, Cracks and Fracture, American Society for Testing and Materials, pp. 33-46

Radon, J.C., C.M. Branco and L.E. Culver (1977 b). Creep in low endurance fatigue of mild steel. Int. J. Fract., 13, 5, 595-610

Rice, J.R. (1968 a). A path independent integral and the approximate analysis of strain concentration by notches and cracks, Trans. A.S.M.E., J. appl. Mech., 35, 379-386

Rice, J.R. (1968 b). In H. Liebowitz (Ed.), Fracture, Vol. III. Academic Press, New York. Chap. 2, pp. 191-218

Rice, J.R., P.C. Paris and J.G. Merkle (1973). Some further results of J integral analysis and estimates. In ASTM STP 536. Progress in Flaw Growth and Fracture Toughness Testing, American Society for Testing and Materials, pp. 231-245