FRACTURE MECHANICS IN THE ELASTIC-PLASTIC REGIME

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ABSTRACT

The topic of elastic-plastic fracture mechanics is reviewed. The suitability of the crack tip opening displacement (CTOD) and the J-integral as elastic-plastic failure parameters that characterise cleavage and ductile fractures is discussed. The introduction of a new parameter, J_e, capable of accounting for load-history dependent cleavage fracture is outlined. The CTOD appears to be relevant to describing ductile crack extension although J is more commonly used. Methods of calculating CTOD and J are briefly mentioned, concentrating on the contribution made by the strip yielding model and finite element methods. Methods of measuring CTOD and J are then briefly reviewed and problems encountered in toughness testing in the elastic-plastic regime are discussed, with particular reference to the effect of stress state on cleavage failures, and the need to understand the interaction of micromechanisms and mechanical quantities in both cleavage and ductile failures. The problems of elastic-plastic failure assessments are discussed. Consideration is given to the treatment of thermal and residual stresses, ductile instability, and elastic-plastic failure assessment procedures which incorporate these. Finally areas of elastic-plastic fracture mechanics that require further attention are mentioned.

INTRODUCTION

The extension of fracture mechanics into the elastic-plastic regime poses a number of important questions over and above those which have previously been asked and mainly answered in linear elastic fracture mechanics (LEFM). In the latter case there is an established and generally accepted failure criterion based on the applied stress intensity factor K_1 equalling or exceeding the fracture toughness, K_c, of the material. There is justification for this criterion from both the engineering and materials standpoints. For example, in the small scale yielding regime all the mechanical parameters (stress, strain, displacement etc.) associated with a loaded crack tip are characterised by K_1 (see, for example, Irwin and Koskinen, 1963, Rice and Rosengren, 1968, Hutchinson, 1968, Rice, 1968, McClintock, 1971, Hutchinson and Paris, 1979). The initiation of the micromechanisms that control the onset of the fracture process may be related via these parameters to a critical value of K_1, which define the toughness K_c. (This has proved particularly successful in the case of cleavage fracture, see, for example, the recent review by Curry, 1980).

In the elastic-plastic regime the situation is no longer so clear-cut, and there are a number of alternative failure parameters to K_1 whichhave been proposed, such as the crack tip opening displacement (CTOD) (Wells, 1961, Cottrell, 1961), the J-integral (Rice, 1968, Begley and Landes, 1972, Landes and Begley, 1972) (and, related to this, the Q or J_e-integral (Bilby, 1973, Miyamoto and Kogeyama, 1978, Chell, Haigh and Vitek, 1979). The meaning of the CTOD is self evident even if in practice it may be difficult to define. J represents an extension of the strain energy release rate, G, to non-linear elastic materials and is usually expressed in the form of a line integral. In particular circumstances these parameters have proved successful in describing elastic-plastic fractures. However their relationships to the micromechanisms of fracture has not always been unequivocably established, and a universal failure parameter which is applicable to all micromodes of failure has not, and probably will not, be discovered.

In elastic-plastic fracture mechanics (EPFM) it is important to differentiate between mechanical and metallurgical effects, although the two may be closely linked. Continuum mechanics may be able to quantify the influence of plastic deformation on mechanically derived quantities, such as the CTOD or J, however it cannot quantify the effects on the critical values of these parameters when yielding produces a change in the micromode of crack extension. These changes are only predictable when combined with micromechanistic considerations, (Hancock and Cowling, 1977, Milne and Chell, 1978). This limitation on the important role played by mechanics is sometimes not fully recognised or is often obscured by semantics. For example, an engineer may define brittle failure as a linear relationship between applied load and displacement up to the failure point. On this definition a failure may be called brittle even though the micromode of crack propagation was ductile. Conversely engineering ductility is associated with large non-linear displacements, even though on the microscale the initiation of crack extension may be by a cleavage (brittle) mechanism. In this paper the phrases "linear elastic", and "small scale yielding" are used synonymously for engineering brittleness, and "elastic-plastic" and "large scale yielding" are used synonymously for ductility, in the engineering sense. The words brittle and ductile refer to the metallurgical mode of failure.

FAILURE PARAMETERS

Ideally a proposed failure parameter should reflect, or characterise, the conditions which control the micromechanisms of fracture while, at the same time, being a useful engineering quantity in that it may be calculated from given structural parameters e.g. applied load, crack length and geometry. To some extent both the CTOD and J satisfy these criteria. It may be expected that the CTOD, which reflects the concentration of strain at the crack tip, is most suited to describing ductile failure, especially if this entails stable crack growth, whereas J, which is essentially a crack extension force, is more suited to stress controlled (brittle) fast fracture (Vitek and Chell, 1977). However, both these parameters have been applied to predicting failure in both of these cases. It is interesting to note, however, that although CTOD and J can be shown to be related through equations of the form (see, for example, Rice, 1968)

$$J = m\sigma_Y \delta ,$$

where δ is the CTOD, σ_Y the yield stress and m a constant usually between 1 and 2, they do not necessarily predict the same failure behaviour (Vitek, 1976). This is borne out by the fact that the constant m depends on structural geometry, the degree of stress triaxiality and possible work-hardening capacity (Robinson, 1976).

In cleavage fracture neither the CTOD or J (as conventionally defined and used) are suitable as generalised failure parameters although they appear adequate to describe isothermal, monotonic load histories. Instead, as recent results reported by Chell, Haigh and Vitek (1989) and Chell (1980) have shown, a similar integral to J, namely J_e (called Q by Bilby (1973), and J_{ext} by Miyamoto and Kogeyama (1978)) is the correct parameter to use. Physically this represents the force exerted on a loaded crack and the plasticity that is enclosed by the contour of integration used to evaluate J_e. For a contour that goes outside the plastic zone J_e equals J, however when the contour cuts through the plastic zone, whereas J appears to be path independent (Hayes, 1970, Sumpter, 1973), J_e is path dependent. This is an important property of J_e.

J_e has been experimentally validated as a failure parameter applicable to cleavage fracture and incremental plasticity (including cases where unloading is performed, and where the yield stress and applied load vary with temperature)(Chell, Haigh and Vitek, 1979, Chell, 1980). Its use is subject to the physical postulate that fracture occurs when J_e equals or exceeds a critical value $J_c(=(1-\nu^2)K_c^2/E$, where ν is Poisson's ratio and E Young's modulus) and provided the contour of integration is chosen to include only the region of the yielded zone where plastic flow can occur. The latter is a consequence of the fact that areas of residual plastic strain may be generated during complex loading histories. It is interesting to note that when J_e is applied to predicting the failure of cracked ferritic steel structures that have been warm prestressed (i.e. prestrained above the ductile-brittle toughness transition temperature) it gives results which are consistent with predictions obtained from micromechanistic considerations (Curry, 1979a). This suggests that J_e, if evaluated according to the requirements mentioned above, characterises the stress field ahead of a loaded crack even after the crack has been subjected to a complicated loading history.

It should be noted that, in the case of warm prestressing, neither the CTOD or J provide a suitable description of the fracture behaviour (Chell, Haigh and Vitek, 1979). However, during ductile stable crack growth the CTOD appears to be the controlling parameter. By experimental measurement is is found that the displacement at the propagating crack tip remains constant, and at a value less than that required for initiation (Garwood and Turner, 1978, Willoughby, Pratt and Turner, 1978). Taking into account the prestraining that a volume of material experiences ahead of the crack tip before the crack tip reaches it, and the fact that ductile failure mechanisms are strain controlled, this observation is perhaps not too surprising.

In the ductile failure regime several analytical attempts have been made to predict crack growth resistance based on CTOD considerations (Wnuk, 1974, Rice and Sorensen, 1978). These can be interpreted in terms of J, where J is determined using a contour distant from the crack tip. If this is done and J calculated as a function of crack extension it is found to increase. However the equation for the slope of the J-resistance curve, dJ/da, where a is the crack length, contains an unknown length parameter which can be assumed proportional to the plastic zone size in small scale yielding, and the remaining ligament in large scale yielding (Rice and Sorensen, 1978). Thus the theory implicitly predicts that dJ/da will depend on specimen size even though local crack tip conditions may remain constant.

Turner (1979a) has proposed that the increase in the J-resistance curve with crack extension is due to plastic work which is performed distant from the crack, although, in agreement with the foregoing experimental observations he assumes that the crack tip "toughness" remains constant during propagation. Alternatively Paris, Tada, Zahoor and Ernst (1979) using an argument based on structural instability arrived at the conclusion that the slope of the J-resistance curve was constant, and was the parameter relevant to the onset of ductile instability. Furthermore Hutchinson and Paris (1979) have shown from non-linear elasticity theory that provided $b(dJ/da)/J >> 1$, where b is the remaining ligament, that crack growth is J dominated.

Clearly the major difficulty in using J as a failure parameter to describe ductile instability after stable growth is the difficulty of relating the value of J calculated, using far field values (i.e. using quantities determined at positions distant from the crack, which is a desirable feature for any engineering application) to local crack tip conditions. This is essentially the same problem as relating the CTOD at the original position of the crack tip, to the CTOD at the instantaneous crack tip during propagation. Under certain circumstances this can be done (Rice and Sorensen, 1978, Curry, 1979b), but in general the problem remains a formidable one.

Whichever elastic-plastic failure parameter is adopted, and in many cases, for example, CTOD and J approaches are equivalent, it should reduce to a K_1 dominated failure criterion in the small scale yielding regime. Thus the efficacity of elastic-plastic failure parameters is inversely proportional to the sensitivity of fracture toughness to the level of plastic deformation which occurs prior to failure. Since fracture toughness is related to the local crack tip mechanical quantities that drive the micromechanisms, and the former are influenced by the degree of macroscopic yielding (which can also alter the state of stress), it should not be too surprising if the critical values of mechanically derived failure parameters show a dependence on the geometry of the crack and structure. This dependence can be strong in cleavage fracture (Milne and Chell, 1979a, Dawes, 1979) and in stable ductile tearing (Green and Knott, 1975, Garwood, Pratt and Turner, 1978), although in the latter case the effect may be partly due to the relative areas of flat and shear fracture on the fracture surfaces.

CALCULATION OF FAILURE PARAMETERS

The most widely known and used analytical model for simulating the effects of plastic deformation ahead of a crack is the strip yielding model (Dugdale, 1960, Bilby, Cottrell and Swinden, 1963). This model has the two great advantages of not only being relatively simple mathematically but also of predicting the salient features of elastic-plastic fracture behaviour that are also observed in practice (Heald, Spink and Worthington, 1972). Although the model may be developed to any degree of sophistication (see, for example, Bilby and Swinden, 1965, Chell, 1976, Vitek, 1976) for engineering purposes perhaps its major importance is in providing a semi-empirical functional form for the J-integral (Chell, 1979a) or as an interpolative formula (Dowling and Townley, 1975). Since within the strip yielding model the CTOD is related to J through $J = \sigma_y \delta$, then these two parameters provide equivalent descriptions of fracture.

The importance of the strip yielding model in the development, understanding and application of EPFM cannot be understated. Its contribution is reflected by the dearth of alternative analytical models of yielding in the published literature.

The most common way of numerically calculating the CTOD and J is to use the finite element method. However, it is not very suited to CTOD determinations, principally because it is difficult to define the position of the CTOD and also because of the numerical errors which are likely to occur in quantities evaluated near the crack tip unless an extremely refined element mesh is used. On the other hand the method is ideally suited to the computation of J values, since a contour may be chosen distant from the crack tip, thus avoiding the uncertainties of the near crack tip area.

Finite element methods have also been used to study ductile crack growth in an attempt to identify the continuum mechanical parameters controlling the process (Kanninen et al, 1979, D'escatha and Devaux, 1979). Once these are known it would be possible, in principal, to simulate and allow for some extent of ductile tearing in failure assessments. However, the calculations, which involve decoupling of the modes at the crack tip are lengthy and costly.

The advantage of finite element methods is that J and plastic collapse loads may be computed for cracks in structures subject to complex loadings using incremental plasticity theory and allowing for work-hardening. The importance of plastic collapse loads in providing an upper bound to elastic plastic fracture predictions is becoming increasingly recognised, and there is certainly a need for a compendium of solutions comparable to the many compendiums that now exist for stress intensity factors.

A disadvantage of finite elements is that at the present time there appears to be no standard procedures for their application to crack problems. As a result a recent comparison of independently computed J values obtained from a number of laboratories produced a disturbingly large variation in answers in the elastic-plastic regime (Wilson and Osias, 1978). Until the reason for these differences is fully understood and the numerical techniques accordingly amended the results of finite element calculations involving extensive yielding should be treated with caution.

MEASUREMENT OF FAILURE PARAMETERS

The analysis of fracture data to obtain values for the CTOD and J from cracked specimens that have failed after extensive plastic deformation is one of the major application areas of EPFM. Such analyses are often required when high toughness, low yield stress materials are tested, and there are a number of useful formulae which have been derived that relate fracture toughness, and the critical values of the CTOD and J to the load-displacement behaviour of the specimen up to failure. (e.g. Rice, Paris and Merkle, 1973, Sumpter and Turner, 1976a, Witt and Mager, 1971, Chell and Milne, 1976, Dawes, 1976, and Merkle and Corten, 1974). However, although the number of available methods is large, it is reassuring to know that for the commonest forms of laboratory specimens, namely three point bend and compact tension specimens containing cracks about half-way through the section, they all produce values of J within a few per cent of each other. Relatively simple modifications to some of the foregoing methods are also available for calculating J during stable crack growth using the instantaneous value of the crack length and the hypothetical load-displacement curve corresponding to it (Garwood. Robinson and Turner, 1975, Milne and Chell, 1979b).

All the formulae used in the above methods for determining J are derived taking the definition of J as the rate of change of potential energy with respect to crack extension (Rice 1968) and assuming non-linear elasticity, rather than incremental plasticity i.e. assuming that the loading and unloading displacement curves are coincident. Although it is not clear that the value of J obtained will be the same as that computed from the integral expression for J using incremental plasticity, never-the-less the two appear to have very similar values (Sumpter and Turner, 1976b).

In cleavage toughness testing crack growth initiation is coincident with fast propagation. However, as previously stated, ductile materials may undergo some stable tearing before instability. This poses two problems, namely, how to identify the point of crack extension, and how to measure the amount of instantaneous growth occurring during the test. Although a number of methods have been proposed to overcome these problems ranging from unloading compliance techniques to electrical potential drop methods (see, for example, Landes and Begley, 1979), at the present time the only method which is reliable for all materials appears to be the interrupted test technique (Curry and Milne 1979).

It has been proposed that the plane strain toughness specimen size requirements

(ASTM, E-399, 1971) can be relaxed for tests where the results are analysed in terms of J. The proposed reduction, which for steels is typically 20 to 40 smaller than the recommended size, may be too great in the case of ferritic steels failing by cleavage near the brittle-ductile toughness transition temperature. Here consideration of the micromechanisms of failure suggests that J_c may depend on crack length and specimen size (Milne and Chell, 1979b). In specimens that fail beyond general yield the extensive plasticity may reduce the level of stress triaxiality required for plane strain conditions, causing an effective decrease in the transition temperature with a concomitant increase in toughness (Sumpter, 1976 Milne and Chell, 1979a, Dawes, 1979). The effect can be very significant, and can even result in a change in failure mode from cleavage to ductile. The ramifications as regards the use of toughness values so obtained in failure assessments involving thick section components is obvious. In cleavage toughness testing, therefore, it is recommended that the smallest specimen thickness tested should at least equal the section thickness of the component to be assessed (Milne and Chell 1979b).

An alternative explanation has been proposed for the apparent cleavage toughness size effect. This is based on a weakest link statistical model (Landes and Begley, 1974, Landes and Shaffer, 1980). Essentially the argument is that as specimen thickness increases, the volume of material sampled by the crack front also increases, and hence there is an increased probability that part of the crack tip will be embedded in material with poor toughness properties. Whereas this cannot be dismissed as a possible contributory cause to the size effect, never-the-less fracture data obtained from specimens where the thickness was held constant, and the specimen width and crack length were varied, also showed increasing toughness with decreasing size. This data, analysed by Chell and Gates (1978), clearly indicates that the effect exists independently of any variation in material properties.

The foregoing increase in toughness is predictable from the reduction in the triaxial stress state ahead of a crack, even if this stress state is characterised by J (Milne and Chell, 1978). For a given applied J the maximum principal stress will be highest in the specimen with the greatest triaxiality. Hence the value of J applied to the former must be increased to equal the stress level in the latter with obvious implications with regard to its critical value in stress controlled (cleavage) fracture.

The effect of size and stress state on the toughness value (measured in terms of J or CTOD) corresponding to the initiation of ductile crack growth does not appear to be so significant. Some workers argue that it should exist because ductile failure is related to void formation, growth, and coalescence, which are influenced not only by strain but also by the state of stress (Hancock and Cowling, 1980). However for reasons that have been mentioned previously e.g. macroscopic plastic work and shear lip formation, a size effect is apparent in crack growth resistance curves that are determined using J as a measure of toughness.

It is clear that three dimensional elastic-plastic finite element calculations could contribute a great deal to the resolution and quantification of the effects of stress state, and hence specimen size, on fracture behaviour.

FAILURE ANALYSIS

Frequently the methods used in failure analyses are dictated by the quality and quantity of the input data (system stresses, defect sizes, material properties etc) rather than accuracy with which EPFM parameters can be calculated per se. Furthermore structures are not intentionally designed to operate in the elastic-plastic regime, design stresses generally being around a third of the yield stress. During service, elastic-plastic effects arise predominantly from the presence of residual stresses in weldments, system stresses, and the local yielding that may occur adjacent to stress concentrators. Indeed, in the former cases the problem may arise because the magnitude and distribution of the stresses are unknown and hence are pessimistically assumed to be or near yield magnitude (Harrison, Dawes, Archer and Kamath, 1979).

EPFM plays an important role in the calculation of critical or tolerable defect sizes made at the design or manufacturing stage. But never-the-less it should be recognised that failure, if it occurs, may be in the elastic-plastic regime, but a crack if detected in service and assessed as safe, is usually not. Milne (1979b) has recently reviewed the problems involved in failure assessment.

Failure analyses can be simplified by recognising that fracture behaviour is bounded by the two extremes, linear elastic and fully plastic deformation (Dowling and Townley, 1975). In the former LEFM is sufficient while in the latter plastic limit analysis is required. Looked at in these terms EPFM is a means of predicting the transitional behaviour in going between these extremes. It is partly because of this that the results of failure assessments are frequently insensitive (in engineering terms) to the detail of the calculation or the choice of failure parameter (given good input data) provided the two extremes appear as a consequence of the choice of elastic-plastic failure parameter. This is the basic philosophy behind the failure avoidance line used in the CEGB's preferred failure assessment procedure (Harrison, Loosemore, Milne, 1976). Indeed it must be the essential background to any procedure which attempts to employ a design or universal failure curve.

Clearly there will be instances when more accurate calculations are required than can be provided by universal failure curves. Furthermore a fundamental understanding of the comments and limitations behind EPFM parameters is essential, not only for the correct formulation of assessment procedures, but also to define their region of applicability and to provide insight into how they may be further developed. These underlying principles have proved particularly invaluable in the treatment of thermal and residual stresses and ductile instability.

Thermal and residual stresses pose a difficult problem for EPFM. In order to maintain the path independence of J and its meaning in terms of potential energy release rate, an area integral must be added to the line integral definition (Ainsworth, Neale and Price, 1978). Whereas thermal and residual stresses may prove extremely deleterious in the EPFM regime, their influence diminishes as plastic collapse is approached and they cannot contribute to collapse itself, which is determined only by the mechanical loads that are present. Even with this complication their effects on elastic-plastic failure analyses may still be quantified using procedures based upon a universal failure line (Chell, 1979c, Milne, 1978), or a design curve approach (Harrison, Dawes, Archer and Kamath 1979, Turner, 1979a). A review of the role of thermal and residual stresses in elastic-plastic fracture mechanics design has recently been published (Chell, 1979).

In the last few years there has been an increasing interest in predicting the stable growth of defects and the conditions leading to ductile instability. This is an important area because a material's resistance to ductile crack extension may increase rapidly in the first few millimetres of growth, providing a large increase in the load bearing capacity of a structure with respect to the load required to initiate growth.

Ductile instability analyses have concentrated on expressing the increase in toughness with crack growth in terms of J, and hence the crack driving force is identified with the applied value of J. Instability occurs when the rate of increase in the applied driving force, dJ/da, equals or exceeds the gradient of the resistance curve. In the elastic-plastic regime the concept was originally formulated in terms of a parameter $T = E(dJ/da)/\sigma_y^2$, where instability is predicted when $T_{applied} > T_{material}$ (Paris, Tada, Zahoor and Ernst, 1979, Hutchinson and Paris, 1979). The original assumption was that $T_{material}$, the normalised gradient of the crack growth resistance curve, was constant, but it is now accepted that this is possibly true only for the first millimetre or so of growth. Turner (1979a,b) has proposed an alternative theory that instability occurs when, under fixed grip conditions, the energy release rate exceeds the energy absorption rate. Given the energy release rate definition of J, it is not surprising that in some circumstances the proposals of Paris and Turner appear to be equivalent (Turner, 1979b).

To predict ductile instability not only must J be known as a function of load and structural parameters, but also its derivative with respect to crack length. This can prove quite a formidable analytical problem, and solutions are often approximate and involved (Hutchinson and Paris, 1979, Tada, Musico and Paris, 1978). However, recently an alternative method of analysis based upon the CEGB failure assessment diagram has been proposed (Milne, 1979a) and shown to be equivalent to a J resistance curve analysis (Chell and Milne, 1979). The great advantage of this technique is that it is simple and quick to use and does not require an explicit calculation of J or its derivative. These are important characteristics in design where often a sensitivity analysis of the input data can reveal much useful information, as demonstrated by the worked examples in Chell and Milne (1979).

CONCLUDING REMARKS

In the last decade elastic-plastic fracture mechanics has become recognised as being of increasing importance. Considering the complexities inherent in the application of fracture mechanics to the elastic-plastic regime the advances have been extensive and highly significant. Considering the newness of the discipline there is every reason to expect equally significant advances in the coming years. It is, therefore, perhaps opportune to mention some of the areas that are, as yet not fully resolved.

One of the outstanding problems of EPFM is the relationship between engineering failure parameters and the micromechanisms of fracture. The interaction between these is essential to the understanding of the effects of specimen size on cleavage toughness, and the geometry dependence of ductile crack growth resistance curves. Furthermore there are two other important aspects of ductile failure that have not been mentioned in this paper up until now. They are the possibility of a change in the micromode of crack extension from ductile to cleavage, and time dependent effects. The latter are known to produce added crack extension and possibly instability in cracked structures held under constant load after crack initiation. Both of these clearly have important ramifications as regards ductile instability analyses, and both require some knowledge of micromechanisms for their understanding. In these areas the type of work being done by Beremin (1979) looks promising.

Another area of importance is the treatment of cracks that are embedded in the plastic enclave around a stress concentrator, or are subject to nominal stress levels that exceed the yield stress. The latter case is a frequent problem when treating non-stress relieved weldments. Attempts to treat this problem have been made using CTOD approaches (Harrison, Dawes, Archer and Kamath, 1979) and a J design curve (Turner, 1979c). However the relevance of fracture mechanics to these situations is questionable. It is difficult to envisage a crack tip characterising parameter surviving the complicated non-linear interaction between the crack tip plasticity, and the plasticity arising from the applied field. Indeed since this situation is physically only realistic for very small defects, a reasonable degree of "notch insensitivity" would be expected.

Finally there is a serious need for experimental data to validate the theoretically predicted effects of thermal and residual stresses on elastic-plastic fracture, and for both experimentally and theoretically derived plastic limit loads for a range of structural geometries and loading systems. As mentioned in the main body of the paper both of these are essential requirements, for the acceptance and application of elastic-plastic failure assessment procedures.

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