FINITE ELEMENT METHOD AND ITS APPLICATION TO TWO- AND THREE-DIMENSIONAL ELASTIC-PLASTIC ANALYSIS OF FRACTURE TOUGHNESS SPECIMENS

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ABSTRACT

The principles of elastic-plastic fracture mechanics are shortly summarized and the special requirements for computational tools are derived. Possibilities to model the crack tip singularities are mentioned. The relevant fracture parameters like J-Integral and COD and their correlation are evaluated from 2D and 3D elastic plastic finite element calculations of standard fracture toughness specimens. The size and form of the plastic zone are shown. The comparison between experiment and calculation is discussed as well as the application of the limit load analysis.

KEYWORDS

Fracture mechanics; elastic-plastic; J-Integral; plastic zones; compact specimen; bend specimen; limit load.

NUMERICAL METHODS

All fracture mechanics variables and fracture parameters are derived from deformations, stresses or strains or from combinations of these variables in the vicinity of the crack. This means that a detailed stress analysis of the flawed structure allowing for the correct material law must always be available in principle for the examination of a specific problem. In addition, assumptions on fracture mechanisms are based on details with regard to the stress and strain condition close to the crack tip which cannot always be obtained with adequate certainty by purely analytical means. Such problems can only be solved analytically in very few special cases and even then for the most part only in the purely elastic range, with usable corrections being possible for small plastifications. In general, the solutions to these special cases cannot simply be transferred to real problems, but they can, however, be used to estimate the anticipated result. Otherwise one is dependent on numerical methods for the solution of such problems. Basically, those methods which are otherwise applied for the determination of deformations, stresses and strains in bodies of any shape and with any kind of load may be used in this case. Non-linear material laws may be treated but only with considerably greater difficulties and as a rule with much greater effort. Due to the presence of a crack in the structure examined, the numerical methods must be able to cover the following with adequate accuracy in addition to the requirements already known:

- singularities of stress and strain
- crack tip blunting and
- large strains in the region of the crack tip.

The finite element method has proven to be suitable for the solution of fracture mechanics problems amongst the common numerical methods (Zienkiewicz, 1977).
It was shown by Henshell (1975) and Barsoum (1977) that strain singularities for isoparametric elements may be introduced solely by the selection of the physical co-ordinates of the nodal points of an element:

$$\varepsilon_{ij}(r) = \frac{a_{ij}}{r} + \frac{b_{ij}}{\sqrt{r}} + c_{ij} + \ldots \quad ,$$

if the degrees of freedom of the collapsed crack tip values are independent, otherwise:

$$\varepsilon_{ij}(r) = \frac{b'_{ij}}{\sqrt{r}} + c'_{ij} + \ldots \quad .$$

Apart from entirely academic examples, the fracture mechanics problems actually arising are always three-dimensional. Even in cases in which it is possible to describe geometry and load in a plane co-ordinate system, assumptions with regard to strains or stresses in the third co-ordinate direction are necessary for a correct determination of the stress and deformation conditions. Such specifications are at first arbitrary. On the whole one considers the condition of plane stress and plane strain as boundary cases. Whilst in a real problem plane stress may occur in thin structures since the normal stresses disappear on free surfaces, the plane strain condition is only to be assumed as a boundary case. Consequently, in a two-dimensional calculation the result is greatly dependent on whether plane stress or plane strain is assumed. In the analysis of a specimen the difference is, for example, 30 % in the load deflection curve. Faults of this kind are thus quite unavoidable due to the limitation "two-dimensional calculation" and cannot be compensated even with extra effort regarding element and nodal numbers.

EVALUATION OF FRACTURE PARAMETERS

Determination of Crack Tip Opening Displacement δ (COD)

As will be shown in detail in the results, the crack tip opening displacement may be determined easily by extrapolation of the crack tip displacements towards the crack front. The accuracy of this method is good (at least in the case of extensive plastic deformation), since here on the one hand the displacements of the crack behave approximately linearly in the vicinity of the crack tip, and on the other hand, the displacements are determined with the utmost accuracy as the primary unknown quantities in the finite element method.

Determination of the J-Integral (plane strain condition)

The variables $W(\varepsilon) = \int_0^{\varepsilon} \sigma_{ij}\, d\varepsilon_{ij}$

and $\vec{t} = \{t_i\} = \{\sigma_{ij} n_j\},\ \{\vec{u}\}$

defining the J integral

$$J = \int_{\Gamma} \left(W dy - \vec{t} \cdot \frac{\delta \vec{u}}{\delta x}\, ds\right) \qquad (1)$$

are immediately accessible by the finite element calculation. Thus J can easily be calculated by way of appropriate program extensions. A further possibility favoured by the authors results from the definition of J as represented by the change of potential energy

$$J = - \frac{\delta U}{\delta A}$$

with the change δA in the crack surface.
In the formulation of the finite elements we obtain the following:

$$U = 1/2 \cdot \{u\}^T \cdot [K] \cdot \{u\} - \{u\}^T \cdot \{F\}$$

Following Parks (1974) we get

$$- \frac{\delta U}{\delta A}\Big|_{L\ =\ const} = - \frac{\delta\{u\}}{\delta A}^T \cdot ([K] \cdot \{u\} - \{F\}) - 1/2 \cdot \{u\}^T \cdot$$

$$\frac{\delta\{K\}}{\delta A}\{u\} + \{u\}^T \frac{\delta\{F\}}{\delta A} \qquad (2)$$

and considering that, according to definition,

$$[K] \cdot \{u\} - \{F\} = 0$$

and

$$\delta\{F\} / \delta A = 0$$

we obtain

$$J = - 1/2 \cdot \{u\}^T \cdot \frac{\delta[K]}{\delta A} \cdot \{u\} \qquad (3)$$

J is actually not defined for three-dimensional cases. According to Parks (1974), it is possible to define a variation of $J' = G = -\delta U / \delta A$ along the crack front at least for the elastic case by way of the definition of J from the change in potential energy. It will be shown in the results that this method leads to reasonable results also in the elastoplastic range. However, it is not possible to demonstrate the validity of the method. Park's method above all offers great advantages with regard to the calculation time since only a few elements must be reassembled in general in the vicinity of the crack tip in the determination of $\delta[K]$.

RESULTS

The FE-analyses were performed by three different types of fracture toughness specimens:
- the compact tension specimen (CT)
- the three point bend specimen (3PB)
- the wedge opening load specimen (WOL)

Geometries and Finite Element (FE) meshes are shown in Fig. 1.

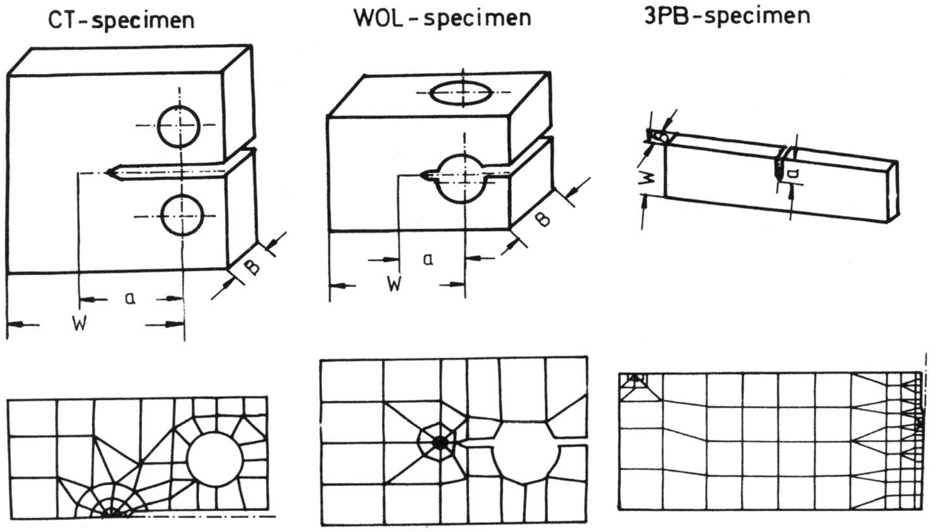

Fig. 1. Geometries and FEmeshes twodimensional of the performed specimens

Compact Tension Specimen

The FE-analyses were done two-dimensionally (2D) and three-dimensionally (3D).

Load displacement curve. Comparing the results of the analysis under plane strain condition leads to an underestimation of the displacements while under plane stress condition the experimental results were overestimated, respectively. Good agreement is shown between the 3D results and experiment.

Plastic zone. We tried to obtain information about the state of stress of the specimen from the plastic zones. Theoretically we consider for small plastic zone sizes

$$r_p \cdot (2\pi \cdot R_{p_{0,2}}^2)/K_I^2 = \cos^2(\theta/2)\cdot[1+3\cdot\sin^2(\theta/2)-4\nu'\cdot(1-\nu')] \quad (4)$$

where

$\nu' = \begin{cases}\nu & \text{Poisson's ratio (plane strain)} \\ 0 & \text{(plane stress)}\end{cases}$

r_p and θ = polar co-ordinates of the plastic zone boundary

$R_{p_{0,2}}$ = yield stress.

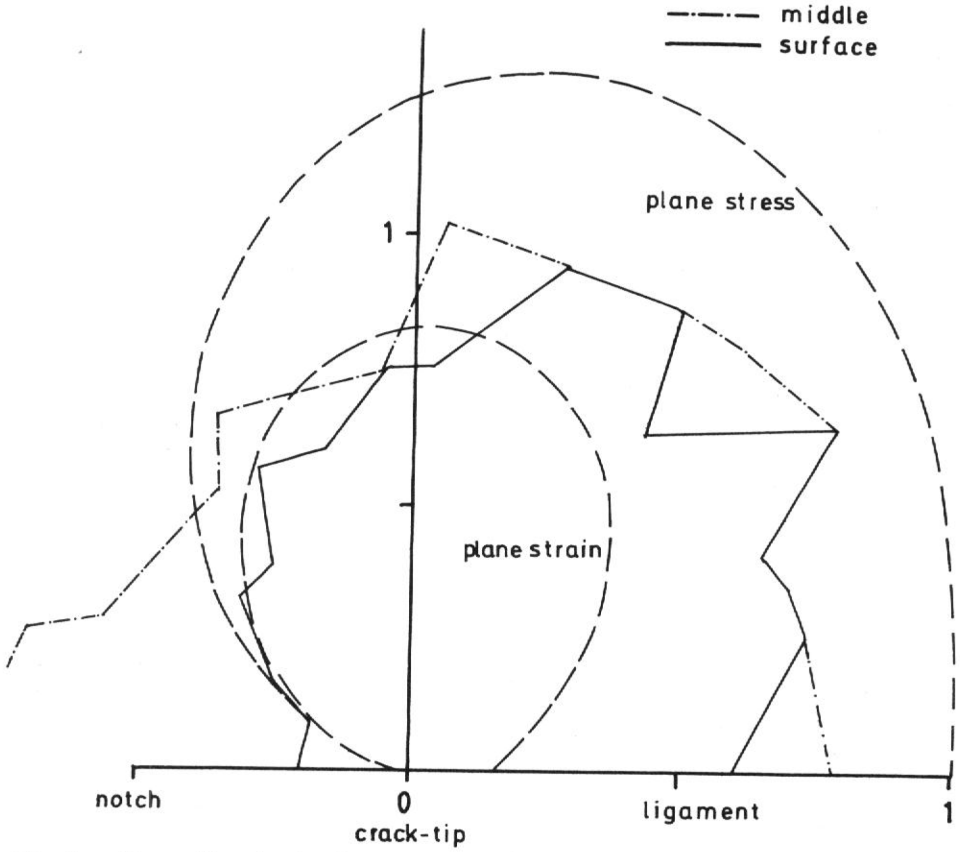

Fig. 2. **Normalised plastic zones at fracture load**

At low loads the form and size of the plastic zone is less different between the specimen surface and middle. In the middle the plastic zone approximates the theoretical form under plane strain conditions. Figure 2 shows the theoretical plastic zone form under plane strain and plane stress conditions as well as the plastic zones at experimental fracture load. Comparing with low loads the plastic zone at the limit load is larger in the middle of the specimen than at the surface. With increasing load the form approximates more and more the theoretical form under plane stress conditions.

J-Integral and crack-tip opening displacement δ (COD). Figure 3 shows the linear relation between $R_{p_{0,2}}$, J and δ following Rice (1970).

$$J = \beta \cdot R_{p_{0,2}} \cdot \delta \qquad (5)$$

From the FE-analysis we obtain for β

$\beta = 1.2$ (surface)
$\beta = 1.64$ (middle).

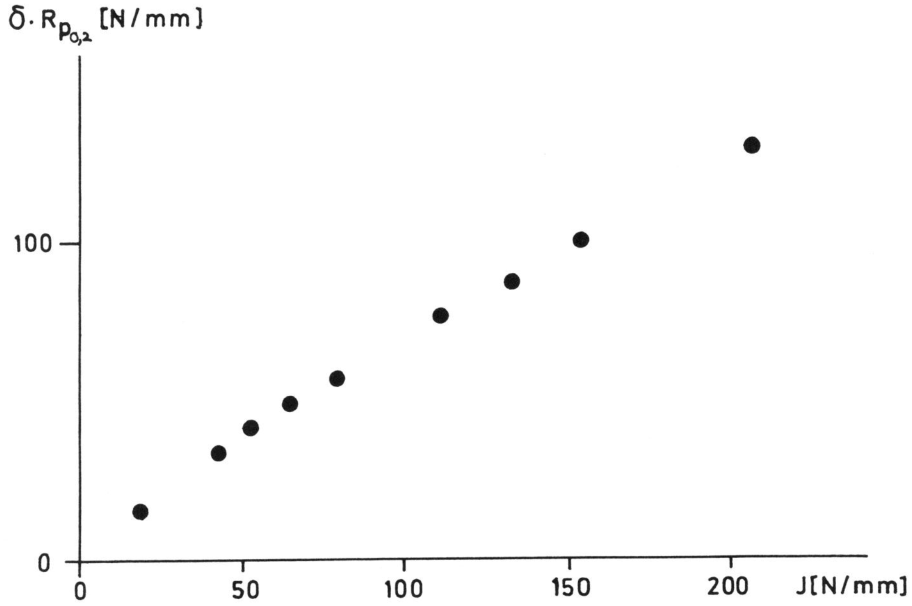

Fig. 3. Correlation between J-Integral and COD (middle)

Three Point Bend Specimen (3PB)

The analysis was done 2D under plane strain conditions.
Load displacement curve. The assumed plane strain condition leads to results which are too stiff with respect to the experimental data. In spite of this the following diagrams are drawn over the clip-gauge displacement (V_g) instead of load.
Plastic zone. Good agreement exists between experiment and calculation for the form of the plastic zone (experimental with stress optical methods). With increasing load the appearing plastic zone at the load point opposite to the crack grows faster than that coming from the crack and it shows at least the same size. Therefore the fracture parameters (J , δ) are probably influenced by this effect and do not describe the real behaviour of the crack (see Fig. 4).
J -Integral and δ . The J -Integral after Sumpter (1973) is most suitable for comparison of the results from calculation and experiment. The elastic and plastic factors are shown in Table 1. Good agreement exists for δ between calculation and experiment. From the FE-analysis we obtained δ by an extrapolation scheme to compare with the measured δ after Wells (1971).
J and δ are correlated by

β = 1.34.

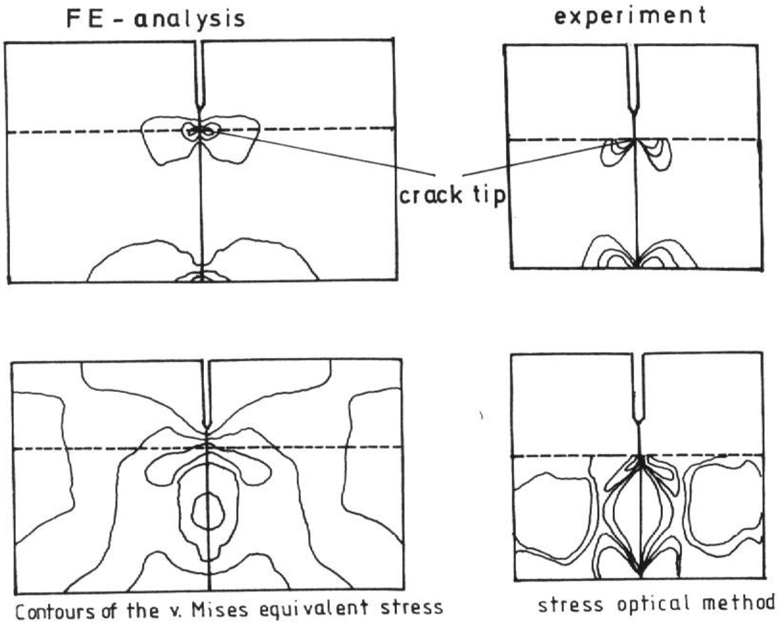

Fig. 4. 3-point bend specimen - contours of the plastic zones (qualitative for two different load levels)

Wedge Opening Load Specimen (WOL)

For comparison of the CT-specimen and the WOL specimen 3D elastic analyses were performed.
A detailed description of the results is not possible here, only limit load estimation is given in Fig. 7 and η_e-values are referenced in Table 1.

Comparison of the Factors for J-Integral evaluated after Sumpter (1973)

Sumpter's method is well applicable for experiments and calculations. He proposed to break up the total energy of the load versus load point displacement curve into an elastic and a plastic part:

$$J_S = \frac{\eta_e \cdot U_e}{B(W-a)} + \frac{\eta_p \cdot U_p}{B(W-a)} \qquad (6)$$

where η_e and η_p are obtained from the results of the analysis and η_e is derived from the load versus load point displacement curve of the experiment, respectively.

TABLE 1 Elastic and Plastic Factors η_e and η_p

Specimen		a/W	η_e	η_p
CT	2D°	0.48	2.34	0.8
CT	2D°	0.45	2.36	0.6
CT	3D°	0.48	2.32	1.7
CT	3D*	0.48	2.33	
CT	3D*	0.53	2.28	
3PB	2D	0.37	1.71	1.2
WOL	3D*	0.41	2.52	
WOL	3D*	0.47	2.17	
CT	Ex+*	0.48	2.34	
3PB	Ex+	0.37	1.68	

° Specimen thickness 70 mm
* Specimen thickness 100 mm
+ Experimental data

For the different specimens we obtained that η_e depends on the relative crack size a/W. An independency is found for the same specimen and different thicknesses.

Evaluation of Limit Load

The plastic limit load is defined as the load at which the external work increment performed by a virtual displacement increment δ_u of this force equals or exceeds the internal work increment consumed.

$$\delta W_{ext} \geq \delta W_{int} \tag{7}$$

The slip lines must be considered with the utmost accuracy in the determination of the internal work. To reach the limit load at least one load-bearing cross-section of the structure must be completely plastified. Limit load estimations for the various geometries and load types are given in the literature. It is important to point out that our estimations of the limit load do not depend on whether the specimen has a notch with a finite notch radius or a fatigue crack. From this it can be concluded that failure of the specimen in the vicinity of the plastic limit load is not influenced by stress or strain singularities in the vicinity of fatique crack. Thus it seems impossible to gain any reasonable fracture parameters (δ_c, J_c, etc.) from such specimens.

CT-Specimen. The following equation was derived based on the slip lines of the specimen:

$$P_L = 0,6 \cdot R_{p_{0,2}} \cdot (1-a/w)^2 \; / \; (1+a/w) \; . \; W \; . \; B$$

Bend specimen. From the contours of the plastic zones we get

$$P_L = 0.15 \cdot R_{p_{0,2}} \cdot (W-a)^2$$

WOL-specimen. For WOL-specimen with a relatively small ratio of a/W we obtain the plastic collapse of the leg before the failure of the ligament by reaching the limit load. From the contours of the plastic zones and from the slip line theory, respectively, we determine

$$P_L = 0.66 \cdot R_{p_{0,2}} \cdot (1-a/W)^2 \;/\; (1+a/W) \;.\; B^2 \quad \text{(ligament)}$$

$$P_L = 1/(4 \cdot \sqrt{3} \cdot \sin^2 \alpha) \;.\; R_{p_{0,2}} \;.\; B^2 \quad \text{(leg)}$$

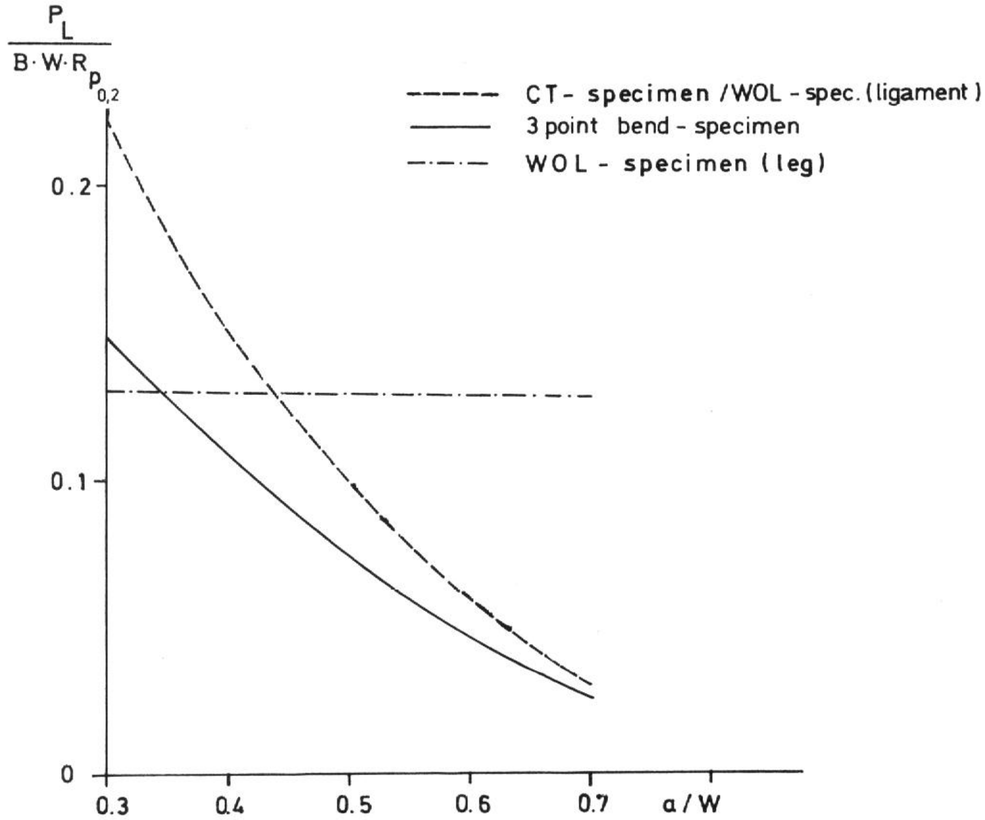

Fig. 5 Evaluated limit load versus a/W

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