EXPERIMENTAL AND THEORETICAL STUDY OF CRACKS IN MIXED MODE CONDITIONS

P. Jodin*, G. Pluvinage*, G. Loubignac and D. Serres****

***Laboratoire de Fiabilité Mécanique, Université de METZ, Ile du Saulcy, 57000 METZ, France**
****CETIM, Boîte Postale 67, 60304 SENLIS, France**

ABSTRACT

Fracture tests were performed on X- H- and L- shaped specimen. Thus, the fatigue precrack is loaded in mixed mode (I + II) condition. The recorded load-displacement curves are compared with those obtained by a finite element computation. The influence of the temperature on the fracture energy of a specimen loaded by tensile and shear stresses is also studied. The contour of the plastic zone at the crack tip is determined experimentally and compared with published formulae.

KEYWORDS

Mixed mode fracture criteria, finite element method, fracture energy, plastic zone.

INTRODUCTION

The most part of published papers concerning the application of fracture mechanics to structures is related to mode I (opening mode) fracture. This mode is usually considered as the most dangerous one but it has been shown (Jodin and Pluvinage, 1978) that a combination of mode I and mode II could lead to a lower fracture stress. The purpose of this paper is to study interactions between mode I and mode II when they are both present in a structure and to establish a comparison with a theory.

An offshore platform rig is an example of a structure where complex loads are present in the junctions (Fig. 1). It is possible to carry fracture tests on these structures with a very large and highly sophisticated equipment. But this is very expensive and time consuming. A simplest approach of this problem is proposed in this paper : small scale test specimen are used representing the geometry of junctions frequently encountered in real structures.

In the same optic to save time and money, the influence of temperature on fracture energy in mixed mode is studied on small steel Charpy-V specimen, the notch of which is inclined so that to produce mixed mode I + II. The results show that the minimum of energy is not for pure mode I. An original method for plastic zone size evaluation is used to estimate the influence of a shear mode on the contour of plastic zone. This method is compared with theoretical formulae.

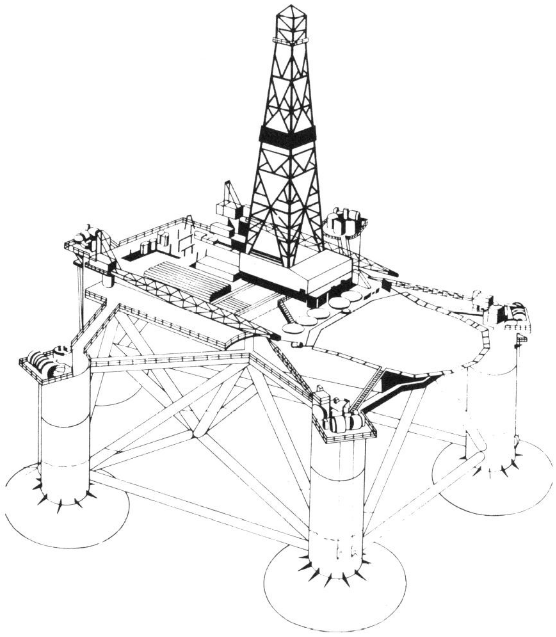

Fig. 1. Example of a complex structure.

SHORT BIBLIOGRAPHICAL REVIEW

A recent work (Gilles, 1977) reported not less than 35 mixed mode fracture criteria. Some of them are described here.

Sih's Criteria

The Erdogan and Sih criterium (1963). This criterium (also called "Sih I") is based on the following hypothesis :

(i) the crack propagates in a radial direction
(ii) this direction is that where the hoop stress $\sigma_{\theta\theta}$ is maximum
(iii) in this direction, when the crack propagates, the relation

$$\sigma_{\theta\theta} \sqrt{2\pi r} = cte = K_{Ic} \quad \text{is verified.}$$

For an inclined crack in an infinite plate, the condition $\sigma_{\theta\theta}$ maximum leads to :

$$\sin \theta_o + (3 \cos \theta_o - 1) \operatorname{cotg} \beta = 0 \quad (1)$$

i. e. $\theta_o = 0$ for $\beta = 90°$ (pure mode I)

$\theta_o = 70.5°$ for $\beta \rightarrow 0$ ($\rightarrow$ pure mode II)

But this model does not take into account the influence of the material properties around the crack tip on the stresses distribution,

The second Sih's criterium. This criterium (Sih, 1974)(also called "Sih II") strikes the balance of the energy localized in an elementary volume near the crack tip. It is shown that the elastic energy dW/dV takes the analytical following form :

$$\frac{dW}{dV} = \frac{1}{r}\,(a_{11}K_I^2 + 2\,a_{12}K_IK_{II} + a_{22}K_{II}^2 + \ldots.) \qquad (2)$$

where the a_{ij} coefficients may be computed from the elastic constants of the material. The quantity in the brackets is called the strain energy density factor. The basic assumptions for this criterium are :

(i) the crack will propagate in the direction where thestrain energy is minimum (or the potential energy is maximum).

(ii) in this direction, propagation occurs when the strain energy density factor S reaches a critical value S_{cr} which is a characteristic of the material.

The crak propagation direction and the critical strain energy density factor are dependant on the Poisson's ratio ν .

Mandel's Criterium (1964)

The stresses at the crack tip are written into the following form :

$$r^{1/2}\,\sigma_{rr} = K_I\,\phi_{rr}(\theta) - K_{II}\,\psi_{rr}(\theta) \qquad (3)$$

$$r^{1/2}\,\sigma_{\theta\theta} = K_I\,\phi_{\theta\theta}(\theta) - K_{II}\,\psi_{\theta\theta}(\theta) = f(\theta) \qquad (4)$$

$$r^{1/2}\,\sigma_{r\theta} = K_I\,\phi_{r\theta}(\theta) + K_{II}\,\psi_{r\theta}(\theta) = g(\theta) \qquad (5)$$

Mandel assumes that a relation between K_I and K_{II} $k(K_I,K_{II}) = 0$ is characteristic of the material's intrinsic toughness. The functions $f(\theta)$ and $g(\theta)$ are represented on the fig. 2.

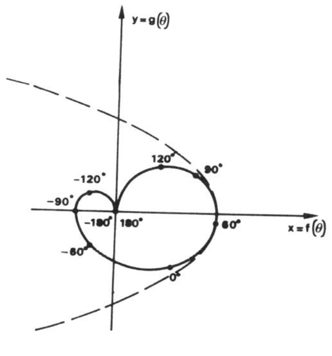

Fig. 2. Mohr's curves and intrinsic toughness curve (Mandel, 1964).

At fracture $f(\theta)$ and $g(\theta)$ is minimum and the intrinsic toughness curve and the $f(\theta) - g(\theta)$ curve are tangent at point N (Fig. 3). Mandel also assumes that the larger the radius of the intrinsic curve at point N, the more ductile the material is .

Strifors' Criterium (1974)

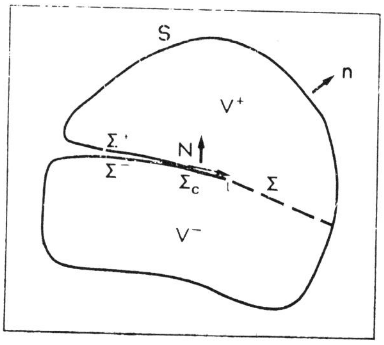

Fig. 3. Cracked body containing a singular surface.

The first principle of thermodynamics applied to a cracked body containing a singular surface Σ(Fig. 3) gives : $\dot{E} + \dot{\Gamma} + \dot{K} = P + Q$ where E is the internal energy, Γ the surface energy, K the kinetic energy, P the power of external loads and Q the heat rate recieved by the body. The dot represents derivation with respect to time. Some hypothesis on these energies allow to compute an apparent crack extension force in a direction defined by vector $\vec{\ell}$.

$$f_\ell(V) = \int_V \ell_\alpha \; \sigma_{i\beta} \; u_{i,\beta\alpha} \; dV - \int_S \ell_\alpha \; T_i \; u_{i,\alpha} \; dS$$

where $u_{i,\alpha} = \partial u_i/\partial x_\alpha$ and the u_i are the components of the displacements field, $\sigma_{i\beta}$ are the components of the stresses tensor and T_i the stress vector.
At this stage, a law of material's behaviour is introduced so that the computation can be carried out in elasto-plastic bodies for instance. The basic assumptions of the criterium are :

(i) the direction $(\vec{\ell})$ where $f_\ell(V)$ is maximum is the direction of crack propagation

(ii) in this direction, propagation occurs when $f_\ell(V)$ reaches a critical value $f_c(V)$.

EXPERIMENTAL WORK

Fig. 4 shows the various geometries for the test specimen. The X, T, H shaped one were used for complex loading experiments. The small modified Charpy V specimen were for resilience tests and the bending specimen with an inclined crack was used for the determination of the plastic zone.

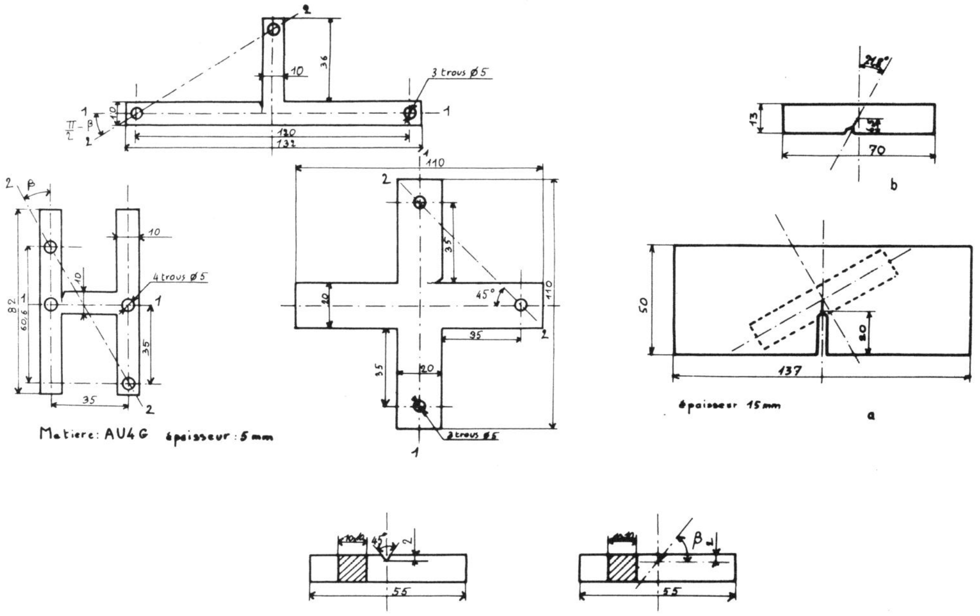

Fig. 4. Test specimens.

Complex Loading Experiments

The specimen were cut from a commercial 2024 aluminium alloy sheet of 5 mm thickness. The mechanical properties of the material are reported in the table 1.

TABLE 1 Mechanical Properties of 2024 Al

Re MN/m^2	R MN/m^2	A %
250	424	11

The slots (2 mm) were machined with an electroslag machine. A razor blade was applied at the notch tip . A fatigue crack was then produced by loading the specimens along the axis 1. The geometries of the specimen are reported in the table 2. The specimen were then statically loaded. The crosshead speed was 0.1 mm/mn and a load-displacement curve was recorded. The crack propagation angle was optically determined after complete fractureof specimen.

TABLE 2 Geometry of Test Specimen

N°	Type	Angle * degree	Crack length mm
1	H	30	4.31
2	H	40	4.25
3	T	64.7	4.12
4	T	70	4.49
5	X	45	5.30

* with respect to crack plane

Resilience Tests

The specimen used wer derived from those designed for standard resilience tests (Fig. 4). The material is a commercial XC 38 steel wire. The inclined slot was machined with an electroslag device using a fine copper wire so that the radius at the notch tip was less than 0.25 mm. The specimens were immersed in a carefully regulated thermostatic bath. The transfer time from the bath to the test place did not excess five seconds. After the test some specimens were cut along a plane perpendicular to the notch tip, the fractured surface being protected by a deposit of nickel. The profile was polished and the structure revealed with picral and nital solutions.

Plastic Zone Size Determination

The following procedure was used to produce an inclined fatigue crack. A notched specimen made in a A316 steel (Fig. 4) was precracked under fatigue with a three-points bending device. The width of the specimen was large enough to obtain a smaller specimen including the fatigue crack, inclined with respect to the plane normal to the longitudinal axis of the specimen. This specimen was then loaded statically in three-points bending (P max = 500 daN), the crack tip being in the plane of symetry.

The surface contraction due to the plastic strain was observed with a microscope and with a surface finish tester (Louah, 1978). The contour reported (Fig. 9) is determined for a normal displacement of 4 μm of the surface finish tester.

RESULTS

Complex Loading Experiments

The different load-displacement curves for each specimen are reported on Fig. 5. These curves are compared with the results of a computer simulation using the finite element programm CA.ST.OR.D2D elaborated at the CETIM. Fig. 6 shows a typical mesh of one of the specimen. The direction and the critical load of crack propagation was theoretically determined at the CETIM with a computer programm using the Strifors' crack extension force concept. They are in good agreement with the experimental observations.

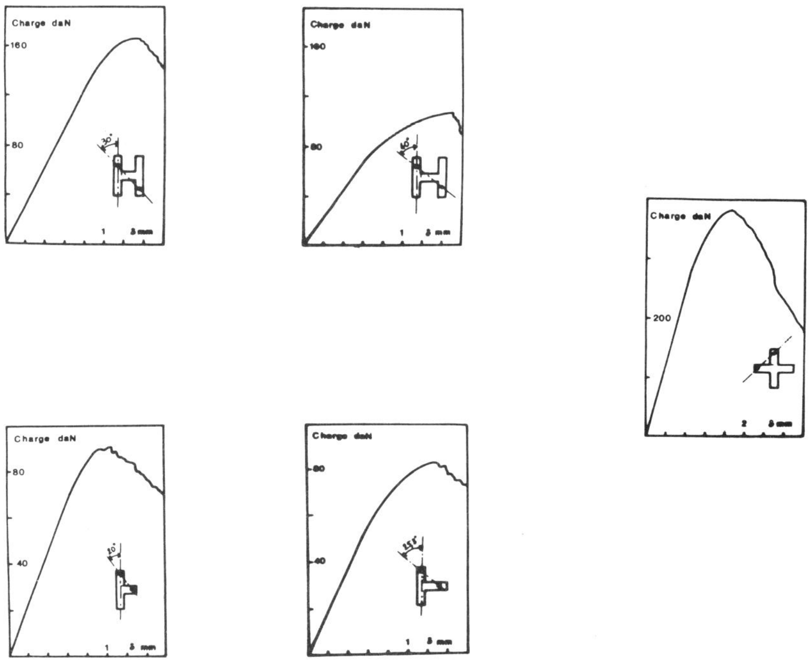

Fig. 5. Load-displacement curves and theoretical results .

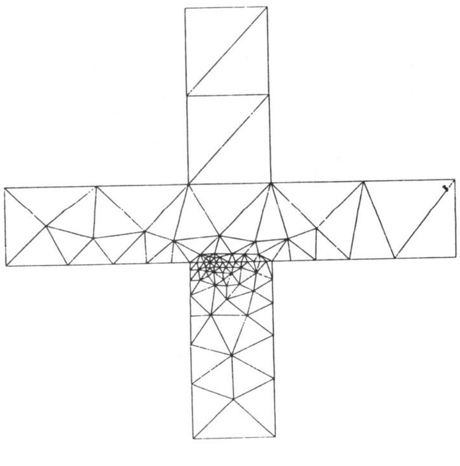

Fig. 6. Typical mesh used with computer program.

Resilience Tests

The results are reported on Fig. 7.

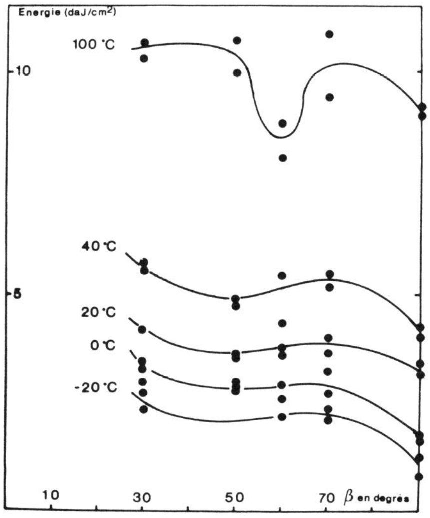

Fig. 7. Results of resilience tests.

The value at 20°C for the standard specimen (90°) is consistant with published results (Lazo La Torre, 1978). A distinct minimum appears on Fig. 7 at 100°C for the 60 degrees specimen. It is no more apparent at lower temperature.

Figure 8 shows the aspect of a micrograph for the 60° specimen at 100°. The existence of a highly damaged zone is evident. The evaluation of crack propagation angle just at the notch tip is not possible.

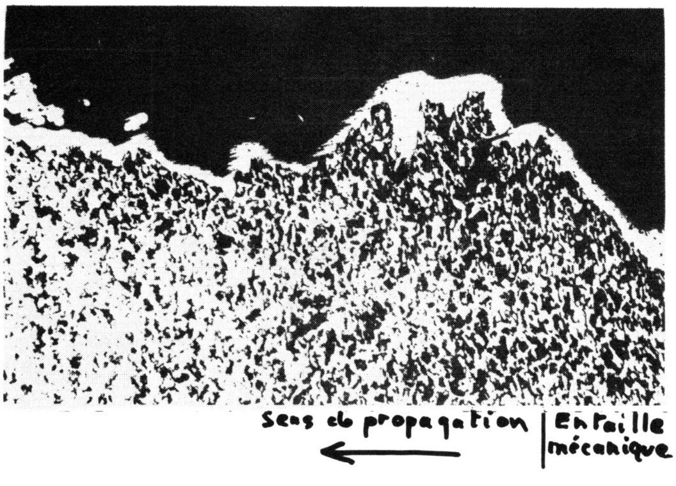

Fig. 8. Micrograph of resilience specimen (angle 60°, temperature 100°C).

Contour of Plastic Zone

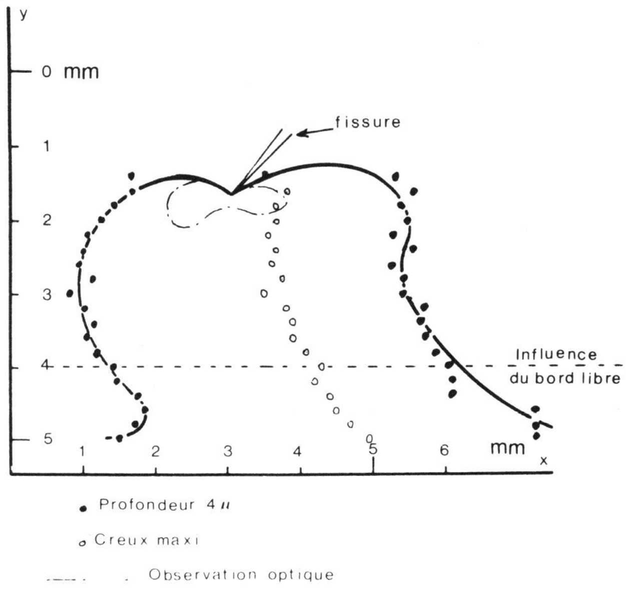

Fig. 9. Compared contour of plastic zone.

Figure 9 shows the contours of plastic zone obtained with a microscope and with a surface finish tester. The large difference between the two contours is probably due to the fact that the error induced by angle of incidence of the light rays is large with respect to the error given by the surface finish tester. A theoretical computation using classical formula (McClintock and Irwin, 1965) gives larger values.

DISCUSSION

Complex Loadings Experiments

The good fit of the results of computation with CA.ST.OR.D2D was obtained by adjusting the stresses applied on the singular element around the crack tip. These stresses , in each case, were found to be approximately equal to the yield stress. This point is reasonnable because, after precracking , a cyclic plastic zone was developed at the crack tip. The influence of other stresses than the remote applied stress was already noticed by Streit and Finnie (1979).

Resilience Tests

Unless there is a relatively large, scattering in the results, it is possible to conclude that there is a combination of mode I and mode II (corresponding to an angle of 60°) when fracture is ductile , which leads to a lower fracture energy. This observation corroborate the phenomenon previously observed (Jodin and Pluvinage, 1978). The observation of the micrographs suggests that, with respect to grain size, the fracture phenomenon is governed by a macroscopic criterium .

Plastic Zone Measurements

When comparing the different results of experimental evaluation and the results of computation, two points can be noticed :
(i) the size obtained experimentally and by computation are quite dissimilar.
(ii) the shape of the plastic zones obtained are quite similar.

The explanation of this two observations is simple. The experimental errors, depending on the techniques used, are negative, that is to say the real plastic zone size is surely higher than that observed. The errors have the same value in all directions, so the plastic zones obtained have the same shape.

CONCLUSION

The results presented in this paper confirm that an elastoplastic computation on small scale specimen is possible and gives a good fit with experimental results. They can be an approach to mixed mode loading in large structures, because the tests and the programm runs are inexpensive and easy. The Strifors' criterium seems to be the best one in this case. The resilience tests confirm that a minimum energy does exist for a combination of about 60 % K_I 30 % K_{II}, i.e. the most dangerous mode of fracture is not necessarily the mode I.

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