DETERMINING THE DEFECT TOLERANCE OF STRUCTURES FAILING BY DUCTILE CRACK GROWTH

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ABSTRACT

A procedure is described for determining the ductile instability of cracked structures subjected to prescribed displacement loading. The procedure is based on the CEGB failure assessment diagram and is illustrated using as an example a cracked beam of various span lengths. It is shown that ductile instability is possible under fixed displacement conditions provided the gauge length over which the displacement is prescribed is long enough. The equivalence of the procedure to a J resistance curve analysis is demonstrated.

KEYWORDS

Stable crack growth; ductile instability; J resistance curve; failure assessment diagram; prescribed displacements.

INTRODUCTION

It is well known that the resistance to stable ductile crack growth measured in terms of J increases as the crack extends. At instability the crack driving force must increase at a rate in excess of the increase in resistance and this may require the accommodation of relatively large displacements. However in structural systems where the geometric constraints are large, or where the loading results from prescribed displacements, the ability to accommodate the extra displacements resulting from crack growth is limited unless significant unloading occurs. Hence the likelihood of crack growth leading to ductile instability is dependent on the compliance of the structure and, in the case of imposed displacements, the gauge length over which they are applied.

In this paper these aspects are investigated by studying the influence of gauge length on the fracture behaviour of a cracked beam subjected to fixed displacements on its ends equivalent to a pure bending moment. This system is analysed using the ductile instability method proposed by Milne (1979a).

DUCTILE INSTABILITY ANALYSIS

Milne's (1979a) proposal for determining ductile instability is based on the failure assessment diagram of Harrison, Loosemore and Milne (1977). Two parameters are evaluated,

$$K_r = \frac{K_1(\sigma, a+\Delta a)}{K_\Omega(\Delta a)} \tag{1a}$$

and

$$S_r = \frac{\sigma}{\sigma_1(a+\Delta a)} \tag{1b}$$

where K_1 is the linear elastic stress intensity factor at the applied stress σ and for a crack of original length a which has grown a distance Δa, σ_1 is the plastic limit stress and $K_\Omega(\Delta a)$ is the crack growth resistance toughness. The latter is obtained from the J resistance curve, $J_R(\Delta a)$, through the formula $K^2_\Omega(\Delta a)=E'J_R(\Delta a)$, where E' is Young's modulus E for plane stress and $E/1-\nu^2$) for plane strain, ν being Poisson's ratio.

To predict the instability stress a locus of coordinates S_r, K_r is plotted at constant applied stress but increasing postulated crack extension, Δa, on the failure assessment diagram, curve ABC in Fig. 1. A load factor F is then evaluated as a function of Δa, either graphically, as shown in Fig. 1, or using the expression (Chell and Milne, 1979).

$$F = \frac{2}{\pi S_r} \cos^{-1} \exp\left(-\frac{\pi^2 S_r^2}{8K_r^2}\right) \tag{2}$$

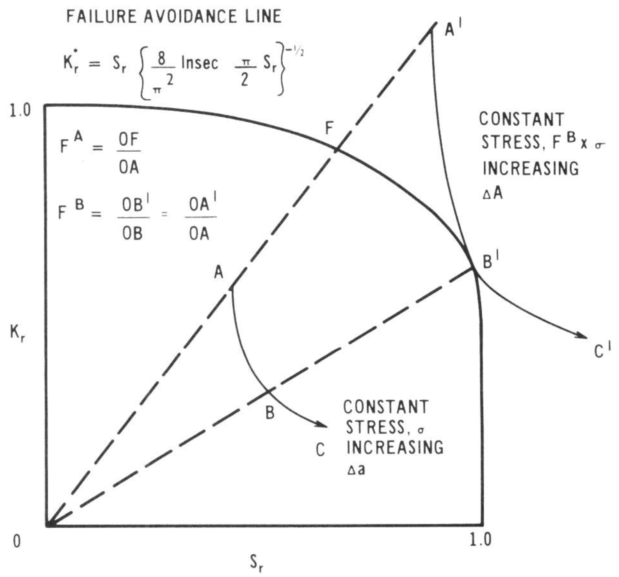

FIG. 1 THE FAILURE ASSESSMENT DIAGRAM AND THE PREDICTION OF DUCTILE INSTABILITY. THE MAXIMUM LOAD FACTOR IS F^B

The point of instability is defined at the maximum value of F, the stress at this point being given by the product of this maximum value of F and the applied stress. The curve A'B'C' in Fig. 1 has been drawn at this maximum stress to demonstrate that the instability occurs when this locus is tangential to the failure line.

It has been shown previously that this method of analysis is equivalent to a ductile instability analysis based on J if the functional form for J is taken as (Chell and Milne, 1979)

$$J = J_1 \cdot \frac{8}{\pi^2 S_r^2} \ln\sec\left(\frac{\pi}{2} S_r\right) \qquad (3)$$

where $J_1 = K_1^2/E'$ is the linear elastic value of J.

PRESCRIBED DISPLACEMENTS

The foregoing analysis applies directly to primary loads e.g. internal pressure. For residual and thermal loadings plastic collapse is not possible and hence their plasticity effects cannot be directly incorporated via S_r; modifications of the type proposed by Chell (1979) and Milne (1979b) are required. When the loading is defined in terms of prescribed displacements, however, additional problems arise because extension of the crack causes a reduction in the effective load, although the applied value of J may be increasing. This requires further special techniques. For displacement controlled cases, once the effective or relaxed load has been determined, S_r and K_r are definable. Thus the problem reduces to determining this load, L(a), as a function of crack length for the given displacement, δ. In two dimensional cases this can be obtained from the equation (Chell and Ewing, 1977, Chell, 1979)

$$\delta = \lambda_o L(a) + B \frac{\partial}{\partial L} \int_o^a J \, da \qquad (4)$$

where λ_o is the compliance of the uncracked structure, B its thickness and the derivative is evaluated at L(a). For fully circumferential cracks in axisymmetric geometries the equation is

$$\delta = \lambda_o L(a) + 2\pi \frac{\partial}{\partial L} \int_o^a (R \pm a) J \, da \qquad (5)$$

Here if the crack is internal, R is the internal radius and the positive sign is taken in the brackets, while for an external crack R is the external radius and the negative sign in the bracket is taken. Thus for any appropriate functional form for J the problem can be solved. The most convenient form for J is that of equation (3). This is consistent with the failure procedures developed using the failure assessment diagram, provided the stress σ is replaced by the effective stress $\sigma(a)$, which is to be determined, and is given by $\sigma(a) = AL(a)$ where A is a geometric term of dimensions $(\text{length})^{-2}$.

EXAMPLE A CRACKED BEAM

Consider a cracked beam subjected to end displacements which are equivalent to a pure bending moment. In this case $\lambda_o = S^3/4EBt^3$, where S is the span, t the width of the beam and $\sigma = 3LS/2Bt^2$. The plastic collapse stress $\sigma_1 = 2.18\bar{\sigma}(1-a/t)^2$ in plane strain (Green and Hundy, 1956) where $\bar{\sigma}$ is a flow stress (e.g. the average of yield and ultimate stress) and $K_1 = \sigma a^{\frac{1}{2}} Y(a/t)$ where Y is a geometric term. For the purposes of calculation we take the following geometric and material values.

	t	=	100mm,
	B	=	50mm,
	a	=	30mm
and	S	=	400, 800, 1600 and 3200mm
	$\bar{\sigma}$	=	400MPa
	E'	=	200GPa
	K_i	=	100MPa $m^{\frac{1}{2}}$
and	T_{mat}	=	25 and $100m^{-1}$.

Here K_i is the toughness at the initiation of crack growth and T_{mat} is the parameter introduced by Paris, Tada, Zahoor and Ernst (1979) which defines a linear $J_R(\Delta a)$ curve through the equation

$$J_R(\Delta a) = J_i + \frac{\bar{\sigma}^2 t}{E'} T_{mat}.\Delta a \qquad (6)$$

or, in terms of $K_\Omega(\Delta a)$

$$K_\Omega(\Delta a) = K_i \left(1 + \frac{\bar{\sigma}^2 t T_{mat}}{K_i^2}.\Delta a\right)^{\frac{1}{2}} \qquad (7)$$

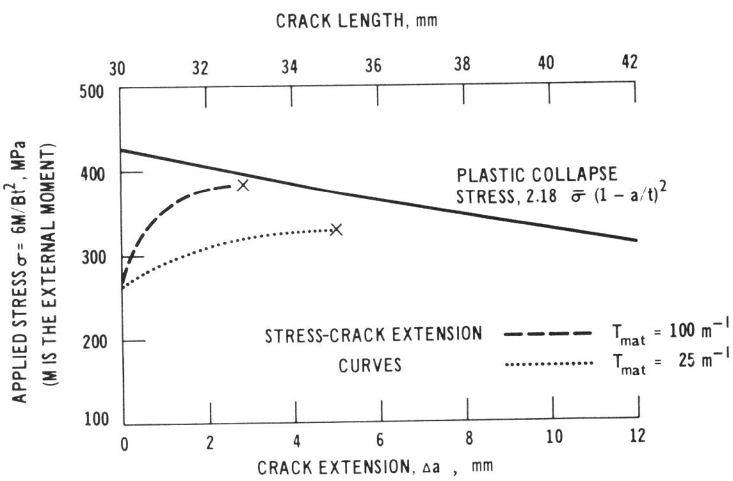

FIG. 2 DUCTILE CRACK EXTENSION IN A BEAM SUBJECTED TO BENDING. INSTABILITY IS MARKED BY X

Following the procedures outlined by Milne (1979a) and Chell and Milne (1979) loci of applied stress as a function of crack extension were calculated for the two T_{mat} values and assuming dead loading. The results are shown in Fig. 2. Crack initiation occurs at a stress level of 263MPa, and the maximum loads (coincident with instability for dead loading) were 328MPa and 382MPa (after crack growth of 5.0mm and 2.8mm) for T_{mat} values of $25m^{-1}$ and $100m^{-1}$ respectively. The corresponding calculated values of the displacements due to the crack, $B\ \partial/\partial L \int_o^a J da$, were 0.81mm and 1.28mm respectively. The total overall displacement at maximum load is given by the crack contribution plus the uncracked beam displacement, $\sigma S^2/6Et$. These were determined for the four span values considered above. Equation (4) was then used to determine the relaxed stress, $\sigma(a)$, and hence the corresponding applied J, as the crack extends from 30mm, assuming the displacements at maximum load were initially prescribed. The results are compared with the $J_R(\Delta a)$ curve for T_{mat}= $25m^{-1}$ in Fig. 3a and for $T_{mat} = 100m^{-1}$ in Fig. 3b. Of course all the J applied curves intersect at the same point, that predicted from the dead loading situation. The relative stability of the beam at this point is determined by the condition for tangency between the J curves and the $J_R(\Delta a)$ curve. From Figs. 3a,b it can be seen that for both values of T_{mat} tangency points are only obtained for the span of 3200mm and for dead loading. Therefore instability is predicted for these cases i.e. the crack will extend without further displacement being applied. For the other three cases further crack extension can only occur if the displacement is increased. Figs. 3a,b therefore illustrate the sensitivity of ductile crack growth to gauge length and also that even under displacement control ductile instability is possible after some amount of crack growth.

In Fig. 4 the $T_{mat} = 100m^{-1}$ results for S = 400 and 3200mm and dead loading are shown plotted on the failure assessment diagram. Here K_r and S_r were evaluated for the instantaneous crack length using the calculated values of $\sigma(a)$. As expected the crack growth locus for S = 3200mm is tangential to the failure line, as is the dead loading curve. In contrast the locus for S = 400mm falls inside the failure curve after intersecting it, showing again that the applied displacement must be increased for further crack extension. It is clear that this description of ductile cracking is entirely equivalent to that of the J resistance curve approach.

DISCUSSION

Because in the elastic-plastic regime displacements are not linearly proportional to load there is no simple way of allowing for prescribed displacement loading. The failure assessment diagram, and the functional form for J which is consistent with it (equation (3)) allow the problem to be treated, but the load displacement curve has to be independently evaluated. If the prescribed displacements are small enough for the structure to remain linear elastic this is not a difficult problem, but if the structure has entered the non-linear regime equations (4) or (5) must be solved.

However a simple pessimistic analysis which can be tried first is to plot the locus of assessment points as a function of postulated crack extension, Δa, at constant displacement assuming linear elastic behaviour. In this case S_r and K_r are evaluated using the elastically determined relaxed stress which is given by equations (4) and (5) with $J = J_1$ as

$$\sigma(a+\Delta a) \quad = \quad \delta/(\frac{\lambda_o}{A} + \frac{A}{E'}\ Z)$$

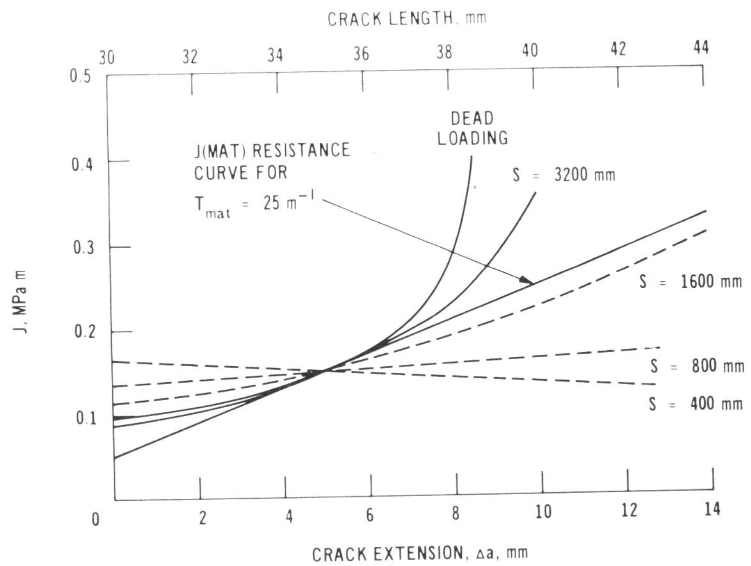

FIG. 3 (a) J RESISTANCE CURVE INSTABILITY ANALYSIS FOR CRACKED BEAM WITH PRESCRIBED DISPLACEMENTS. T_{mat} = 25 m^{-1}

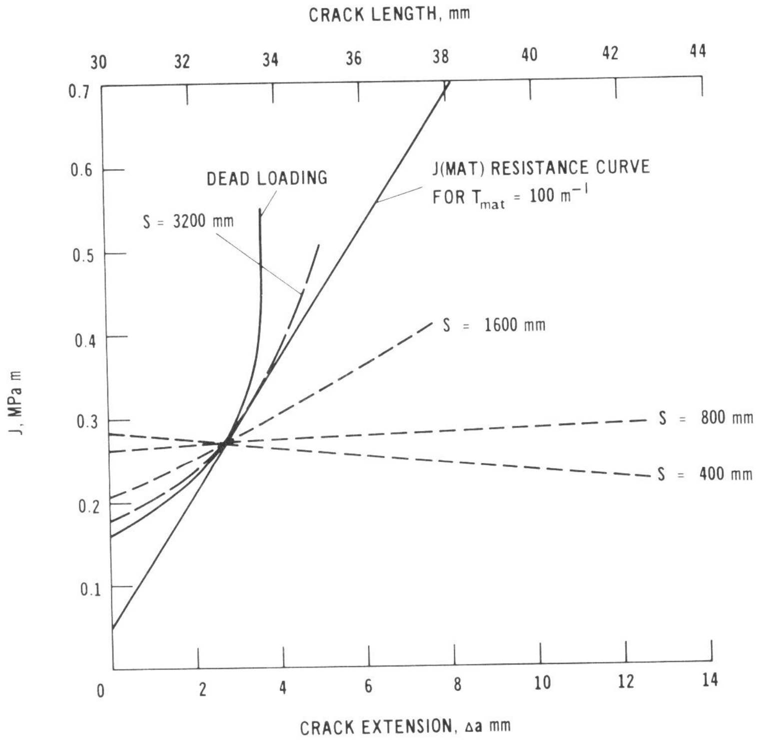

FIG. 3 (b) J RESISTANCE CURVE INSTABILITY ANALYSIS FOR CRACKED BEAM WITH PRESCRIBED DISPLACEMENTS. T_{mat} = 100 m^{-1}

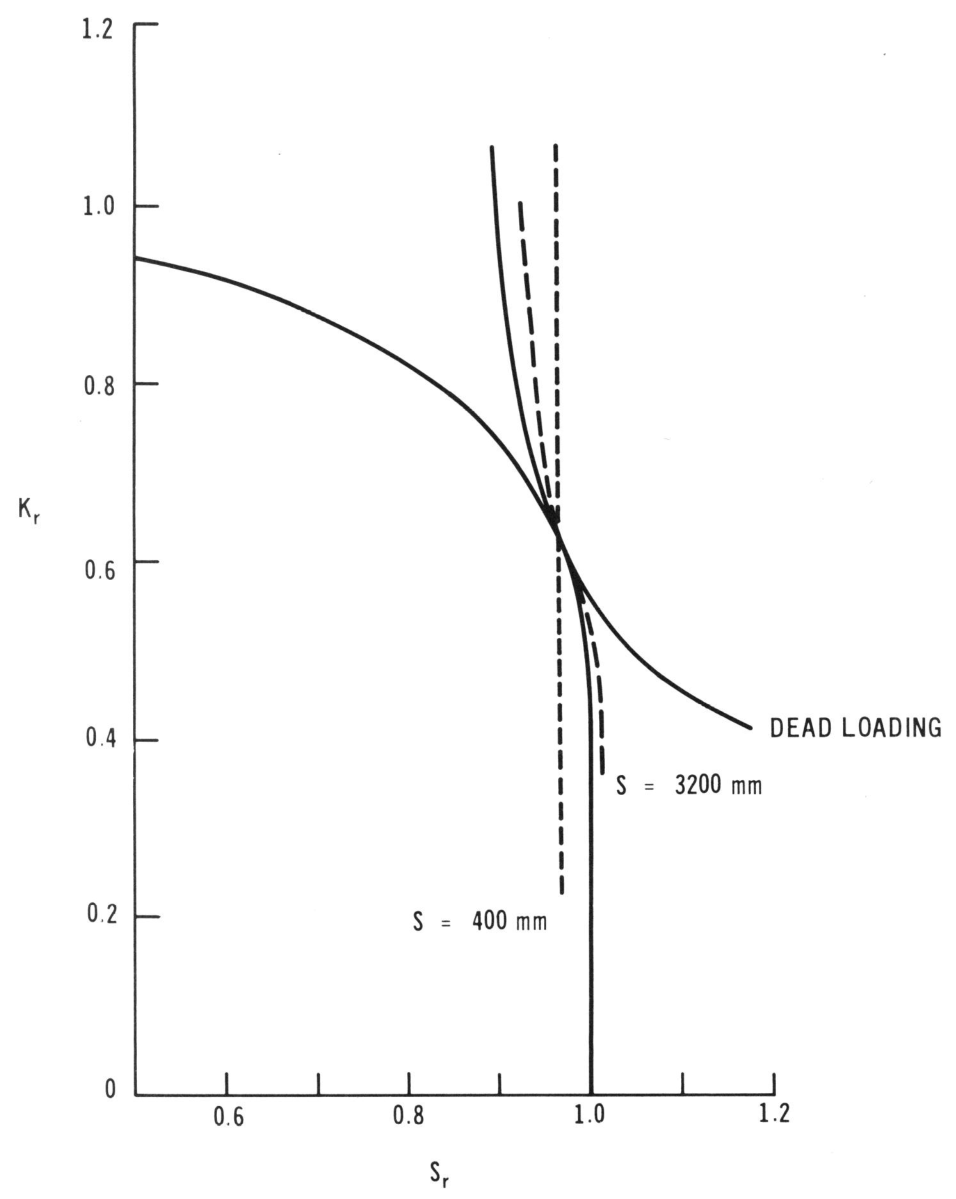

FIG. 4 THE STABILITY ANALYSIS FOR A CRACKED BEAM SUBJECTED TO PRESCRIBED DISPLACEMENTS IMPOSED OVER DIFFERENT SPANS

where $$Z = 2B\int_{0}^{a+\Delta a} aY^2 da$$

for the two dimensional case and

$$Z = 2\pi\int_{0}^{a+\Delta a} (R\pm a)\, aY^2 da$$

for axisymmetric geometries. Chell (1979) has demonstrated that the failure line in Fig. 1 is a lower bound to failure curves determined under prescribed displacement conditions using the foregoing approximation. Because of the assumption of a linear relation between load and displacement, a failure analysis can be performed on the locus of assessment points in a similar manner to the case for dead loading. If this analysis proves that adequate margins of safety exist then no further work is required. However, if adequate safety margins are not obtained and an elastic-plastic analysis is required then similar procedures to those described in the cracked beam example should be used.

CONCLUSIONS

(1) A ductile instability analysis can be performed under prescribed displacement loading using a modification to the procedure developed by Milne (1979) for prescribed loads.
(2) For prescribed loads the analysis requires the plotting of a locus of assessment points, as a function of postulated crack growth at the applied load, on the CEGB failure assessment diagram.
(3) For prescribed displacements the analysis requires the plotting of a locus of assessment points, as a function of postulated crack growth at the applied displacement, on the CEGB failure assessment diagram.
(4) In both cases stability is predicted if part or all of this locus falls within the failure assessment line.
(5) The instability condition is determined as the load or displacement at which this locus becomes tangential to the assessment line.

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REFERENCES

Chell, G.G., (1979), ASTM STP 668, 581.
Chell, G.G., and Ewing, D.J.F. (1977), Int. J. Fract. 13, 467.
Chell, G.G., and Milne, I. (1979), CSNI Specialists Meeting on Plastic Tearing Instability, NUREG/CP-0010, CSNI Report No. 39, 376.
Green, A.P., and Hundy, B.B., (1956) J. Mech. Phys. Solids 4, 128.
Harrison, R.P., Loosemore, K., and Milne, I. (1977), CEGB Report R/H/R6-Rev 1/1977.
Milne, I. (1979a) Mats. Sci. and Eng., 39, 65.
Milne, I., (1979b), 5th Int. Conf. on Structural Mechanics in Reactor Technology, Berlin, SMIRT 5 paper G1/3.
Paris, P.G., Tada, H., Zahoor, A. and Ernst, H. (1977) ASTM STP668, 5.