

CURRENT ASPECTS OF CRACK GROWTH UNDER MONOTONIC LOADING

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ABSTRACT

Fracture processes ahead of a crack tip, in a number of materials and testing conditions, can be divided into two classes: cracking and void-growth. Cracking is commonly observed in brittle solids, such as glasses, ceramics, some glassy polymers; and in transgranular and intergranular cleavage fractures in steel. In a number of cases, the macroscopic fracture-toughness, if greater than that corresponding to the elastic work-of-fracture, can be related to microscopic fracture events, in terms of a local fracture stress, generated at a position where a suitable crack-nucleus exists. Void-growth is the operative fracture mechanism in the fibrous fracture of steels, and non-ferrous alloys, in creep-crack growth and, perhaps, in the coalescence of "voids" in the crazed material ahead of a crack tip in some amorphous polymers.

KEYWORDS

Brittle Fracture; Fibrous Fracture; Void-Growth. Crazes. Fracture Toughness Intergranular Fracture. Steels; Ceramics; Glassy Polymers.

INTRODUCTION

The aim of the paper is to review briefly the micromechanisms by which fracture occurs ahead of a pre-existing crack tip and the ways in which they affect fracture toughness, drawing attention to similarities and differences between the behaviour of different materials. These topics have been the subject of the recent conference "Micro-mechanisms of Crack Extension" (Metals Society/Inst. of Physics, 1980). A general approach is to divide micromechanisms of fracture under monotonic loading into two groups - one involving cracking and the other involving void formation and linkage. The first group is typified by transgranular and intergranular cleavage fracture in both metallic and ceramic materials, whilst the second group includes ductile tearing and creep-crack growth in metallic materials and in a sense, craze-rupture in polymers.

For brittle materials, such as refractory metals, glasses and hard ceramics, the fracture toughness K_{IC} simply reflects the elastic work of fracture, which is conventionally equated to the surface energies of the two free surfaces created:

$$K_{IC} = \sqrt{2E' \gamma_s} \quad (1)$$

where γ_s is the true surface energy and E' is Young's modulus, E , in plane stress or $E/(1-\nu^2)$ in plane strain, ν being Poisson's ratio. In most structural metals, polymers and some polycrystalline ceramics, further energy-absorbing processes operate at a crack tip so that the fracture toughness is no longer determined simply by the true surface energy of the material. These energy-absorbing processes may be plastic-flow in metals, crazing in amorphous polymers or microcrack formation in poly-phase ceramics. In the first two cases they allow an initially sharp crack to blunt, thereby relieving the stress singularity at the crack tip. This can also be reduced by the formation of a number of microcracks. In many instances, fracture can still occur at a critical stress intensity factor (K_{IC}) provided that the conditions are such that stress intensity characterises the crack tip stress- and strain-fields. These conditions are generally termed "small-scale yielding" in metals. Dimensional considerations show that, if fracture occurs at a critical K value, the local fracture criteria must be satisfied over some microstructurally-determined "characteristic distance". This characteristic distance can be thought of as representing the smallest element of material in which the fracture process may operate. The relationships between the characteristic distance and microstructure will be used to illustrate the sources of fracture toughness for various micromechanisms of fracture.

Cracking Processes

The cleavage fracture of steels appears to follow a critical tensile stress criterion (Knott, 1966). Since tensile stress is associated with propagation of crack nuclei, this implies that cleavage fracture in steels is governed by the propagation of microcracks. In mild steels, grain-boundary carbide particles crack, either under the influence of an impinging slip band (Smith, 1966) or by a fibre-loading mechanism (Lindley, Oates and Richards, 1970), to give potential cleavage nuclei. In quenched and tempered steels, spheroidal carbides act as nuclei, producing "penny-shaped" microcracks (Hodgson and Tetelman, 1969; Curry and Knott, 1978). Carbide microcracks propagate into the ferrite matrix under the combined influence of the applied tensile stress and dislocation pile-up stresses, although the latter component makes a rather small contribution to the total fracture stress in typical microstructures. For both mild steels and quenched- and-tempered steels, experimental results indicate a value for the effective surface energy of ferrite of about 14Jm^{-2} . This is about an order of magnitude greater than the surface energy (Curry and Knott, 1978) and indicates that some dislocation movement must be associated with propagation.

Cleavage fracture also occurs in steels which possess a microstructure composed of lath-martensite or bainite. In low-carbon steel, these do not obviously contain discrete carbide particles but cleavage fracture still seems to obey a critical tensile stress criterion (Baldi and Buzzichelli, 1978; Curry, 1980a). The experimental results of Brozzo et al (1977) show that the cleavage fracture stress in low-carbon bainites is proportional to the reciprocal of the square-root of the bainitic packet size. If these results are interpreted by assuming the critical event to be the propagation of packet-sized microcracks, then an effective surface energy of 120Jm^{-2} is deduced. A microcrack crossing a single packet of bainitic laths may be arrested at the packet boundary when it encounters a bundle of differently oriented laths and Knott (1979) has proposed an upper bound to the toughness, in which an arrested microcrack propagates through unfavourably oriented laths by an internal necking process, in which the C.O.D. at the microcrack tip is set equal to the lath width. It was shown that this fracture micromechanism would result in an apparent surface energy for ferrite of the same order as that deduced from the experimental results of Brozzo et al (1977).

Intergranular fracture is frequently observed in steels in which residual elements such as P, Sb or Sn have segregated to prior-austenite grain boundaries. Recently the intergranular fracture stress has been measured as a function of the grain-boundary concentration of embrittling species (Kameda and McMahon, 1980). It was shown that the local fracture stress, as measured in a notched-bar test, was a function of the largest expected concentration of embrittling species at the grain boundary. The segregation of embrittling species is not, however, sufficient to produce by itself the reduction in effective surface energy, γ_p , necessary to explain the observed reduction in fracture stress. Jokl, Kameda, McMahon and Vitek (1980) have studied in a general manner the relationship between the effective surface energy, γ_p , and the chemical surface energy, γ_s , by assuming that dislocation emission at the tip of a propagating microcrack is a necessary part of the fracture process. Their model shows γ_p to be sufficiently sensitive to γ_s to explain the observed reduction in fracture stress in terms of the segregation-induced reduction in grain boundary work-of-fracture, γ_s . It also predicts that γ_p for transgranular cleavage in iron should be about an order of magnitude greater than the chemical surface energy, γ_s , in agreement with experimental observations on mild steels and quenched-and-tempered steels.

A criterion for the onset of cleavage fracture ahead of a sharp crack is that the maximum tensile stress exceeds the fracture stress over some microstructurally determined characteristic distance (Ritchie, Knott and Rice, 1973). The fracture toughness of a steel is then the stress intensity that must be applied to achieve the critical combination of stress and distance. Many aspects of the fracture behaviour of ferritic steels can be explained by applying this fracture criterion (Curry, 1979a,b; Milne and Chell, 1978). The "characteristic distance" was originally interpreted as a measure of the amount of material needed to be sampled to find a cracked carbide particle of greater than the critical size. In more detail, the local fracture stress around a cracked carbide particle is determined by the size of the particle. Since there is inevitably a range of carbide sizes in any microstructure, there are, clearly, spatial variations in the fracture stress. In the absence of microstructurally-significant stress-gradients (i.e. in tests made on bars containing blunt notches) the measured fracture stress is simply the lowest stress that could produce fracture and is therefore determined by the size of the largest microcrack. A carbide-microcrack can propagate across one or two ferrite grains before it encounters a large change in cleavage-plane orientation. Except at very low temperatures, therefore, the propagating crack arrests at the first grain boundary and is blunted by plastic deformation in the next grain. The initiation process has to be repeated ahead of this crack before it can repropagate and cleavage crack propagation therefore tends to involve a series of dynamic reinitiation events. Catastrophic crack propagation requires that fracture is nucleated, more or less sympathetically at many points across the crack front. The statistical sampling processes involved in crack propagation are restricted to a volume element of perhaps one or two grains in thickness so that they become essentially two-dimensional.

The stress gradients ahead of a loaded, sharp crack lead to significant variations in stress over typical carbide particle (microcrack) spacings and fracture is therefore controlled by a statistical competition between crack nuclei (cracked carbides) of different sizes. Fine, numerous carbide particles can cause fracture only when subjected to high tensile stresses and hence when found closer to the crack tip than the less-numerous, larger carbides. A statistically-based model of cleavage fracture (Curry and Knott, 1979) considers the contribution to the fracture probability arising from the full range of carbide sizes present in the steel. This model gives K_{IC} predictions that are in reasonably good agreement with experimental results for a range of different testing temperatures and microstructures in quenched-and-tempered steels. It has recently been shown (Curry, 1980b) that the Ritchie, Knott and Rice model of cleavage fracture is compatible

with this statistical analysis. So long as the characteristic distance is less than the plastic-zone-size at fracture, a combination of fracture stress and characteristic distance can always be chosen to represent the statistical competition between different sized crack nuclei at a loaded crack tip.

The Ritchie, Knott and Rice model can also be applied to analyse intergranular fracture (Ritchie, Geniets and Knott (1973); Jokl et al. 1980)) when the characteristic distance is again a largely empirical parameter determined by the microstructure in a rather complicated fashion. It is of interest to note that the fracture stress relevant to intergranular crack growth is apparently governed by the mean grain boundary concentration of the embrittling species, rather than by the highest expected concentration (Kameda, to be published). As for transgranular cleavage fracture, it seems that this could be a result of statistical sampling. Intergranular fracture proceeds along one or two grain-boundaries only, before encountering changes in grain-boundary orientation and surface energy (resulting from changing grain-boundary concentration of segregant) which cause the crack to arrest in the steep stress gradient ahead of the main crack. This behaviour should be contrasted with that of timber and of ceramics to which the classical three-dimensional 'weakest link' ideas apply (Weibull, 1951; Davidge, 1979; Barrett, 1976). In these materials, increasing the specimen thickness increases the volume of material sampled at the crack tip and so the mean fracture toughness decreases with increasing specimen thickness even when the specimen is large enough to provide plane-strain conditions across virtually all the crack fronts.

The micromechanisms of fracture in ceramics and their relationships with fracture toughness have been reviewed by Pratt (1980). In the absence of energy-dissipating crack-tip processes the fracture toughness is determined directly by the surface energy, cf. equation (1) (e.g. Gilman, 1960). Plastic deformation can, however, occur at a crack tip both at high temperatures and in the softer, cubic ceramics, giving rise to an increase in toughness above that predicted by the true surface energy (Freiman, Becher and Klein, 1975). This behaviour is similar to that in steels. In low-ductility, coarse-grained polycrystalline ceramics failing predominantly by cleavage the fracture toughness can be determined by the single-crystal cleavage surface energy adjusted to allow for grain-to-grain misorientation effects. For example, CaF_2 of 2mm grain size when failing predominantly by cleavage has an apparent surface energy ~ 3 times the single crystal value (Freiman et al. 1975). Generally, however, polycrystalline, anisotropic ceramics fail in an intergranular fashion over a wide range of grain sizes and the fracture toughness is then determined by the grain-boundary cohesion. In contrast to metallic grain-boundaries, boundaries in ceramics can be strengthened by the segregation of impurities (Class and Machlin, 1966) although they are also weakened by the presence of porosity (Ku and Johnston, 1964) and by the precipitation of second-phases. Internal stresses arising from anisotropy in thermal-expansion can contribute to fracture, thereby reducing the apparent fracture toughness. An interesting toughening process, observed in many ceramics, involves the formation of microcracks ahead of the main crack (Evans, Heuer and Porter, 1977). Microcrack formation may be assisted by thermal expansion stresses alone (Kuzyk and Brandt, 1973) or by density changes associated with phase transformations in second phase particles (Claussen, Steeb and Pabst, 1977). The microcracks influence toughness in two ways analogous to the effects of crack tip plastic deformation in metals. Firstly their formation can act as an energy-absorbing process at the crack tip: secondly, they give rise to a region of reduced elastic modulus which lowers the crack tip stresses and so causes an elevation in toughness.

Fracture Processes Involving Void Formation and Growth

Ductile fracture in metals proceeds by the linkage of microvoids formed at second-

phase particles. In low-strength steels, voids form by the separation, often at very low local strains, of the interface between the ferrite matrix and non-metallic inclusions, which are often sulphide particles. Carbide particles, whether spheroidal, rod-like or lamellar, can also nucleate voids either by interfacial separation or by particle-cracking (Curry and Pratt, 1979). Similarly the intermetallic dispersoids in aluminium alloys crack or cavitate to provide sites for void nucleation (Gurland and Plateau, 1963; Knott, 1980). In high work-hardening capacity metals the voids grow under an applied stress until they coalesce by the process often termed 'internal necking' (Cottrell, 1959). Generally this failure process requires the development of high strains in the matrix around and between the voids. When the matrix has a low work-hardening capacity, plastic-flow can become localised so that the voids link by shear decohesion along planar slip bands, giving rise to the so called 'zig-zag fracture' (Beachem and Yoder, 1973; Clayton and Knott, 1976). An intermediate form of void-linkage involves shear-decohesion around the logarithmic-spiral slip lines that form at a blunting crack tip.

The process of ductile fracture at a crack tip can be represented by the requirement that a critical strain, dependent on the local hydrostatic tension, should be exceeded over a characteristic distance (McKenzie, Hancock and Brown, 1977). The characteristic distance might be expected to be the void-spacing, and therefore the inclusion spacing, but it has generally been found to be some small multiple of the inclusion-spacing (Pandy and Banerjee, 1978; Ritchie, Server and Wullaert, 1979). Void-growth is strongly dependent on hydrostatic tension (McClintock, 1971; Rice and Tracey, 1969) and in consequence, when the void growth strain (as opposed to the void nucleation-strain) dominates, the critical strain is expected to be similarly dependent on hydrostatic tension. Experimental observation supports this prediction (McKenzie et al. 1977). Plastic strains decrease rapidly with increase in distance away from a blunting crack tip whilst simultaneously the hydrostatic tension increases from the crack tip out to the tip of the blunting region. The combination of decreasing strain and increasing hydrostatic tension results in a peak in the (non-interacting) void-growth-rate a distance of $1.1 \times \text{COD}$ ahead of the crack tip (Knott, 1980).

The growth of a pre-existing void and its coalescence with a blunting crack tip have been modelled in two dimensions by Rice and Johnson (1970). The predictions of this model seem to provide a good representation of initiation toughness in low-strength steels, but the experimentally-determined initiation toughness in high-strength or pre-strained steels can be much lower than the predictions (Knott, 1980). Several conflicting factors operate to cause discrepancies between the prediction of this model and experimental results. Firstly, the nucleation-strain has been neglected and where it is finite this would lead to an underestimate of the true toughness. Secondly, as previously mentioned, void-growth may be interrupted when shear decohesion allows premature void linkage and so prevents the development of the full predicted toughness. The toughness will therefore be reduced by those factors, such as prestraining (Clayton and Knott, 1976) or neutron irradiation (Loss et al. 1979) that reduce work-hardening capacity and so promote shear decohesion. Shear decohesion often involves the separation of carbide-matrix interfaces in sheets of fine carbides in high strength steels. Segregation of residual elements to these interfaces can reduce the interfacial strength and is observed to cause a significant degradation in the already low toughness, producing in effect ductile "temper-embrittlement" (King, 1979). Finally the Rice and Johnson model considers void growth in two dimensions only. Some aspects of the growth in three dimensions of spherical voids are discussed by Knott (1980); difficulties include the definitions of void-spacing and of fracture initiation in three dimensions. The presence of elongated inclusions introduces a strong orientation dependence of fracture toughness (Willoughby, Pratt and Turner, 1978). Elongated inclusions perpendicular to both the crack propagation direction and the crack front act as crack stoppers in a

manner similar to that of weak interfaces in composite materials (Harris, 1980) and are associated with high toughness levels. In contrast, low-toughness "lamellar-tearing" occurs when the inclusions are elongated in the direction of crack propagation.

Creep-crack growth in metals at high temperatures involves the formation and growth of intergranular cavities. When high stress-levels promote power-law creep deformation of the bulk material, creep cavities grow by viscous flow (Beere and Speight, 1978; Edward and Ashby, 1979). The rate of void growth and the strain to fracture in a region of hydrostatic tension such as that ahead of a crack tip have been predicted by Cocks and Ashby (1980). The total strain to fracture depends on the initial volume fraction of cavities, their spacing and the grain size. Although it has not been investigated, it would seem reasonable to take the characteristic distance, necessary to relate local fracture strain to crack growth-rate, as the cavity spacing. When stress levels are low, void growth proceeds by stress-assisted diffusion of vacancies into the cavities (Hull and Rimmer, 1959; Speight and Harris, 1967; Raj and Ashby, 1971. Vitek (1980) has analysed diffusional cavity growth ahead of a crack tip and concludes that this is likely to be controlled by grain-boundary diffusion. The diffusional-controlled creep-crack growth-rate was predicted to be proportional to K^n , where $2 < n < 4$. There is, of course, also an intermediate regime in which both power-law creep and diffusional processes assist the growth of the cavities, and, here, it would be expected that shorter times to failure and higher crack-growth rates would be found than would be predicted for either growth mechanism in isolation (Beere and Speight, 1978; Edward and Ashby, 1979; Rice and Needleman, 1980).

A failure mode involving stable void formation is observed in "crazes" in thermoplastic polymers (Kinloch, 1980). Voids form ahead of a crack tip and grow until the regions between the voids have been drawn out to give "fibrils" of strongly oriented and heavily work-hardened material. These fibrils transfer load across the voided region. The craze extends either by further void formation and subsequent fibril-drawing at the craze tip or by the meniscus instability' mechanism proposed by Argon and Salama (1977). Craze-thickening may also occur under load, both by straining of the fibrils and by drawing further material from the craze surfaces into the existing fibrils (Kramer, 1979). Eventually the fibrils break down and the crack extends within the craze. Perhaps because of the porous nature of the craze allowing ready access to the crazed material with its high surface area to volume ratio, environmental effects on crazing are pronounced (Williams, 1980). The toughness of polymers failing by a crazing mechanism can be predicted by assuming that fracture occurs at a critical COD, which is typically 1-2 μ m (Williams, 1980). The craze profile is predicted by a Dugdale's (1960) model of a yielding crack tip if a constant craze stress is assumed to act across the craze surfaces (Weidmann and Doll, 1978). The temperature-dependence and crack-velocity-dependence of K_{IC} can be explained from those of Young's modulus; the craze stress is less than the yield stress and largely temperature independent. If fracture toughness is interpreted in terms of a critical stress and a characteristic distance, the stress becomes the craze-stress and the distance the length of craze ahead of the crack tip (Williams, 1980). These parameters are analogous to the yield stress and the plastic zone size in metallic materials. They are controlled by the stress intensity factor needed to achieve the critical condition for craze breakdown, which is probably strain-controlled. In the absence of crack-tip crazing, Williams has shown that polymeric fracture can again be interpreted in terms of a critical stress and distance. Then the stress is of the order of the Van der Waal's bond strength in polymers and the distance reduces to the bond spacing.

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