

# CORRELATION BETWEEN THICKNESS AND CRACK LENGTH AS A MEANS OF ACHIEVING PLANE STRAIN CONDITIONS

**P. M. S. T. de Castro\* , J. Spurrier\*\*  
and P. Hancock\*\***

**\* Departamento de Engenharia Mecânica, Faculdade de Engenharia da  
Universidade do Porto, Portugal**

**\*\* Department of Materials, Cranfield Institute of Technology, Cranfield,  
Bedford MK43 0AL, UK**

## ABSTRACT

Both crack opening displacement (COD) and J integral tests indicate that the measured fracture toughness depends on the crack length/specimen width (a/W) ratio for temperatures at and above the transition temperature. A correlation between the levels of through thickness strain and the corresponding levels of fracture toughness, together with a graphical technique relating thickness and crack length indicate that there may be a possibility of using deeply cracked specimens to determine lower bound levels of fracture toughness in the elastic plastic regime.

## KEYWORDS

elastic plastic fracture, state of stress, through thickness strain, crack opening displacement, crack length/specimen width (a/W) ratio

## INTRODUCTION

In previous publications by the present authors (de Castro, Spurrier and Hancock, 1979, 1980), results obtained by testing a low sulphur BS4360 50D steel were presented and the dependence of fracture toughness measurements on the crack length/specimen width (a/W) ratio was analysed using the two most widely accepted elastic plastic fracture toughness characterizing parameters, i.e. the crack opening displacement (COD) and the J integral.

Although their theoretical and experimental background are widely different, it was possible to conclude that at low temperatures both  $J_c$  and  $\delta_c$  were found to be insensitive to a/W, whereas at higher temperatures (near the transition temperature)  $J_c$  and  $\delta_c$  presented a dependence on the value of a/W of the same type as that found for  $J_m$  and  $\delta_m$  measured at upper shelf temperatures, i.e. fracture toughness was found to decrease with an increase in a/W ratio.

As pointed out by Broek (1974) crack opening displacement tests are especially used for tough materials where fracture occurs after general yield. As far as J measurements are concerned, Turner (1978) comments that if plasticity reaches the far boundary of a finite cracked body, there are intuitive reasons why this may affect the whole nature of the plastic zone and hence the crack tip. However, Turner recog-

nizes that there is not so far a definite statement one way or the other on this subject, although the computational results tend to support the view that  $J$  is still meaningful even if the plasticity has spread to the far boundary of the test piece.

The tests described in the work by de Castro, Spurrier and Hancock (1979, 1980) were carried out using three point bend specimens of square cross section, and the plastic deformation was found to spread to the far boundary of the test pieces. This was clearly displayed by the dulling of the polished surfaces of the specimens, and it is schematically represented in Fig. 1. An important feature of this deformation is the development of triaxial stresses that arise at or just below the notch tip. These are caused by the plastic component of strain at the crack tip attempting to contract with a contraction ratio of 0.5 (as required by the constant volume condition of plastic deformation) whilst the surrounding still elastic material contracts with a Poisson's ratio of 0.3. The restraint exercised by the elastic material in the, at first, small volume of plastically deforming material induces stresses across the width and through the thickness of the plastic zone. Neglecting the elastic strains in relation to the plastic, a condition of plane strain is approached. As the plastic zone spreads the restraining effect of the elastic material is removed further from the notch root so that more through thickness straining is allowed and some relaxation of stress through the thickness occurs. Conditions in the plastic zone relax some way towards plane stress conditions, although not attaining it except for very thin plate (and very thin plate bend specimens are likely to buckle and therefore are not usually tested). In the case of shallow notches ( $a/W$  less than approximately 0.2), it was observed that the plastic deformation spreads to the top surface (Fig. 1b) and this predictably removes the stress-strain conditions further from plane strain conditions. This is well documented, for example, in Fig. 2, where the high values of through thickness deformation attained for shallow cracked specimens are indicated.

As illustrated by the slip line yield solutions, triaxial stresses are induced in the case where the slip line pattern, as traced from a free boundary, is curved; and this is certainly so in the case of deeper cracks. Knott (1969) indicates that for a crack depth ratio greater than the critical value  $W/(W-a) = 1.42$  (i.e.  $a/W = 0.3$ ), a zero degree notch, i.e. a crack, will ensure that general yield is confined to the hinge field.

It is well known that minimum values of fracture toughness are to be found for plane strain conditions. Generally, measurements of fracture toughness may be correlated with the type of plastic deformation observed at fracture, as illustrated in Fig. 2. It is therefore interesting to explore further the relationship between thickness  $B$  and crack length as a means of achieving low values of through thickness deformation in fracture studies.

One way of achieving this consists of carrying out measurements of through thickness strain,  $e_z$ , on specimens of constant  $a/W$  ratio and varying thickness  $B$ , and on specimens of constant thickness  $B$  and varying crack length/specimen width ratio. By determining the thickness or crack length necessary to achieve the same value of through thickness strain  $e_z$  it is possible to express the effects of crack length in terms of the more familiar idea of thickness (or equivalent thickness, in this case). Some technique must then be developed to eliminate  $e_z$  in the relationship between  $B$ ,  $a/W$  and  $e_z$ , such that finally an  $a/W$  versus  $B$  relationship is obtained. This is achieved using a graphical technique, which will be explained in the following section.

## DATA FOR THE ANALYSIS

Measurements of through thickness deformation near the pre-existing crack tip at fracture, for square cross section three point bend specimens with constant  $a/W = 0.3$  are presented in Fig. 3. These measurements were carried out using a micrometer.

Measurements of  $e_z$  were carried out on specimens of constant thickness  $B = 18\text{mm}$  and varying  $a/W$  ratio. Several series of such tests were conducted at  $-80^\circ\text{C}$ ,  $-90^\circ\text{C}$ ,  $-100^\circ\text{C}$  and upper shelf temperatures ( $+20^\circ\text{C}$ ,  $-10^\circ\text{C}$  and  $-50^\circ\text{C}$ ). These results are presented in Fig. 4 and Fig. 5 as explained in the following paragraphs.

It is necessary to determine, for a given temperature of interest, the pair of values  $a/W$  (for constant  $B$ ) and  $B$  (for constant  $a/W = 0.3$ ) corresponding to the same value of through thickness strain  $e_z$ . Data obtained experimentally at a temperature of  $-80^\circ\text{C}$  may be found in Fig. 4. Figure 4a presents the through thickness strain  $e_z$  of specimens of constant thickness and varying crack length. Curves of  $e_z$  versus thickness  $B$  for standard specimens with  $a/W = 0.3$  must also be determined. This can be achieved by intersecting the curves of Fig. 3 with vertical lines, which correspond to a given temperature of interest. The data so obtained may afterwards be plotted as through thickness strain against thickness, for a given temperature. A similar pair of figures could naturally be obtained for  $\delta_c$  instead of  $e_z$  measurements.

Since the vertical axis of both Fig. 4a and Fig. 4b are plotted using the same scale, a straightforward graphical construction enables the determination of the desired equivalence between crack length and specimen thickness. This is exemplified in the case of specimens with  $a/W = 0.3$ . Entering the horizontal axis of Fig. 4a with  $a/W = 0.3$ , the corresponding value of through thickness strain  $e_z$  may be read on the vertical axis. Entering Fig. 4b with this value of  $e_z$ , the horizontal axis of this Figure will provide the corresponding value of specimen thickness,  $B = 18\text{mm}$ .

Since the data for specimens of constant thickness and varying crack lengths (Fig. 4a) was obtained by testing  $18\text{mm}$  thick specimens, and since the data for standard specimens of varying thicknesses (Fig. 4b) was obtained by testing a geometry with  $a/W = 0.3$ , it was obviously to be expected that these two points should be interconnected. More interesting are the relationships to be established elsewhere. For example, a further two values of  $a/W$  and  $B$  are related in Fig. 4 : entering Fig. 4a with a  $a/W = 0.85$ , one finds, following the same procedure, an equivalent thickness of  $B = 36\text{mm}$ .

Figures similar to Fig. 4 were obtained for other temperatures. Data of this type is collected together in Fig. 5, corresponding to the temperatures of  $-100^\circ\text{C}$ ,  $-90^\circ\text{C}$ ,  $-80^\circ\text{C}$ ,  $-50^\circ\text{C}$ ,  $-40^\circ\text{C}$  and  $-30^\circ\text{C}$ .

The use of the technique for correlating  $a/W$  ratio and thickness effects described above enables Fig. 6 to be plotted. In this Figure curves relating crack length/specimen width ratio and specimen thickness for the temperatures of  $-100^\circ\text{C}$ ,  $-80^\circ\text{C}$ ,  $-50^\circ\text{C}$ ,  $-40^\circ\text{C}$  and  $-30^\circ\text{C}$  are presented.

Bearing in mind that the data on specimens of varying crack lengths was obtained for a thickness  $B = 18\text{mm}$ , and that data for specimens of varying thickness was obtained

using the common ('standard') value  $a/W = 0.3$ , the following two points are clearly understandable : (i) the curves corresponding to the relationship between  $a/W$  ratio and thickness  $B$  present a common point of coordinates at  $a/W = 0.3$  and  $B = 18\text{mm}$ ; (ii) since no  $a/W$  effect was found at  $-100^{\circ}\text{C}$ , the relationship of  $B$  versus  $a/W$  for this temperature is a straight line parallel to the horizontal axis up to  $a/W = 0.85$ , the highest value of crack length actually tested.

#### SUGGESTED MODEL FOR THE INTERPRETATION OF THE DATA

The parts of the curves of Fig. 6 shown as continuous lines describe actual experimental measurements, and a clear trend can be identified : in general an increase of crack length in a thin specimen has the same effect as far as through thickness deformation is concerned as an increase in thickness in a 'standard' specimen of  $a/W = 0.3$ . It may be observed from Fig. 6 that, for  $a/W$  values greater than 0.3, an increase in  $a/W$  always corresponds to an increase in specimen thickness for those temperatures where  $a/W$  effects were detected. Comparing the curve obtained at  $-30^{\circ}\text{C}$  with that corresponding to  $-80^{\circ}\text{C}$ , however, although the same overall trend is displayed, the  $-30^{\circ}\text{C}$  curve is a different shape, containing inflexions which are not displayed by the  $-80^{\circ}\text{C}$  curve. This difference is attributed to the fact that at  $-30^{\circ}\text{C}$  two different forms of failure behaviour were found, i.e. unstable fracture by cleavage for the thicker test pieces and ductile tearing for the thinner test-pieces, whereas at  $-80^{\circ}\text{C}$  all the data correspond to unstable fractures.

Practical difficulties prevent experiments with higher  $a/W$  values than approximately 0.85. These difficulties are related to the very low values of maximum load associated with pre-fatiguing of the specimens. Since the calibration of stress intensity factor  $K$  versus crack length in this region rises sharply, keeping a constant  $\Delta K$  level requires very frequent readjustments of maximum load, which rapidly fall outside the capabilities of conventional pre-fatiguing equipment. It was, however, considered interesting to make an educated guess of what would happen in this region. This is expressed by the interrupted lines in Fig. 6, which are traced to connect the experimentally obtained curves with the thickness necessary to obtain plane strain fracture toughness values at the temperatures of interest.

The plane strain fracture toughness data was obtained from a recent paper by Dawes (1977) where valid  $K_{IC}$  values for BS4360 50D steel are presented. It should be noted that Dawes' work is concerned with 50D steel with 0.018 sulphur, whereas the present work is based on 50D steel with a sulphur content less than 0.005. Since toughness increases with decreasing levels of sulphur, Dawes' data may constitute a lower bound of fracture toughness values for the present case.

From the  $K_{IC}$  data it is then possible to determine a plane strain value of thickness using the equation

$$B \geq 2.5(K_{IC}/\sigma_Y)^2$$

The values of plane strain thickness for  $-100^{\circ}\text{C}$ ,  $-80^{\circ}\text{C}$  and  $-50^{\circ}\text{C}$ , so obtained, are plotted on the vertical axis of Fig. 6. The axis is also expressed in terms of multiples of the thickness used as the basis of this work, 18mm. Hence it can be seen that at  $-100^{\circ}\text{C}$  the LEFM valid plane strain thickness is approximately twice 18mm, at  $-80^{\circ}\text{C}$  five times 18mm, and at  $-50^{\circ}\text{C}$  approximately thirty five times 18mm.

The extrapolation of the curves presented in Fig. 6 to high values of  $a/W$  suggests

that plane strain levels of 'equivalent thickness' may be attained if sufficiently deeply cracked testpieces are used. Since there is a correlation between the levels of fracture toughness and the mode of deformation, the present study suggests that the use of very deeply cracked testpieces may enable values of fracture toughness approaching those found under plane strain conditions to be obtained from specimens which are substantially thinner than those required by LEFM methods.

#### CONCLUSIONS

Measurements of plastic through thickness strain corresponding to fracture in the crack tip region of specimens of constant thickness and various fatigue crack lengths  $a/W$ , and on specimens of constant  $a/W$  ratio but varying thickness  $B$ , suggested a correlation between the thickness and crack length necessary to achieve a similar value of through thickness strain. It is suggested that these measurements could be related to the degree of plane strain at fracture.

Equating through thickness strains, an increase in the  $a/W$  ratio is equivalent to an increase in thickness for a specimen with  $a/W$  fixed at 0.3. The extrapolation of this relationship to very high values of  $a/W$  suggests that plane strain levels of 'equivalent thickness' may be attained if sufficiently deeply cracked specimens are used. Further work is continuing to identify the relationships, limitations and scope of this conclusion.

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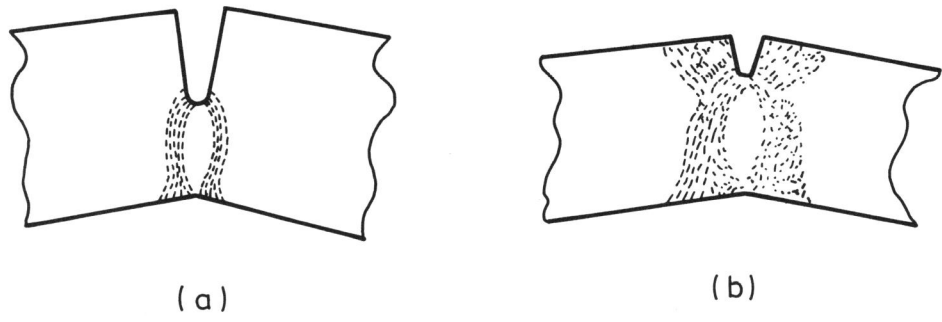


Fig. 1. Dulling of the polished surfaces generally observed (a) for  $a/W$  greater than 0.2, (b) for lower values of  $a/W$

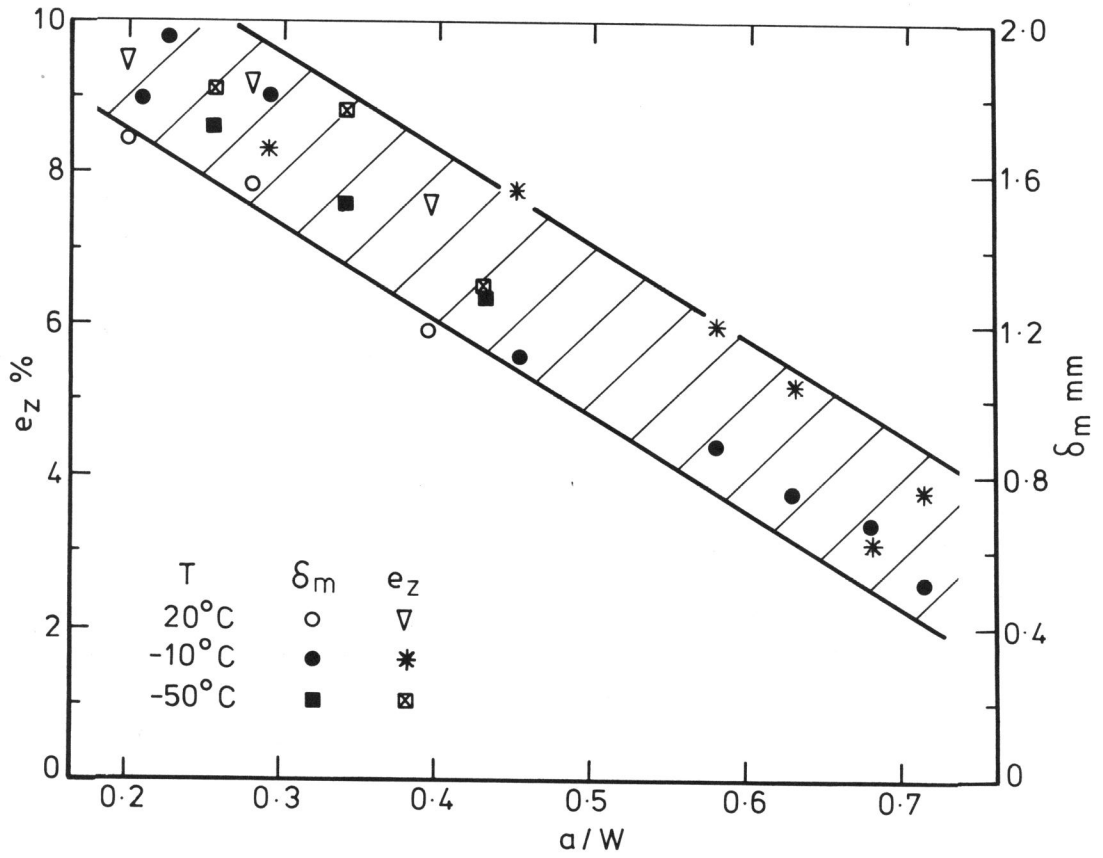


Fig. 2. Through thickness strain  $e_z$  and COD fracture toughness data for upper shelf temperatures

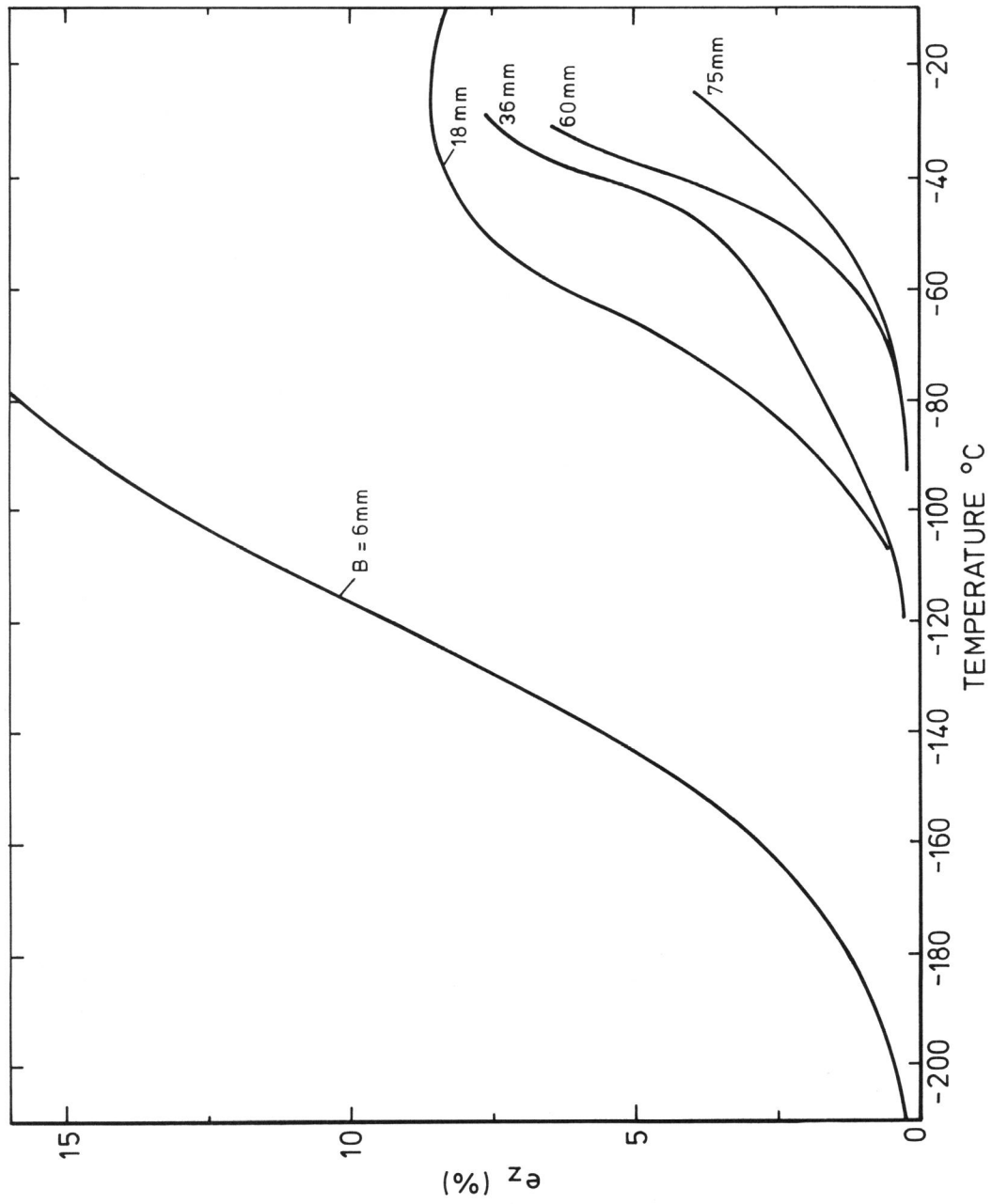


Fig. 3. Plastic through thickness strain  $e_z$  at fracture for specimens of various thicknesses

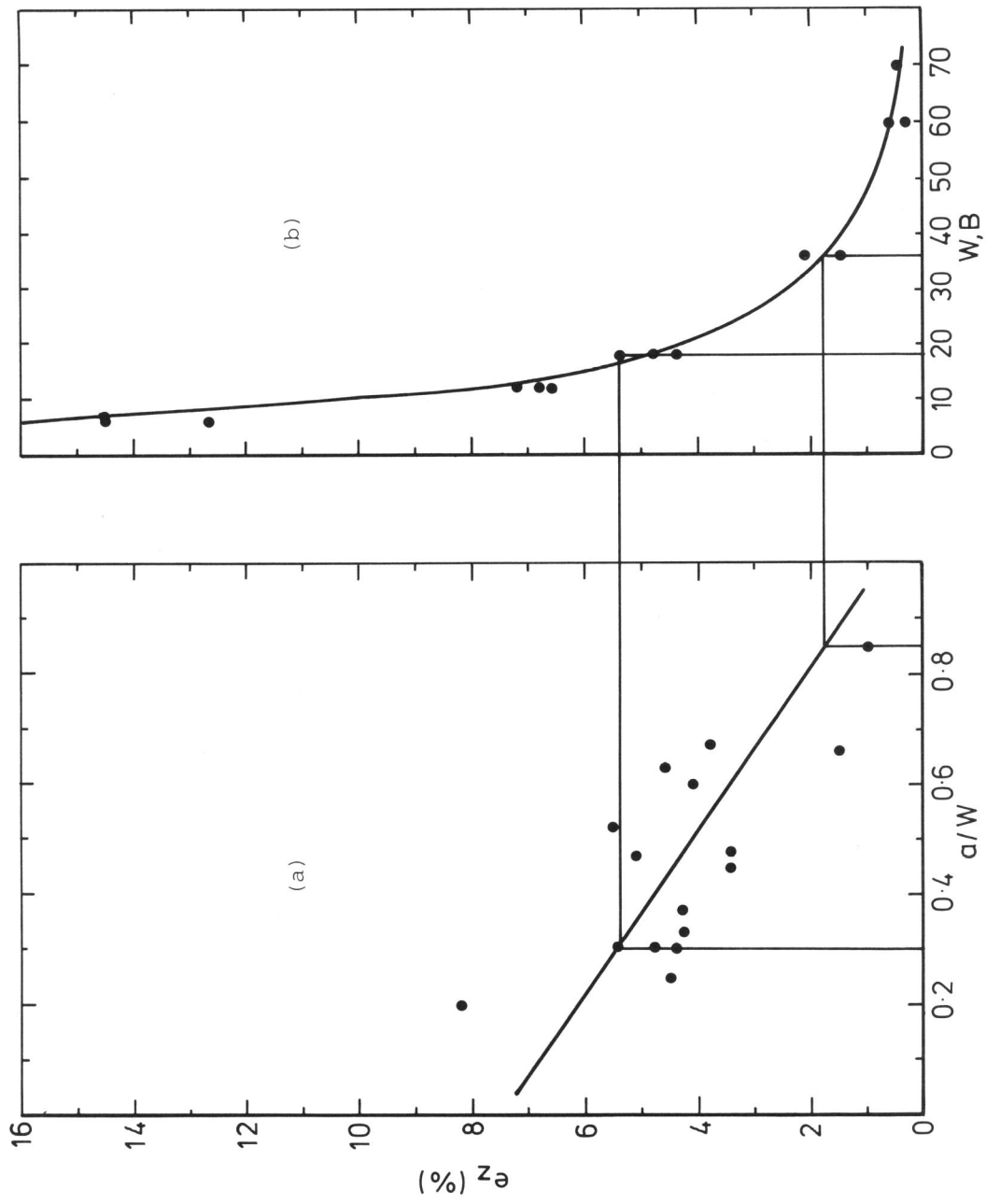


Fig. 4. Plastic through thickness strain at fracture at  $T = -80^{\circ}\text{C}$   
 (a) for specimens of constant thickness  $B = 18\text{mm}$  and various  $a/W$  values  
 (b) for specimens of constant  $a/W$  ratio ( $a/W = 0.3$ ) and various thicknesses.



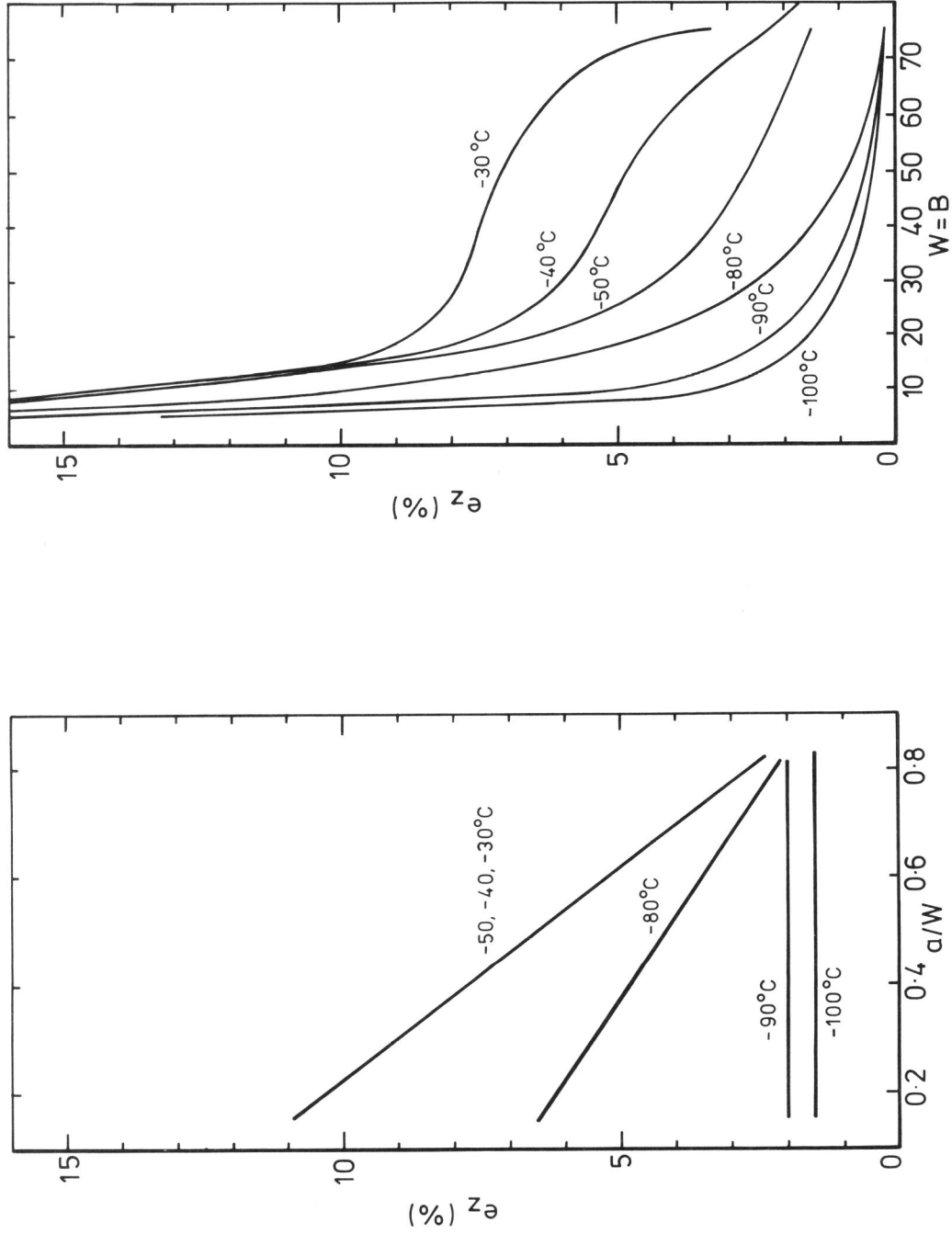


Fig. 5. Plastic through thickness strain  $e_z$  at fracture for tests at various temperatures

