# 15. THE GRAIN SIZE DEPENDENCE OF CLEAVAGE FRACTURE TOUGHNESS IN MILD STEEL

by

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# SUMMARY

Asymptotic crack tip stress distributions are combined with a recently proposed micro mechanistic fracture model to provide an analytical expression for the cleavage fracture toughness of mild steel in terms of microstructurally determined properties. The microstructural variation of these properties is represented by simply expressed grain size dependences. These dependences are used to predict analytically the grain size dependence of the fracture toughness of mild steel.

It is predicted that the toughness passes through a maximum at grain sizes in the region of 5µm and that further grain refinement leads to reduced toughness. The implications of this prediction are discussed in relation to the use of ultra fine grained controlled-rolled steels.

### 1. INTRODUCTION

A model has been proposed by Ritchie, Knott and Rice (1) that relates the cleavage fracture toughness of a mild steel to the steel's yield and fracture stresses through a microstructurally determined characteristic distance. This model quantitatively predicted the temperature dependence of the cleavage toughness in a mild steel and Parks (2) has since demonstrated its applicability to a low alloy steel. Ritchie, Knott and Rice investigated the characteristic distance for only one grain size. Curry and Knott (3) have since determined its value for a range of grain sizes in mild steel.

It has recently been demonstrated that there exists a general relationship between ferrite grain size and cleavage fracture stress in mild steels (4) and that this relationship is consistent with the accepted micro mechanistic model of cleavage in these steels (5). This note reports the results of an attempt to predict the grain size dependence of fracture toughness in mild steels using the Curry & Knott fracture stress relationship (4) together with their reported characteristic distance (3).

# CRACK TIP STRESS ANALYSIS

Hutchinson (6) has presented an asymptotic small scale yielding solution for the stress distribution at a crack tip in a non-linear elastic material. Considering a point at a distance x from the tip of a crack of length a, the maximum principle stress  $\sigma_{11}(x)$  at that point is given, in plane strain, by

$$\frac{\sigma_{11}(x)}{\sigma_{y}} = \theta \left(\frac{J}{\alpha I} \frac{1}{r}\right)^{1/N+1}$$
 (writing  $r = \frac{x}{a}$ ) ... (1)

where J is a non-dimensionalised value of the J contour integral to which the crack is loaded,  $\alpha$  the hardening constant and N the hardening exponent

in the Ramberg-Osgood stress strain law  $\left(\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \alpha(\frac{\sigma}{\sigma_y})^N\right)$  (7).

I is a numerical constant (weakly) dependent on N and  $\theta$  an angular dependence term whose maximum value is again dependent on N. This expression can be converted to dimensional quantities, with the loading expressed in terms of stress intensity K, to give

$$\frac{\sigma_{11}(x)}{\sigma_{y}} = \beta \left\{ x / \left( x / \sigma_{y} \right)^{2} \right\}^{-1/N+1} , \beta = \theta \left( \frac{(1-v^{2})}{\alpha I} \right)^{1/N+1} \dots (2)$$

where  $\beta$ , the amplitude of the stress singularity is available from Hutchinson's solutions.

The Ritchie, Knott and Rice model predicts that fracture will occur when  $\sigma_{11}(x)$  first exceeds  $\sigma_{\mathbf{f}}$ , the cleavage fracture stress, over a distance  $\mathbf{X}_{\mathbf{O}}$  equal to the characteristic distance. Making the appropriate substitution, equation (2) can be rearranged to predict the fracture toughness,  $\mathbf{K}_{\mathbf{TC}}$ , as

$$\kappa_{\text{IC}} = \beta^{-\left(\frac{N+1}{2}\right)} \qquad \kappa_{o}^{\frac{1}{2}} \left(\frac{\sigma_{f}^{\frac{N+1}{2}}}{\frac{N-1}{2}}\right) \qquad \dots (3)$$

### MATERIAL PROPERTIES

The grain size dependence of the yield stress in mild steel is well established, following the Hall-Petch relationship:-

$$\sigma_{\mathbf{y}} = \sigma_{0} + k_{\mathbf{y}} d^{-\frac{1}{2}} \qquad \dots \tag{4}$$

where  $\sigma_0$  is the internal friction stress,  $k_y$  a constant of value 0.74 MPa  $m^{\frac{1}{2}}$  (8) and d the grain size. It is generally held that the temperature dependence of  $\sigma_y$  arises solely from  $\sigma_0$ , with the constant  $k_y$  being independent of temperature while yielding occurs by dislocation motion (8).

McMahon and Cohen (9) demonstrated microscopically that cleavage failure occurs in mild steel when a crack in a grain boundary carbide propagates into the ferrite matrix. Smith analysed this situation theoretically to arrive at a microstructurally based cleavage fracture criterion (5).

$$\left(\frac{c_o}{d}\right)\sigma_f^2 + \tau_{eff}^2 \left\{1 + \frac{4}{\pi} \left(\frac{c_o}{d}\right)^{\frac{1}{2}} \frac{\tau_i}{\tau_{eff}}\right\}^2 \geqslant \frac{4E\gamma_p}{\pi (1-\nu^2)d} \dots (5)$$

where  $c_{o}$  is the carbide thickness, d the grain diameter,  $\tau_{eff}$  the effective shear stress,  $\tau_{i}$  the lattice (shear) friction stress, and  $\gamma_{p}$  the effective surface energy of ferrite. Noting that  $\tau_{eff} = k_{y}^{s}$  d<sup>-1</sup>, this fracture criterion can be expressed as

$$\sigma_{f}^{2} + \frac{k_{y}^{s^{2}}}{C_{o}} \left(1 + \frac{4}{\pi} \sqrt{C_{o}} \frac{\tau_{i}}{k_{y}^{s}}\right)^{2} \geqslant \frac{4E\gamma_{p}}{\pi(1-\nu^{2})C_{o}} \dots (6)$$

Thus it is predicted that  $\sigma_{\mathbf{f}}$  be independent of the ferrite grain size. By compiling all published data on fracture stresses at different grain sizes (Fig. 1), it can be shown that  $\sigma_{\mathbf{f}}$  varies markedly with changing grain size. Curry and Knott (4) have reconciled this discrepancy by demonstrating the existence of a general, non-linear relationship between ferrite grain size and carbide thickness in mild steels. When this empirical relationship is combined with Smith's fracture criterion, the predictions of Smith's model are seen to be in good agreement with the experimental data (Fig. 1). The scatter in the experimental data is probably due to differences in chemical composition between the different steels giving rise to fluctuation about the mean carbide thickness: grain size relationship.

Also shown in Fig. 1 is the line

$$\sigma_{\mathbf{f}} = k_{\mathbf{f}} a^{-\frac{1}{4}} \qquad \dots \tag{7}$$

for a value of  $k_f$  of 80 MPa  $m^{\frac{1}{4}}$ . This line provides a good description of both experimental results and the derived relationship, and this expression will be taken to represent the grain size dependence of  $\sigma_f$ .

Curry and Knott (3) have investigated experimentally the grain size dependence of the characteristic distance,  $X_{O}$ , in mild steel. It was shown that  $X_{O}$  was independent of grain size for grain sizes of less than 40µm: the following discussion is restricted to such grain sizes.

Lastly, it is noted that the work hardening exponent may be weakly dependent on grain size (refs. 10 and 11 present conflicting evidence on the grain size independence of N). As discussed below, the results of this analysis are insensitive to N and so this possible grain size dependence is neglected. If, in the Ramberg-Osgood stress-strain law  $\alpha$  is set to 1, then N becomes equal to the reciprocal of the conventional Hollomon work hardening exponent and hence is of the order of 10. Fig. 2 compares real and assumed work hardening behaviour.

# 4. FRACTURE TOUGHNESS PREDICTIONS

The recorded grain size dependences for  $\sigma_{\bf f}$  and  $\sigma_{\bf y}$  are substituted into the eqn. (3) to yield the predicted grain size dependence of the fracture toughness:

$$\kappa_{1C} = \beta^{-(\frac{N+1}{2})} \times_{0}^{\frac{1}{2}} \frac{(k_{f} d^{-\frac{1}{2}})^{N+1/2}}{(\sigma_{0}^{+} k_{y} d^{-\frac{1}{2}})^{N-1/2}} \dots (8)$$

Taking typical values of d = 20µm,  $X_O$  = 180µm,  $\alpha$  = 1,  $\theta$  = 2.4, I = 4.5,  $\sigma_O$  = 220 MPa, and N = 9, this predicts  $K_{1C}$  to be  $\simeq$  40 MPam when a value of 30 MPam would be more appropriate (3). It can be seen that for realistic values of work hardening exponent the predicted toughness is very sensitive to the chosen value of the singularity amplitude,  $\beta$ . A 50% increase in  $\beta$  leads to an 8-fold change in the toughness prediction for N = 9, more than sufficient to explain the above discrepancy. Since Hutchinson's stress analysis considers elastic strains to be negligible when compared with the plastic strain in the plastic zone, such an error in the singularity amplitude

is not improbable. In consequence it is unreasonable to use eqn. (8) to predict the absolute magnitude of fracture toughness as a function of grain size. However, the grain size dependence is independent of the amplitude,  $\beta$ , and so should be reliably predicted. Fig. 3 shows the product  $K_{\rm IC}\beta^{\,(N+1/2)}$ , calculated taking N = 9, as a function of grain size for three different  $\sigma_{\rm O}$  values, corresponding to temperatures at which cleavage is the probable fracture mode.

A maximum in toughness is predicted for all three temperatures, the grain size at which this maximum occurs increasing with increasing temperature. Differentiation of eqn. (8) shows this maximum to be located at

$$d^{-\frac{1}{2}} = \frac{\sigma_{o}}{k_{y}} \left( \frac{N+1}{N-3} \right) \qquad \dots \qquad (9)$$

The temperature dependence of the optimum grain size clearly arises from that of the friction stress,  $\sigma_{_{\scriptsize O}}$ . Although the turning point in toughness vanishes for N < 3, this does not affect the discussion since such a high work hardening exponent is not displayed by mild steels. The predicted optimum grain size is insensitive to N for realistic work hardening exponents (i.e. N  $\simeq$  10). Taking N = 9, the optimum toughness is given by a grain size of  $\sim$  3.5µm for  $\sigma_{_{\scriptsize O}}$  = 220 MPa (i.e. T  $\simeq$  -110°C), and for  $\sigma_{_{\scriptsize O}}$  = 110 MPa (i.e. T  $\simeq$  -50°C) the maximum is found at d  $\simeq$  15µm.

#### DISCUSSION

The prediction of a maximum toughness beyond which further grain refinement leads to a decrease in toughness is at the same time both surprising and rather reassuring. Surprising in that experience has suggested that grain refinement always produces a tougher as well as harder steel, and reassuring in that every other hardening process causes a reduction in toughness. This anomalous behaviour arises because in mild steels the cleavage fracture

stress is determined primarily by the carbide thickness, which, although metallurgically related to the grain size, is not directly proportional to it. In consequence the ratio of yield to fracture stress is seen to vary with grain size (cf. eqns. (4) and (7)).

Clearly one should not regard the predictions of this simple analysis as being anything other than speculative. It is based on an approximate stress analysis and, to some extent, on the extrapolation of existing experimental data. Nevertheless, dimensional considerations show the functional dependence of toughness on grain size to be correct and hence the location of the maximum toughness is unlikely to be significantly in error.

The implications of this study for the production and structural utilization of mild steels are considerable. Firstly, Fig. 3 shows that the grain size for maximum toughness varies with temperature. Thus it is predicted that an optimum grain size specified for one temperature may maximize toughness at that temperature, whilst leading to embrittlement at higher temperatures. Secondly, it is predicted that the optimum grain size lies in the range 3 to 15µm for typical temperatures at which failure would be expected to occur by cleavage, further grain refinement leading to hardening at the expense of reduced toughness. Controlled-rolled steels are currently being produced with grain sizes of the order of 5µm as a possible alternative to alloy steels for structural use. The prediction of the present study is that, at the upper end of the cleavage temperature range, a mild steel with a grain size of  $5\mu m$  will be less tough than one of  $40\mu m$  grain size. If the carbide - grain size and hence the fracture stress - grain size relationship in controlled-rolled steels is the same as that observed in normalised and annealed steels, then the efficacy of controlled-rolling as a general route to higher toughness steels is called into question.

Obviously the present predictions do not apply to controlled-rolled steels alloyed in such a way as to change significantly the carbide particle dispersion, for example giving  ${\rm V_4C_3}$  instead of Fe $_3$ C and hence changing the fracture stress-grain size relationship.

Knott (12) has drawn attention to the need for an understanding of the micro mechanisms of fracture in the design of materials with improved resistance to fast fracture. This study illustrates how quantitative microstructural modelling of failure can be combined with fracture mechanics analyses to make predictions about optimising a steel's toughness.

#### CONCLUSIONS

- Grain refinement may not always provide an inexhaustible source of enhanced toughness in steels. It is predicted that the grain size dependence of cleavage toughness passes through a maximum.
- 2. The grain size giving maximum toughness is predicted to vary with temperature within the range 3 to 16µm. If this is the case then the current quest to produce ultra fine grained (< 5µm) mild steels as an alternative to alloy steels for structural use may be unrewarding.
- There is an obvious need for thorough experimental investigation of the microstructural dependence of fracture toughness in the new fine grained steels.

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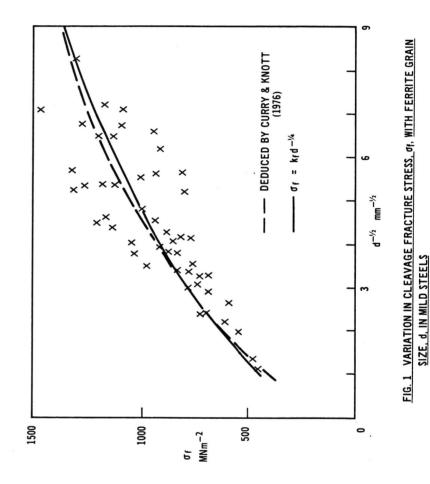
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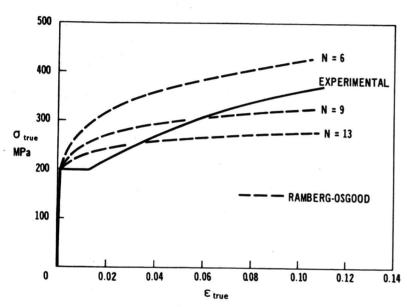


FIG. 2 COMPARISON BETWEEN REAL AND ASSUMED STRESS-STRAIN CURVES

EXPERIMENTAL DATA FOR MILD STEEL AFTER MORRISON (1Q)

