14. The determination of COD with the Dugdale model

K.-H. Schwalbe, DFVLR, D-5000 Köln 90

For sheets of infinite width the crack tip opening displacement can be calculated using the wellknown relation [1,2]

$$\delta = \frac{8\sigma_S}{\pi E} a \ln\left[\sec \frac{\pi\sigma}{2\sigma_S}\right] \tag{1a}$$

$$= \frac{8\sigma_S}{\pi E} a \ln\left[\frac{\omega}{a} + 1\right] \tag{1b}$$

where ω is the length of the plastic zone. The latter relation is assumed to be valid also in the case of finite widths provided the proper expression for ω is inserted. For finite widths Rice [3] gave the relation

$$\omega = a \left\{ \frac{2W}{\pi a} \arcsin\left[\sin \frac{\pi a}{2W} \sec \frac{\pi\sigma}{2\sigma_S}\right] - 1\right\} \tag{2}$$

Combining (1b) with (2) we obtain

$$\delta = \frac{8\sigma_S}{\pi E} a \ln \left\{ \frac{2W}{\pi a} \arcsin\left[\sin \frac{\pi a}{2W} \sec \frac{\pi\sigma}{2\sigma_S}\right]\right\} \tag{3}$$

i.e. the crack tip opening displacement for a sheet of finite width. Fig. 1 shows a normalized plot of δ. The influence of a/W is obvious. These results agree very well with the numerical calculations of Erdogan and Bakioglu [4].

Now this result will be used in the COD concept. Usually the crack mouth opening is measured and an extrapolation to the crack tip is made to determine a critical COD. In linear elastic fracture mechanics we have calibration functions for the determination of critical stress intensities from critical loads and specimen geometries. For the COD concept we have now also a calibration function for tension specimens of finite width, so on this specimen type we can determine a critical COD by measuring the critical load and the specimen dimensions. By this procedure we avoid the uncertainties of the extrapolation techniques.

To check the applicability of this idea residual strength test results on center cracked panels of aluminum alloys were used. From the test data of one crack length the critical stress was calculated as a function of a/W:

$$\sigma_c = \frac{2\sigma_S}{\pi} \arccos \left[\frac{\sin \frac{\pi a}{2W}}{\sin \left\{ \exp\left(\frac{\delta_c E\pi}{8a\sigma_S}\right) \frac{\pi a}{2W} \right\}} \right] \tag{4}$$

For σ_S the average value of yield strength and UTS was used to account for work hardening.

Fig. 2 shows some results. The fracture behaviour can be reasonably well predicted by Eq.(4).

In the case of thin sheets fracture occurs often under quasi linear elastic conditions at intermediate crack lengths. But at short and long cracks the net fracture stress approaches the yield strength.

The Feddersen concept offers one possibility to treat fracture problems in the elastic and in the elastic-plastic regime. The COD concept based on Eq.(4) offers another possibility with the advantage that a single relationship predicts fracture in both regimes. It is intended to extend this modified COD concept on compact and bend specimens.

References:

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[3] J.R. Rice, Mechanics of crack tip deformation and extension by fatigue, in: ASTM STP 415 (1967) 247

[4] F. Erdogan and M. Bakioglu, Crack opening stretch in a plate of finite width, Int. Journ. of Fracture 11 (1975) 1031

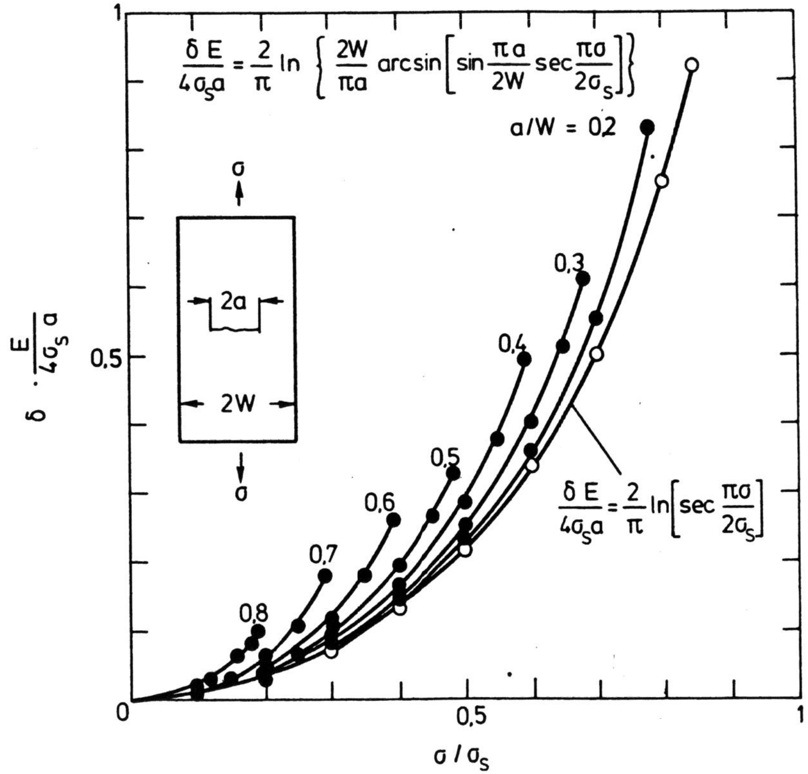

Fig. 1: Normalized crack tip opening displacement

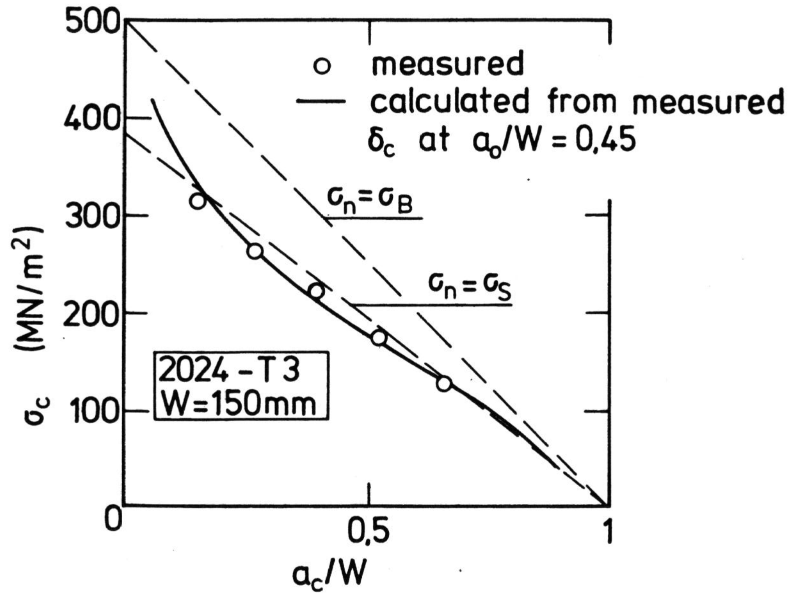

Fig. 2: Calculated and measured critical stresses for an aluminum alloy