13. Numerical Experiences with an Energy-Based Fracture Criterion

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This short contribution wants to show some numerical disadvantages of an energy-based fracture criterion. The criterion is a generalized Griffith criterion and was mentioned in the literature by several authors under different names. Light and Luxmoore made large numerical investigations on it in 1975 /1/. As it is not so commonly known as for example the J-integral, the theory will be explained in a few sentences.

Let 'a' be the original crack length of any specimen. We now let the crack extend by a small increment Δa. During crack-extension the displacement at the load point remains constant. In a FEM calculation this crack-extension can be realized by relaxing the nodes at the crack tip.

Looking at the energy before and after crack extension it is now possible to formulate a global energy balance which leads to the following equation:

$$Q = \left| \frac{\Delta U_e}{\Delta a} \right| - \left| \frac{\Delta U_{pl}}{\Delta a} \right| \tag{1}$$

ΔU_e = elastic energy released during crack extension

ΔU_{pl} = plastic work done during crack extension

Q = work to extend this crack

This is the notation used by Light and Luxmoore for their definition of the so-called Q-value.

It has to be noted that for the purely elastic case the following relationship is valid:

$$Q = J = G \quad (2).$$

A physical interpretation of the Q-value can be given as follows. Referring to fig. 1 we have one energy store which is the elastic energy. This store transfers during crack-extension some of his energy to two different consumers, namely to the plastic work done and to the energy necessary to extend the crack.

Light and Luxmoore found that the Q-value is more geometry independent than the J-integral.

In Aachen we made 2- and 3-dimensional elasto-plastic calculations on CT-specimens, using isoparametric parabolic elements and a first order displacement theory. A mesh generator enabled us to modify the FEM mesh over a wide range. Unfortunately, the results of this mesh variation show a big disadvantage of the Q-value. As it is plotted in fig. 2, the Q-value is a function of Δa. Q seems to go to zero for Δa against zero, as other authors found [3,4,5,6]. At this point it has to be mentioned that Luxmoore and Bleackley did not find this effect during their investigations of double edge notched and centre notched specimens /2/. They also modified Δa and found that the Q-value is nearly independent of Δa. At the moment we are not able to explain this contradiction.

Fig.2 shows that the Q-value,though it is rich in meaning from the mechanical point of view,looses some of its general validity. It only can be used with restrictions.

For a constant Δa we also find that Q behaves better than J. In fig. 3 the variation of Q and J with the thickness is compared. It can be seen that the Q-value is nearly independent of the thickness, whereas the J-integral which was calculated from a line integral shows a strong dependency. So, the Q-concept still can be used but not with the physical background, that was given above.

A Q-value which is dependent of Δa, can only be explained as an energy difference effected by a <u>defined</u> disturbance of the length Δa. This disturbance is given on to the displacement field and its influence is defined as a fracture criterion. From the numerical point of view this procedure might be justified, but it must be pointed out, that any physical interpretation of a so calculated 'Q-value' is without meaning.

References

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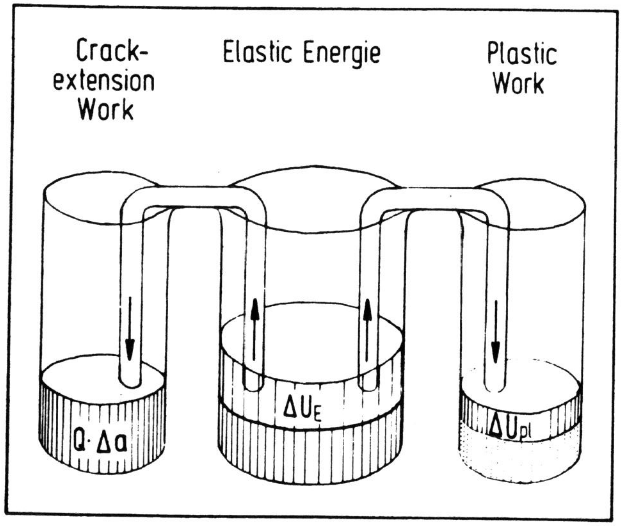

Fig. : 1

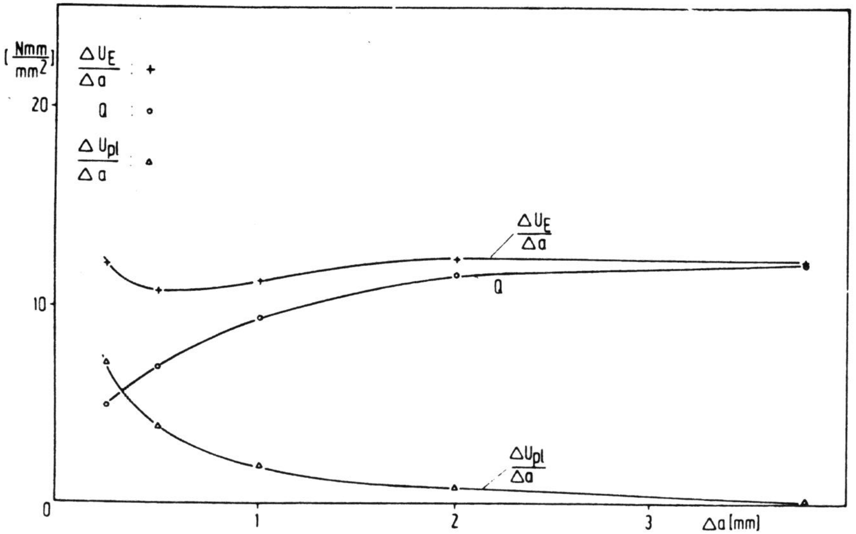

Fig. 2

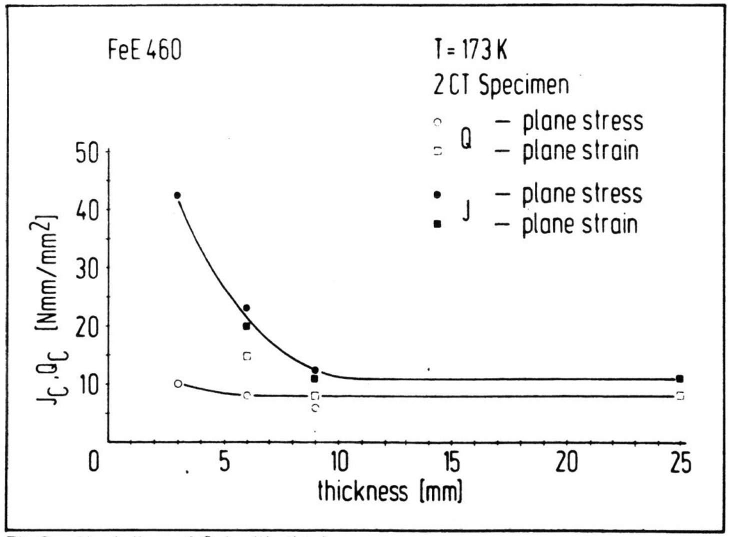

Fig.3 : Variation of Q,J with thickness