

# THE PHILOSOPHY OF CONSTRAINT CORRECTION BY THE GLOBAL APPROACH

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**Abstract.** It is now admitted that mechanical properties are sensitive to specimen geometry and loading conditions. This phenomenon is also well-known in modern global fracture mechanics as transferability problem and reflects the effect of specimen configuration, crack size, thickness and loading conditions on the Material Failure Curve (MFC). Fracture toughness dependence is often referred to the effect of crack and/or notch tip constraint. The fracture toughness is normally measured from tests of deeply edge cracked bend or compact tension specimens according to well-known testing procedures. Local conditions ahead of the notch tip are assumed to be plane strain with high constraint. However, crack tip constraint is reduced and fracture toughness is increased for specimens of small thickness or with notches. In this work, we have attention to create a framework for including constraint effects in global fracture mechanics approaches to explain the geometry and loading condition dependencies of the fracture toughness of specimens and structures with cracks and/or notches. The two-parameter fracture mechanics methodology (K and T-stress) seems to be very attractive for this purpose. This methodology suggests employing a constraint parameter in addition to the classical notch tip one parameter.

## Introduction

It is now admitted that mechanical properties are sensitive to specimen geometry and loading conditions. One of the most ancient confirmations of this phenomenon is the so-called scale effect on tensile properties which evidence has been noted by Galileo Galile and Leonardo da Vinci. This phenomenon is also well-known in modern fracture mechanics as transferability problem and reflects the effect of specimen configuration, crack size, thickness and loading conditions on the fracture toughness. Fracture toughness dependence is often referred to the effect of crack tip constraint. The fracture toughness is normally measured from tests of deeply edge cracked bend or compact tension specimens according to well-known testing procedures. Local conditions ahead of the crack tip are assumed to be plane strain with high constraint. However, crack tip constraint is reduced and fracture toughness is increased for specimens of small thickness or with notches. In recent years, there has been considerable effort to create a framework for including constraint effects in fracture mechanics approaches to explain the geometry and loading condition dependencies of the fracture toughness of specimens and structures with cracks. The two-parameter fracture mechanics methodology seems to be very attractive for this purpose (e.g. [1-4]). This methodology suggests employing a constraint parameter in addition to the classical crack tip one parameter, for example, the stress intensity factor or the J-integral. The stress intensity factor can be characterized by the following solution

$$K = f(a/w, \rho, T - stress, A_3, T^0, loading, \dots) , \quad (1)$$

where  $K$  is the stress intensity factor,  $a/w$  the depth of notch,  $T$  the constraint,  $T^\circ$  the temperature. The third term  $A_3$  is sometimes used as a transferability parameter like the T-stress.

### **Constraint**

The starting point in fracture mechanics analysis is to consider a crack of a certain size located in a component or specimen. An external load is applied and the component is loaded until it fails [5]. During loading a plastic zone develops from the crack tip, and at a certain load net section yielding occurs as the plastic zone reaches the through thickness surface. As long as the plastic zone at the crack tip is limited compared with the geometry of the component or specimen, so-called small scale yielding, a single parameter fracture mechanics approach can be applied.  $K$ ,  $J$  or CTOD characterizes the crack tip conditions and can be used as geometry independent fracture criterion. The pure tensile specimens have the lowest constraint, while specimens dominated by bending have the highest constraint. Standard fracture mechanics testing procedures are based on the specimens with high constraint in order to reproduce the worst case conditions. However, the single parameter fracture mechanics breaks down in the presence of excessive plasticity, and fracture toughness will now depend on the size, geometry and mode of loading. Several authors [6-10] have examined the near crack tip stress field under fully elastic or elastoplastic conditions for various specimen geometries and non-hardening materials. The history of constraint is how to deal with crack tip stresses under any fully conditions. The aim is to find different parameters that characterize the stress-strain fields, so that results from one test geometry can be transferred to another geometry. A more basic approach has been to define the crack tip triaxiality as the ratio between the hydrostatic stress and the Mises effective stress. Another constraint parameter is the T-stress, Larsson and Carlsson[11], Du and Hancock [12]. This is a non-singular linear elastic stress component parallel to the crack. The T-stress characterizes the local crack tip stress field for linear elastic material, and the global in-plane constraint of a specimen with respect to predominantly local small scale yielding conditions. The Q parameter, like the T stress, is supposed to characterize the geometry dependent constraint. Both quantities affect the hydrostatic stress in the same way, i.e. negative values lower, positive values raise the hydrostatic stress.

The standard gives literature references for constraint correction methods, based on T and Q, but none of these are included in the standard. When toughness is measured using standard procedures, it is possible to modify the material failure curve (MFC) to account for lower constraint. Alternatively, it is possible to maintain the use of a high constraint of MFC and account for lower structural constraints using appropriate test geometries. There is evidently a need to present a framework for a practical application of constraint corrections. The global approach, presented by the volumetric method [13], quantifies the notch tip stress fields in dependence of geometry (size, crack depth, global geometry and mode of loading). The Approach is based on the existing  $K$ - $T_{ef}$  theory and the RKR brittle failure criterion. The constraint of the different fracture mechanics specimens can now be presented as  $K$  vs  $T+A_3$  (Williams's equation). The fracture toughness obtained under standardized high constraint conditions can be transferred to more structural relevant lower constraint conditions. The methodology will now be presented with reference to the X52-steel investigated in the paper.

### **Effect of T-stress on the Fracture Toughness**

The non-singular term  $T$  represents a tension (or compression) stress. Positive  $T$ -stress strengthens the level of crack tip stress triaxiality and leads to high crack tip constraint while negative  $T$ -stress leads to the lost of constraint. Therefore, the  $T$ -stress can be suggested to be a constraint crack tip

parameter. The following effects characterizing the T-stress should be noted. Rice [17], Larsson and Carlsson [18] have shown that sign and magnitude of the T-stress substantially change the size and shape of the crack tip plastic zone. Positive or negative the T-stress increases the plastic zone size comparing with no T-stress situation. Analytical and experimental studies carried out by Sumpter [19] demonstrate that the T-stress can be used as a measure of constraint for plastic yielding ahead of the crack tip. Chao et al [20] and Hancock et al [21] reported that the fracture toughness increases with the increase of  $(-T)$ . At the same time, T-stress estimations ahead of the notch tip and the T-stress effect on the notch fracture toughness are strongly limited in the literature.

### Influence of the notch depth

The ASTM E-399 [22] testing procedure recommends certain types of specimen geometries and  $K_{IC}$  can be considered as the plane-strain fracture toughness. All specimen geometries recommended by ASTM E-399 are high constraint. Using the recommendation specimen geometry for testing creates an “ASTM Window” since their corresponding  $T$  or  $A_3$  values are within a certain range ( $0.45 < a/w < 0.55$ ). A  $K_{IC}$  value is believed to represent a lower limiting value of fracture toughness and the ASTM E-399 may not be generally valid. Increasing the size of a specimen shifts the stress distribution closer to the K-stress. Consequently, larger specimens tend to possess better K-dominance. This may explain why a large specimen is better suited for ASTM fracture toughness  $K_{IC}$  testing in addition to the reason for the plastic zone size. This phenomenon limiting the recommendation of ASTM and can be explained using the analytical  $K-T$  or  $K-A_3$  relation for common the effects of specimen geometries.

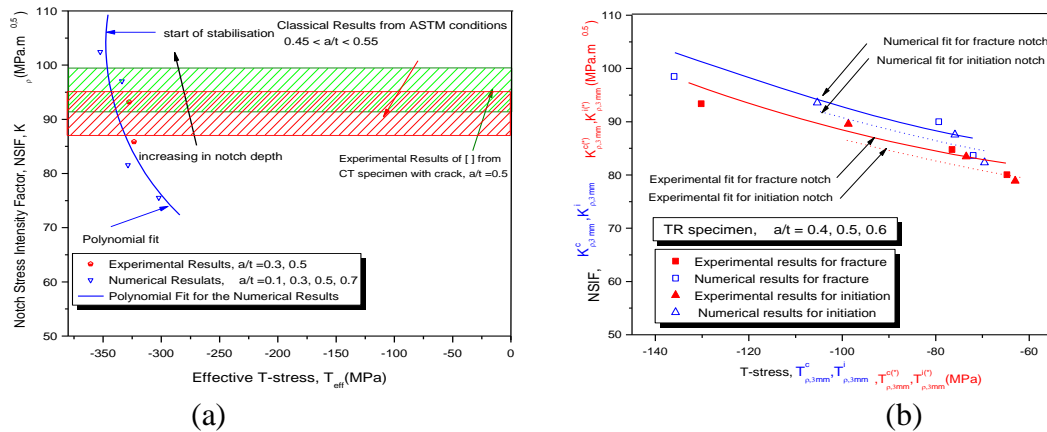


Figure 1. The results of  $K_{\rho,c} - T_{ef,c}$  estimations by numerical and experimental methods for the (a) CT and (b) RT specimens.

The influences of the notch depth on the material failure curve are presented in figure 1. The results of  $K_{\rho,c} - T_{ef,c}$  estimations by numerical and experimental methods, at the initiation and fracture, are presented in Fig. 2 for the CT and RT specimens with the notch aspect ratio  $a/w = 0.1; 0.3; 0.5$  and  $0.6$ . It can be seen that the line method compared with experimental estimations gives the lowest constraint value (greater  $-T$  value) and consequently the highest notch fracture toughness. The maximum difference in the notch fracture toughness as well as the effective T-stress does not exceed 10%. This difference can be explained by the experimental scatter.

### Influence of the geometry of specimen

The main idea of this section is that Irwin's fracture criterion should be modified introducing  $K_{IC}$  as a function of T-stress (or, in this case, effective T-stress measured some distance ahead of the crack

tip) for different laboratory geometry specimens. Since end of 50's when Griffith-Irwin criterion was introduced and LEFM was formed in its present state we all observed numerous attempts to "correct" critical stress intensity factor. There were attempts to introduce "geometric" corrections or corrections accounting for more accurate representation of the asymptotic field surrounding the crack tip. So far, all these corrections (though sometimes very useful for specific problems, materials or geometries) are not able to provide anything improving Irwin's approach in "general situation". In this section, reviewed solutions are doing exactly the same thing. Should some authors [23-25] establish range of problems (materials, geometries) where the proposed  $K_{IC}$  correction can significantly improve critical load predictions, the result could be very useful. Unfortunately, some others [26-30] are not trying to do this, but trying to claim that they had found a kind of "universal" correction to critical stress intensity factor concept. This is done using experimental results from one single material. Notch fracture toughness transferability has been proposed as a  $K_{\rho,c} - T_{eff,c}$  curve and established from the tests of four specimen types (CT, SENT, DCB and RT) made from X52 pipe steel. A material failure curve  $K_{\rho,c} = f(T_{eff,c})$  is established for the specimens under consideration. Fracture conditions are then given by the intersection of the material failure curve and fracture driving force curve for gas pipes with the surface notch. An example of the evolution of the notch stress intensity factor with the presence of the T-stress parameter is given in the Figure 2 for CT, SENT and TR laboratory specimens. The experimental assessment points ( $K_{\rho,c}, T_{eff,c}$ ) for four specimen geometries (CT, SENT, RT and DCB) with several notch aspect ratio are summarized in Figure 2. These experimental assessment points allow constructing a material failure curve called also a material master curve which is approximated by the following expression  $K_{\rho,c} = aT_{eff,c} + b$ , where  $a = -0.069$  and  $b = 77.28$  for the X52 pipe steel.

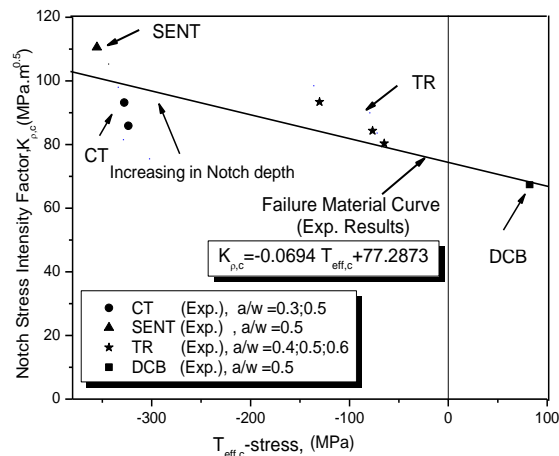


Figure 2. The experimental assessment points ( $K_{\rho,c}, T_{eff,c}$ ) and the material failure curve  $K_{\rho,c} = f(T_{eff,c})$  for X52 pipe steel.

The present results demonstrate that tensile notched specimens show lower constraint than bending specimens and, as a result, higher notch fracture toughness. Moreover, the DCB specimen exhibits positive value of the  $T_{eff,c}$ -stress. Eisele and al [31], and Kabiri [32] draw same conclusion. In addition to these conclusions, the experimental results reported in the literature [33-34] confirm the effect of constraint on the fracture toughness of different specimen geometries with cracks. Thus, the master curve is a way to take into account the effect of constraint on the notch fracture toughness and is very attractive to establish fracture conditions for components with various constraint values.

## Effect of Hydrogen

The  $K_{\rho} - T_{ef}$  curve is built in order to create a material characteristic taking into account specimen geometries, ligament sizes, type of steel and loading conditions. To get different assessment points ( $K_{\rho}, T_{ef}$ ), four specimen geometries (CT, SENT, RT and DCB) with several notch aspect ratio were tested with and without hydrogen.

### Determination of Material Failure curve ( $K_{pc} - T_{ef,c}$ )

The exploited of the  $K-T$  crack approach which was derived from a rigorous asymptotic solution has been developed for a notch two-parameter fracture to determine the Material Failure Curve (MFC). With  $K_{pc}$  as the driving force and  $T_{ef,c}$  a constraint parameter, this approach has been successfully used to quantify the constraints of notch-tip fields for various proposed geometry and loading configurations. We suggest extending the  $K_{pc} - T_{ef,c}$  to different steel and with the presence of hydrogen. The different specimens geometries are presented with the notch depth of 0.5 ( $a/t = 0.5$ ) after emerged in the hydrogen environment for 30 days and compared with the results of [33]. The experimental assessment points ( $K_{\rho,c}, T_{ef,c}$ ) for four specimen geometries (CT, SENT, RT and DCB) with notch aspect ratio ( $a/t = 0.5$ ) are summarized in Figure 3. These experimental assessment points allow constructing a material failure curve called also a material master curve which is approximated by the following expression. The degradation of the notch stress intensity factor with the presence of constraint is in the range 5.8 – 9.8 % for the different specimens. The shift between the virgin SENT specimen and the hydrogenated is small, however a concrete difference noted in DCB specimen (about 10 %). This deviation is ascribed to the exploit of specimens between the tension and flexion loading. The decreasing of the notch stress intensity factor values for differed specimens can be explained by the degree of constraint. Increasing the yield stress increase the constraint parameter. Figure 3.(a) and 3.(b) are presented in order to examine the effect of the second terms of Williams' solutions, i.e. effective  $T$ -stress on the notch stress intensity factor for the different shape of specimen's with the presence of hydrogen. The Figure 3.(a) and 3.(b) pertain to a fixed value of the depth of  $a/t = 0,5$  with the three steel X52, X70 and X100.

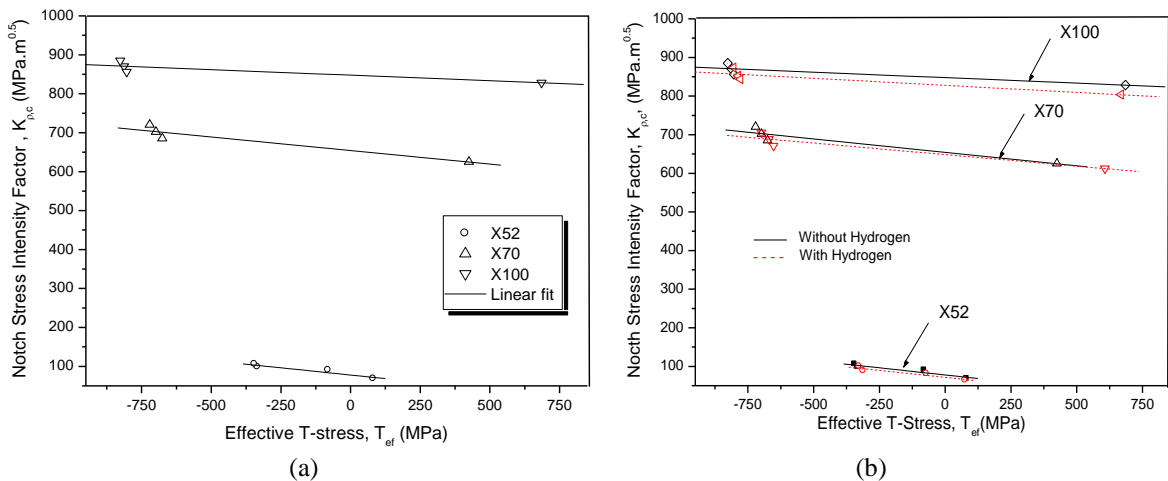


Figure 3. (a) The Material Failure Curve for the X52, X70 and X100 steel without and (b) with hydrogen.

## Effect of Notch Radius

The characteristic length was associated with the notch radius firstly in the Creager and Paris [36] analysis of the stress distribution at notch tip. For rounded V-notches, analytical expression of notch tip stress distribution for elastic material was developed by Filippi et al [37]. They introduces in this analytical expression the distance between the origin of the polar coordinates system and the notch tip  $r_0$ . This distance  $r_0$  depends on notch radius and notch angle. For the particular case of a zero. Fracture toughness have been made on steel specimens. The material is a ductile steel (French name X38) with the following mechanical properties, yield stress  $R_e= 304$  MPa, ultimate strength  $R_m = 430$  MPa. Stress distribution at notch-tip has been computed using Finite Element Method. the CASTEM™ code was used for this purpose. SENB specimen exhibits a symmetry axis. in order to reduce the number of elements and saving time computing, only half the specimen has been represented by a mesh work. Loading conditions are represented by non displacement along y axis in the ligament section.

## Influence of the notch radius on the stress intensity factor

The numerical results of the notch stress intensity factor (NSIF), for SENB specimens with different notch depth  $a/w$ , are compared to the results of Akouri et al.[38]. The numerical assessment points ( $K_{\rho,c}, T_{ef,c}$ ) for SENB specimen geometry with notch aspect ratio ( $a/w = 0.2, 0.5$  and  $0.7$ ) are summarized in Figure 4. These points allow constructing a material failure curve called also a material master curve for different notch radius. The increasing of the notch stress intensity factor with the presence of constraint is in the range 27%– 49 % with the increasing of the notch tip for  $\rho = 0.15$  to  $\rho = 2$ mm. One note that the material failure curve for the X38 steel is very sensitive to notch tip.

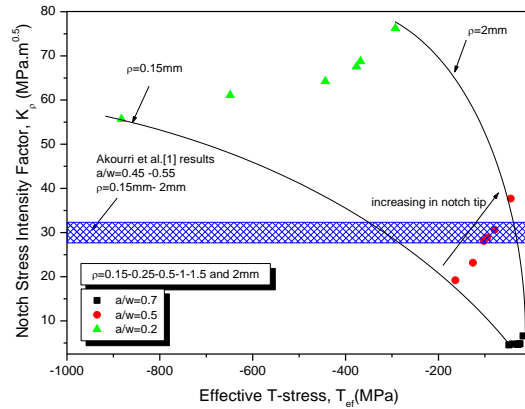


Figure 4. Material failure curve for the SENB specimen with  $a/w = 0.2, 0.5$  and  $0.7$ .

A increasing of the notch stress intensity factor  $K_\rho$  with constraint parameter has been noted by the Figure 4 for SENB specimen with non dimensional crack length  $a/W = 0.2 ; 0.5$  and  $0.7$ . These figure show a decreasing of the fracture toughness with ligament size for different notch radius. The transferability problem has been expressed as a curve  $K_\rho = f(T_{ef}, \rho)$  where  $T_{ef}$  is the constraint parameter and  $\rho$  the notch radius. The concept of brittle crack-extension resistance is based on the assumption that stress intensity factor  $K$ -dominance exists at a notch-tip. Then, in a region surrounding the notch-tip; the stress fields can be characterized by the mathematical solution limited to the second term [33]:

$$K_\rho \approx \sigma_{ij} \sqrt{2\pi r} \cdot f_{ij}(\theta) + T \cdot r^0 \delta_{xi} \delta_{xj} \quad \text{as } r \rightarrow \infty, \quad (2)$$

where  $K$  is the Stress Intensity Factor,  $f_{ij}(\theta)$  defines the angular function,  $\delta_{ij}$  is the symbol of Kronecker's determinant. A polar coordinate system  $(r, \theta)$  with origin at the notch tip is used. the notch stress intensity factor evolution with the presence of constraint is presented in the Figure 5.b with the different notch depth. The increasing of the NSIFs are about 15.8% for short notch to 7% for the very depth notch. This remark confirm that the strong effect of constraint.

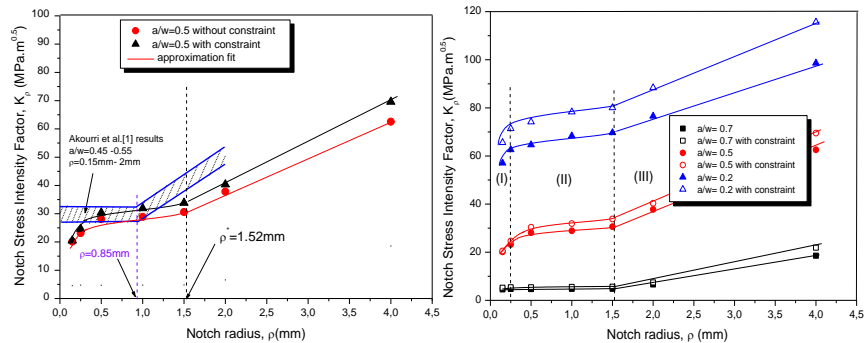


Figure 5. (a) Evolution of the notch stress intensity factor ( $K_\rho$ ) with the presence of constraint ( $T$ -stress) for SENB specimen ( $a/w = 0.5$ ) and (b) with different  $a/w$  with constraint.

The present results demonstrate that tensile notched specimens show lower constraint than bending specimens and, as a result, higher notch fracture toughness. Thus, the master curve is a way to take into account the effect of constraint on the notch fracture toughness and is very attractive to establish fracture conditions for components with various constraint values as it will be shown below.

## Conclusion

The concept of the constraint in the case of the crack stress distribution has been extended to the notch stress distribution. We have adopted the first nonvanishing term from the series solutions of Williams' for different situations. Its applicability for every effects of constraint with the presence of type of specimens, notch length, hydrogen embrittlement and notch radius are studied. We found that the next term may not be negligible for notch.

The failure mechanisms caused by constraint in the thickness direction alleviate the stress intensification near the crack tip and result in elevated apparent  $K$ . The  $K$ - $T$  methodology is used and  $T$ -stress is identified to quantify the constraint at the notch-tip. Procedures to shift the mechanical properties curve between Pipelines of different in plane constraint levels are developed which enables the determination of the transition curve of non-standard flawed structures from the experimental results of standard specimens for different situations.

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