The Experimental And Numerical Study Of Elastoplastic Deformation Processes, Limit States And Failure Of The Structure Elements Under The Monotonic Static And Dynamic Loading

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Abstract. General axisymmetric problems with torsion and 3-D processes of elastoplastic deformation of structure elements, loss of stability problems, damage cumulating and limit states under the static and dynamic loading are considered. A mathematical model is based on a continual approach to the description of non-elastic material failure processes and generalized theory of plastic flow. The original method of determining deformation and strength characteristics of metal and alloy structural components under an inhomogeneous stress-strain state has been developed. The results of numerical and natural experiments for the processes of failure and limit states of the standard specimens and elastoplastic structure elements in the conditions of tension, compression, torsion and combined simple and complex loading are described.

Introduction. By now a broad spectrum of mathematical models describing the nonlinear behavior of elastoplastic materials under the simple and complex loading has been developed. However, experimental verification has only been received by the models whose domain of application is bounded by the class of small elastoplastic strains and loading trajectories of small and medium curvature [1–5]. Under the complex stress-strain state (SSS) conditions, the equipment of these models with material functions and constants (true stress-strain diagrams, SSS type, fracture parameters) on the basis of direct measurements by the available tools is difficult even in the case of small strains. Under the strains preceding failure, an inhomogeneous stress-strain state arises in laboratory specimens and structure elements. Therefore it is reasonable to use mathematic modeling methods of deformation processes in laboratory specimens or structure elements in combination with experimental methods to identify the strain and strength characteristics of materials. The present paper aims at methodology for numerical solving axisymmetric problems with torsion and 3-D problem of elastoplastic deformation and damage cumulating. Limit states and failure of the structure elements in the conditions of the static and dynamic loading are analyzed.

Numerical model. A mathematical model is based on a continual approach to the description of non-elastic material destruction process and modified defining relation, which describes failure as a lost resistibility of the material under the certain conditions [7-9]. The deformation process is described by the generalized flow theory with combined kinematic and isotropic hardening. The failure process is supposed to be quasi-equilibrium and it doesn’t allow an explicit dependence of the parameters and characteristics of the failure on the time. Common strength criterions are applied to predict the failure conditions; the history of the SSS is described by the hypothesis of the cumulative damage. A damage measure is determined by the non-negative function $\omega \leq 1$ [10].

The solution of the problem is based on a finite element method and an explicit finite-difference scheme of CREST type [7, 8, 11], realized in the “Dynamic-2” and “Dynamic-3” software packages [12, 13].

Identification method of the strain and strength characteristics of materials under the conditions of complex SSS and large strains. Conventionally, the deformation and strength properties of the material are defined with experimental and analytical approaches, starting from experimental data and simplifying hypotheses that impose limits on the specimen shape and loading type. Such methods allow determining the characteristics of elastoplastic materials only in a homogeneous uniaxial SSS, which is not fulfilled in real experiment conditions at high strains. To study the strength properties of materials, especially at high strains, it is reasonable to develop an experimental-computational approach largely free from the limitations of experimental and analytical methods. This approach implies a combined analysis of experimental findings and full-scale (within continuum mechanics) computer simulation of deformation of laboratory specimens or structural elements without assuming a priori force or kinematic hypotheses.

In the paper [14] the experimental-computational approach is applied to develop techniques and algorithms of studying the strain and strength properties of isotropic and composite materials under static and dynamic loading of structural components. The basis for the development are identification theory methods in combination with iterative schemes of successive refinement of material characteristics in the specimen which account for an inhomogeneous SSS, high strains and strain rate dependence. For all types of experiments implemented in system objective functions of comparison parameters which describe the deviation of physical quantities measured in nature experiment from corresponding values of computational experiment are derived. Then, a converging iterative process of refinement of current material function values in the specimen through minimizing the objective function by a sequence of computational experiments is constructed. The developed techniques allow reducing the inverse problem solution to the successive solution of a series of direct problems and finally obtaining a set of mathematical model parameters. Concurrently with identification, according to the proposed technique, the sensitivity of the calculated parameters of comparison with experimental data to model parameter variation is analyzed. The applicability domain of the experimental-computational methodology is thus defined by the applicability domain of the mathematical model of elastic-plastic media, because the unconditional convergence of the iterative process guarantees that the sought model parameters will be found with a given accuracy.

Deformation and failure of cylindrical specimens in the conditions of torsion and tension. In the paper [7] numerical and experimental studies of deformation processes in an axisymmetric samples (of steel 12X18H10T) of variable thickness with a cylindrical working part under monotonic kinematic torsional/tensile loading are considered. Experiments were conducted under the different torsional/tensile ratio. Based on the calculation results the stability domain of plastic deformation under the combined torsional/tensile loading was described. There is no loss of stability of plastic deformation with the formation of a neck in numerical and nature torsion experiments, unlike the case of tension. So the line, which separate stability and instability domains, has a point of inflection. In the case of prevailing tension, loss of stability and localization of plastic deformations occur on earlier stage of torsion.

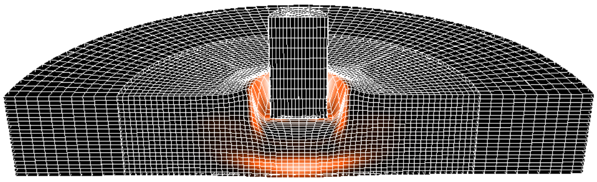
The experiments with loading along two-link broken deformation trajectories, combining two types of loading: torsion and tensile, are usually used to study the behavior of material under the complex loading. There are a wide set of such investigations and material models in the case of small plastic strains [1 – 5]. Authors of this paper have modified a variant of theory of plasticity [6] to the case of large strains [8]. To check the reliability of it quasi-static tests of solid axisymmetric samples (of steel 12X18H10T) along two two-link broken deformation trajectories: torsion until shear strain $(2/\sqrt{3})\ e_{\beta z} \approx 1$, followed by tension until failure and tension until axial strain $e_{zz} \approx 0.2$, followed by torsion until failure, were performed. The experimental and computational data obtained in radial torsional or tensile loading practically coincide. After

the trajectory break, the discrepancy between the computational results with combined hardening and the experimental results does not exceed 6%. Using the model with the isotropic hardening alone leads to an earlier formation of a neck in tension followed by torsion and the effect is significantly smaller if torsion is followed by tension. This can be explained by high sensitivity of the plastic deformation stability to the modulus of hardening. Under small elastoplastic strains, the delay trace (the width of the stress drop on the graph $\sigma_i - \kappa$) for steel is 1% in strains; it increases depending with increasing degree of strain. According to the above results, for the pre-strains $\kappa = 0.36$, 0.54, and 0.73, the drop width makes up 4–6% in strains, which means that under complex loading, the material memory due to the kinematic hardening is restricted by the decay trace in the region of small strain variations in the current state. Since in the studied generalized axially symmetric problem, the formulas for the strain rates don't contain rotations of material particles as a rigid body, the tensor of active stresses is determined by the same relations as in the case of small elastoplastic strains. As the computations show, for shear strains up to 150%, no oscillations in the shear stresses are observed. This is because, in contrast to [15, 16], the active stress tensor is determined by integro-differential relations with decaying memory rather than by the tensor of plastic strains.
Figure 1 displays the photograph of samples after tests until failure under monotonic torsional (digit 1), tensile (2), and combined torsional-tensile loading (3). One can see that unlike the case of tension with the formation of a neck, there is no loss of stability of plastic deformation in torsion, and the failure occurs due to shear strains in the plane perpendicular to the sample axis. In combined torsion and tension, a neck is formed whose cross-sectional radius is greater and length is smaller than the respective parameters in the case of pure tension. Under the combined loading, the failure occurs along a helical surface with maximal shear strains, depending on the relation between tension and torsion.

Fig. 1

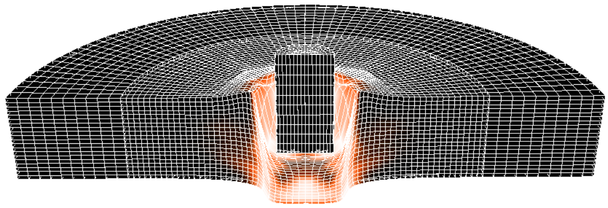
Penetration of solid striker into a steel plate. The 3-D axisymmetric problem of lengthwise penetration of the elastic cylinder (R=1,275 cm, L=4,7 cm, density ρ=7,8 g/cm^3, modulus of elasticity E=2x106 atm, Poisson ratio μ=0,3) into the round steel plate (R=13 cm, H=4 cm) with rigidly fixed boundary is considered. The results of problem solving at impact velocity 750 m/s are shown in Fig. 2, 3 in the form of finite element grids of the calculation domain on different time steps with marked damaged areas.
Figures show that three zones of failure appear nearby face and back surfaces of the plate during penetration of the striker. Failure nearby the face, localized on the sample axis, occurs due to interaction between loading and unloading waves, which go from free surface of the plate. This zone of failure is being suppressed during penetration of the cylinder. The second failure zone on the face, caused by shear deformations, is distal from axis of symmetry on the distance of 1.2 radius of the striker. Active shear deformations lead to appearance of a circumferential crack along the boundary of a forming disk in the upper half of the plate. In the bottom half of the plate

there is a failure on the back, which is characterized by fast specific volume cracking. Active shear deformations, occurred in weaken area, lead to growth of the circumferential crack in the vertical dimension. The process of perforation of the plate has finished by the moment the circumferential crack reaches the failure zone, which goes from the face. The upper and the bottom zones of failure have connected and a plug goes out. Allocation of the plug's material defects (Fig. 3) points on its fragmentation on two parts due to a spalling crack, which is in the good agreement with experiments [17].



t=20 mc

Fig.2



t=100 mc

Fig.3

Limit states of round plates under the pressure. A lot of papers devote to investigation of static and dynamic process of deformation and limit states of thin structures (beams, plates, shells) by using rigid-plastic models [18, 19]. However, applicability of the rigid-plastic models for design calculation of the metal structures, which have elastoplastic properties, hasn't been studied enough. It is well known that elastic strain energy has to be one order less than work of plastic strain to use the rigid-plastic model [19], i.e. strains have to be one order greater then the yield strength. It is need to take into account the non-linear geometry effects of deformation in presence of such bending deformations of thin structures, especially in the case of beams and shells.

So investigations of applicability of the rigid-plastic models have been conducted through the problems of quasi-static and dynamic bending of round plates under the low and big deflection [9], based on a comparative analysis of the results of numerical calculation of the problems of plate bending with hinged movable and immovable supports in the different cases: geometry linear and non-linear problem statements, using the elastoplastic and rigid-plastic models.

The problems of quasi-static and dynamic bending of thick and thin round plates with boundary fixed on hinged movable and immovable supports have been studied. Geometry characteristics of the plates are R=1 m, a thickness h1=0.2 m, h2=0.04 m. In the case of the elastoplastic statement Young modulus was $E=2.1*10^5$ MPa, Poisson ratio was $\mu=0.3$, in the case of the rigid-plastic statement Young modulus increased a thousand times. Low linear hardening was set for ideal plasticity modeling. It should be noticed that in the case of the absence of elastic zone and hardening a resolving system of equations lose the property of hyperbolicity [20], and a numerical scheme becomes unstable.

Fig. 4 shows relative discrepancy between the maximum deflections of the plate with thickness h=0.04 m, fixed on immovable support, which have been get for the rigid-plastic model in a geometry non-linear statement (RPN) and for the elastoplastic model in a geometry linear statement (EPL), and the deflections, which have been calculated by using the elastoplastic model in a geometry non-linear statement. The values of the plate deflections on the rotation axis max u/R (R is an initial radius of the plate), get by using the elastoplastic model in the geometry non-linear statement under the external pressure, are denoted on the abscissa. As figures shows, for deflection less than 0.025 R physically correct solutions can be get only by taking into account the elastic properties of the material, but in this case considerable discrepancy between the linear and non-linear statements is appeared, it becomes more than 10% for the deflection bigger than 0.012 R. For the deflection bigger than 0.025 R the rigid-plastic solution in the non-linear statement becomes closer to the elastoplastic solution, the difference does not exceed 10% when the deflections have been bigger than 0.05 R.

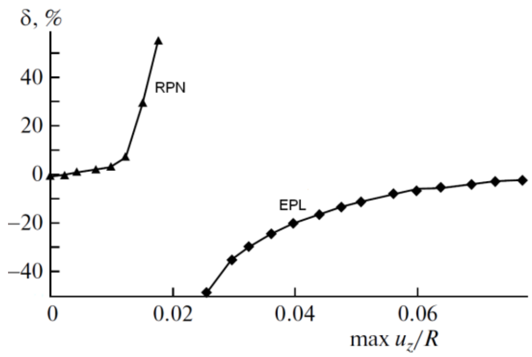


Fig. 4

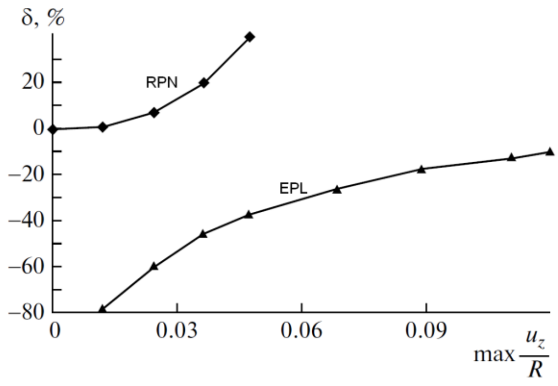


Fig.5

Fig. 5 performs the results of modeling of bending of the plate with thickness h=0.04 m, fixed on immovable support in the notation of Fig. 4. One can see that when the deflection is low the rigid-plastic solution differs from the elastoplastic solution considerable due to no registered elastic properties of the material. Further the results begin to approach. The discrepancy between the rigid-plastic and elastoplastic solutions don't exceed 10% when the deflections are more than 0.12 R. At that time significant differences between the linear and non-linear statements begin to appear. The discrepancy becomes more than 10% for the deflections bigger than 0.03 R.
Based on the conducted research, it is obvious that the rigid-plastic analysis isn't applicable to estimate the deflections of the elastoplastic plates in the general geometry linear theory. Its using in the geometry non-linear statement is possible only for cases of the big deflections, for example, in a working operation of pulse processing of thin-walled specimens by pressure. Errors of the rigid-plastic analysis of the deflections decrease when the plate's thickness increases; but the Kirchhoff-Love model is applicable only for the plates with the thickness one order less than the diameter. The rigid-plastic analysis can be used for the quasi-static loading to a rough estimate the magnitude of "safe" plate loading, because the deflections, corresponding to this loading, don't exceed 1-2% of a diameter of the plate. The real limit loading differs in times from the loading, which has been found by using the rigid-plastic model, because of missing the geometry non-linear effects.

Failure of balls under the contact compression. The set of experiments, based on test machine URC-20/6000, including the compression of two balls (with diameter D_b=7.98 mm), which are made of high-strength ball bearing steel and loaded into open steel race (Fig. 6) [21], have been developed to determine the strain and strength characteristic of material of the ball. The tests were conducted with intermediate unloading up to the ball failure. Compressing plates were made of steel 02N18K9M5T-VI with diameter 14 mm and thickness 7.07 mm. The load passed through the steel 02N18K9M5T-VI plates. Failure load during the tests was 77.62 kN.

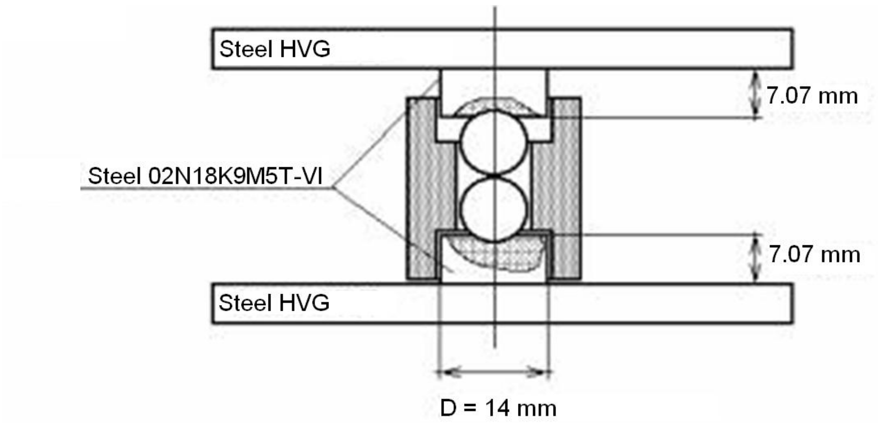


Fig. 6

Figs. 7, 9 depict the results of the experimental-calculation investigations. Fig. 7 gives the dependences of compressing loading on displacement of the support compressing plate, according to the experiment (solid line) and numerical modeling (dashed line). The good agreement of experimental and calculation data justifies the correctness of the obtained strain and strength characteristic of material. The numerical calculation shows that the failure has begun when plastic strain rate achieves 8% in the case of one ball compression and 20% in the case of two balls compression.

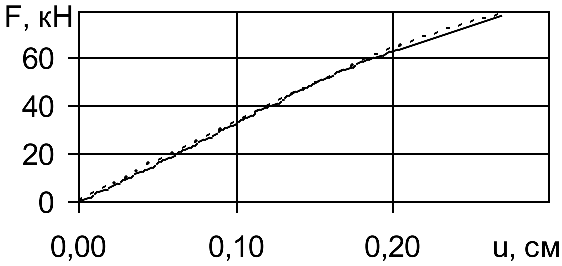


Fig. 7

The failure mode, which has been get in the test, is demonstrated in Fig. 8. Fig. 9 illustrates the allocation of parameter of damaged at the end of calculation. Only the bottom ball has been broken down in the experiment; the size of the cone formed due to the ball destruction is: the diameter of cone basis D_c=4.1 mm, cone height H_c=3.0 mm. The surface of the cone is smooth, which shows primary shearing failure behavior.

During the numerical experiment of the compression of two balls the cone was symmetrically formed in each of them. Evidently, the difference from the nature experiments connected to heterogeneity of the material and instability of the process of symmetric failure. The behavior of damage initiation and growth is in the good quality agreement with test results.

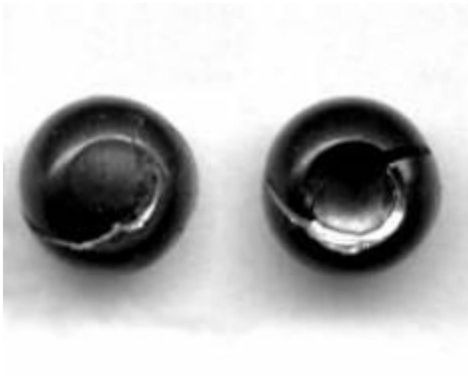


Fig. 8

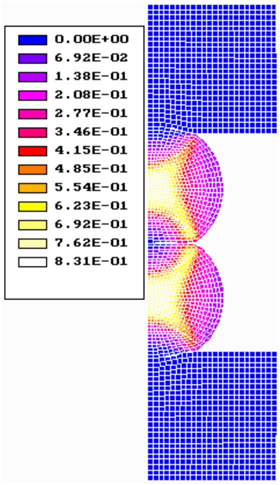


Fig. 9

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Summary

The developed techniques allow the strain and strength characteristic of materials to be defined independently of the specimen shape and loading type for high strains and with regard to stress-strain state inhomogeneity up to the moment of fracture, without the attraction of simplifying force or kinematic hypotheses. It seems possible to obtain the limiting fracture surface depending on the stress state type numerically calculated at the moment of fracture. The high information capacity and accuracy of determining the strain and strength characteristics of materials allow more reliable diagnostics of the material state and lifetime in structural components under service and alarm conditions.

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