The Block Element Method for a Block Structure

V.A. Babeshko [1,a], O.V. Evdokimova [2,b], O.M. Babeshko [3,c]

[1] 350040, Krasnodar, Stavropolskia, 149, Kuban State University, Russia

[2] 344006 Rostov on Don, Chehov av, 41, Southern Scientific Center of the Russian Academy of Sciences, Russia

[3] 350040, Krasnodar, Stavropolskia, 149, Kuban State University, Russia

[a] (babeshko@kubsu.ru), [b] (evdokimova.olga@mail.ru), [c] (olg.@kubsu.ru)

Keywords: boundary-value problems, factorization, block element, functional equation, pseudo-differential equation, generalized factorization, block structure, exterior form, matrix-function

The block element method described in [1-5] as applied to an individual domain is extended herein to a collection of neighboring domains, which are referred to as block structures. As applied to boundary value problems for such collections of domains, this method has specificity features that distinguish it from traditional approaches.

1. By block structures, we mean materials occupying bounded, semibounded, or unbounded domains, which are called contacting blocks. It is assumed that each block in a block structure has its own specific behavioral in response to physical fields of a various nature. It is also assumed that these fields are described by boundary value problems for systems of coupled partial differential equations with constant coefficients. Media of this type are typical of the earth's crust, structural materials under complex physical-mechanical conditions, nano materials, crystal structures of various arrangements, and electronics materials. A similar structure is also possesse by various materials, including those created by combining only nanoscale components or macro- and nanoscale components.

We consider structures with three-dimensional blocks. The absence of considerable constraints on boundary value problems describing the properties of individual blocks suggests that these block structures can have a wide variety of properties. In the general case, the concept of a block requires that the boundary of the domain a boundary value problem, including multiply connected domains, be unchanged and piecewise smooth. Each block can be bounded or unbounded and can involve coupled processes related to solid and fluid mechanics and electromagnetic, diffusion, thermal, acoustic, and other processes. Block structures are more general objects than piecewise homogeneous structures, in which the physical parameters of the medium are assumed to change in jumps in the transition from one block to another with the preservation of the medium material. The last property means that certain coefficients in the differential equations of a boundary value problem undergo jump variations in the transition from one block to another with the type of the boundary value problem being preserved.

Block structures have a wider range of properties than piecewise homogenous structures. This follows from the variety of blocks' properties, their shapes, and the character of interblock interactions and also results from the interaction of physical fields, some of which are produced or transformed by blocks. A special case of block structures is layered structures. Such structures with plane boundaries for linear boundary value problems can be viewed as fairly thoroughly investigated. Block structures are studied primarily by numerical methods, for which unbounded domains always present difficulties. The block element method, which is a generalization of the integral transform method, gives answers to questions concerning the properties of physical fields in each block even at the stage of solving boundary value problems.

2. We formulate the following boundary value problem for a block structure. Assume that the block-structure domain Ω consists of subdomains Ω_b, $b=1,2,...B$ with boundaries $\partial\Omega_b$ It may happen that a portion of the block's boundary is shared with another block, in which case it is a contact boundary. The remaining non-contact portion can be free or subject to external forces. It is assumed that a boundary value problem for systems of partial differential equations with (their own) constant coefficients is set in each domain Ω_b.

For each block, the boundary value problem for the system of P partial differential equations in the three-dimensional block domain Ω can be written as

$$\mathbf{K}_b(\partial x_1,\, \partial x_2,\, \partial x_3)\varphi_b = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{p=1}^{P} A^b{}_{spmnk}\varphi_{b\,p,\,x_1\ x_2\ x_3}^{\ (m)(n)(k)} = 0, \qquad s=1,\, 2,\, ...,\, P_b,$$

$$A^b{}_{sqmnk} = const, \quad \varphi_b = \{\varphi_{b1},\, \varphi_{b2},\, ...,\, \varphi_{bP}\}, \quad b=1,2,...,B.$$

$$\boldsymbol{\varphi} = \{\boldsymbol{\varphi}_s\}, \qquad \boldsymbol{\varphi}(\mathbf{x}) = \boldsymbol{\varphi}(x_1,\, x_2,\, x_3), \qquad \mathbf{x} = \{x_1,\, x_2,\, x_3\}, \qquad \mathbf{x}\in\Omega_b. \qquad (1)$$

The following matching conditions are set on the common contact boundary $\partial\Omega_b \cap \partial\Omega_d$

$$\mathbf{R}_b(\partial x_1,\, \partial x_2,\, \partial x_3)\varphi_b + \mathbf{R}_d(\partial x_1,\, \partial x_2,\, \partial x_3)\varphi_d =$$

$$\sum_{m=1}^{M_1}\sum_{n=1}^{N_1}\sum_{k=1}^{K_1}\sum_{p=1}^{P}[\ B^b{}_{spmnk}\varphi_{b\,p,x_1\ x_2\ x_3}^{\ (m)(n)(k)} + B^d{}_{spmnk}\varphi_{d\,p,x_1\ x_2\ x_3}^{\ (m)(n)(k)}\ ] = f_{bds}, \qquad (2)$$

$$s=1,\, 2,\, ...,\, s_{b0} < P, \quad \mathbf{x}\in\partial\Omega_b \cap \partial\Omega_d, \quad M_1 < M\ , \quad N_1, < N\ , \quad K_1 < K\ .$$

$$b,d = 1,2,...,B.$$

The boundary value problem is studied in the spaces of tempered distributions described in [6].

In the general form, the above boundary conditions describe the contact of blocks with the relevant components of physical fields coinciding on the common boundaries as dedicated by the corresponding physical laws. The scheme for applying the differential factorization method to such domains can be described as follows.

Following the differential factorization method [1], the boundary value problem is reduced to a system of functional equations with each domain Ω_b considered separately. As a result, we obtain the system of functional equations.

$$\mathbf{K}_b(\boldsymbol{\alpha})\mathbf{\Phi}_b = \iint_{\partial\Omega_b}\boldsymbol{\omega}_b, \qquad \mathbf{K}_b(\boldsymbol{\alpha}) \equiv -\mathbf{K}_b(-i\boldsymbol{\alpha}_1,\, -i\boldsymbol{\alpha}_2,\, -i\boldsymbol{\alpha}_3) = \left\|k_{bnm}(\boldsymbol{\alpha})\right\|, \qquad (3)$$

$$b=1,2,...,B.$$

Here, we used the notation adopted in [1] with additional indices b. For example, is the vector of exterior forms of the boundary value problem in Ω_b.

3. According to the differential factorization method, the next step consists of factorizing the matrix function $\mathbf{K}_b(\boldsymbol{\alpha}_3^{\nu})$ given by (3). For this purpose, we choose a matrix function

$\mathbf{K}_b^*(\alpha_3^\nu, m)$ of order $P-1$ obtained by deleting the row and column indexed by m in the adjoint matrix function $\mathbf{K}_b^*(\alpha_3^\nu)$ such that the zeros ξ_n^ν of its determinant $Q_b\left(\alpha_3^\nu\right) = \det \mathbf{K}_b\left(\alpha_3^\nu, m\right)$ do not coincide with the zeros z_{s+}^ν, z_{s-}^ν .

The elements of the inverse matrix function are denoted by .
$\left[\ \mathbf{K}_b^*(\alpha_3^\nu, m)\ \right]^{-1} = \left\| Q_b^{-1} Q_{psb} \right\|$

Then the elements of $\mathbf{K}^{-1}\left(\alpha_3^\nu, -\right)$ given by

$$\mathbf{K}_b^{-1}\left(\alpha_{3,}^\nu -\right) = \left\| \begin{matrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & \ddots & & & \\ S_{m1} & S_{m2} & \dots & S_{mm} & \dots & S_{mN} \\ & & & & \ddots & \\ 0 & & & & & 1 \end{matrix} \right\| \qquad (4)$$

can be represented in integral the form

$$S_{mp}(\alpha_3^\nu) = \frac{1}{2\pi i} \oint_{\Gamma_\mp} \sum_{s=1}^{N} {}' \frac{Q_{psb}(u_3) M_{sm}(u_3) du_3}{Q_b(u_3) K(u_3)(u_3 - \alpha_3^\nu)} - \left(\frac{1}{2} \mp \frac{1}{2}\right) \frac{R_{mp}(\alpha_3^\nu)}{K(\alpha_3^\nu)}, \quad m \neq p,$$

$$\frac{R_{mp}(\alpha_3^\nu)}{K_b(\alpha_3^\nu)} = \frac{Z_{mp}(\alpha_3^\nu)}{Q_b(\alpha_3^\nu) K_b(\alpha_3^\nu)} + \sum_n \frac{Z_{mp}(\xi_n^\nu)}{Q_b'(\xi_n^\nu) K_b(\xi_n^\nu)(\xi_n^\nu - \alpha_3^\nu)} \qquad (5)$$

$$S_{mm}(\alpha_3^\nu) = K_b^{-1}(\alpha_3^\nu), \quad \alpha_3^\nu \in \lambda_\mp$$

$$Z_{mp}(\alpha_3^\nu) = \sum_{s=1}^{N} {}' Q_{psb}(\alpha_3^\nu) M_{sm}(\alpha_3^\nu)$$

Here, Γ_+ is a closed contour such that the domain λ_+ contains only the zeros z_{s+}^ν, z_{s-}^ν and , while the domain λ_- contains only the zeros ξ_n^ν . The closed contour Γ_- encloses a domain containing all the zeros z_{s+}^ν, z_{s-}^ν , and ξ_n^ν . Representation (5) implies that the elements of $\mathbf{K}_b^{-1}\left(\alpha_3^\nu, -\right)$ are rational functions with their only singularities being and . The term $K_b^{-1}\left(\alpha_3^\nu\right)$ containing them is given explicitly.

In the case of noncontact boundaries, the boundary conditions in the differential factorization method are set according to the rules described in [1].

The boundary conditions are fulfilled according to the following scheme. First boundary conditions on the noncontact boundary of each block are taken to the corresponding vectors of exterior forms in functional equations (3). For contact blocks, matching conditions (2) hold on the common boundaries of neighboring blocks. Depending on the properties of the described fields, these conditions can include some relations for the solutions and their derivatives. In the simplest case, this is the equality of the solutions and their derivatives on the common boundary in the transition from one block to another. These relations are taken to the corresponding vectors of exterior forms of functional equations (3), which are preliminary solved for the unknown normal derivatives on the boundary. The last procedure ensures the fulfillment of contact boundary conditions (2) in the solution to pseudodifferential equations.

Assume that the blocks are convex. Omitting the intermediate transformations, which can be found in [1], we find that the solution in each block is represented as

$$\boldsymbol{\varphi}_b(\mathbf{x}^\nu) = \frac{1}{8\pi^3}\iiint_{-\infty}^{\infty} \mathbf{K}_{rb}^{-1}\left(\boldsymbol{\alpha}_3^\nu\right)\mathbf{K}_b^{-1}\left(\boldsymbol{\alpha}_3^\nu,-\right)\iint_{\partial\Omega}\boldsymbol{\omega}_b e^{-i\langle\alpha_3^\nu x_3^\nu\rangle} d\boldsymbol{\alpha}_1^\nu d\boldsymbol{\alpha}_2^\nu d\boldsymbol{\alpha}_3^\nu, \quad \mathbf{x}^\nu \in \Omega_b.$$

To illustrate this solution, we evaluate the integral with respect to α_3^ν by applying Leray's residue form theory to obtain

$$\boldsymbol{\varphi}_b\left(\mathbf{x}^\nu\right) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\!\!\int \sum_s e^{-i\left(\alpha_1^\nu x_1^\nu + \alpha_2^\nu x_2^\nu\right)}\left[\mathbf{K}_{rb}^{-1}\left(i\frac{\partial}{\partial x_3^\nu}\right)\mathbf{T}_{+b}\left(\boldsymbol{\alpha}_1^\nu, \boldsymbol{\alpha}_2^\nu, \mathrm{z}_{s+}^\nu\right)e^{-iz_{s+}^\nu x_3^\nu} -\right.$$

$$\left. - \mathbf{K}_{rb}^{-1}\left(i\frac{\partial}{\partial x_3^\nu}\right)\mathbf{T}_{-b}\left(\boldsymbol{\alpha}_1^\nu, \boldsymbol{\alpha}_2^\nu, \mathrm{z}_{s-}^\nu\right)e^{-iz_{s-}^\nu x_3^\nu}\right] d\boldsymbol{\alpha}_1^\nu d\boldsymbol{\alpha}_2^\nu$$

Here, the boundary $\partial\Omega_b$ for the chosen $x_3^\nu < 0$, $\mathbf{x}^\nu \in \Omega$ is divided as follows:

$$\iint_{\partial\Omega_b}\boldsymbol{\omega}_b = \iint_{\partial\Omega_{+b}}\boldsymbol{\omega}_b + \iint_{\partial\Omega_{-b}}\boldsymbol{\omega}_b,$$

$$\iint_{\partial\Omega_{+b}}\boldsymbol{\omega}_b\, exp(-i\boldsymbol{\alpha}_3^\nu x_3^\nu) \to 0, \quad Im\,\boldsymbol{\alpha}_3^\nu \to \infty,$$

$$\iint_{\partial\Omega_{-b}}\boldsymbol{\omega}_b\, exp(-i\boldsymbol{\alpha}_3^\nu x_3^\nu) \to 0, \quad Im\,\boldsymbol{\alpha}_3^\nu \to -\infty.$$

If a block degenerates into a half-space or a layered medium, the pseudodifferential equations appearing in the course of solving the boundary value problem degenerate into algebraic equations. The latter are inversed, and the solution is constructed in a finite form [1].

4. The possibilities of the block-element method are displayed by its use in a number of polytypic problems presented below.

In [1-4], the concept of a block element is introduced, and a number of examples of particular block elements are given for certain boundary-value problems. It is shown that the block elements are determined by the boundary-value problem and can always be constructed for an unambiguously solvable boundary-value problem formulated for a set of partial differential equations of a finite order with constant coefficients in the region with a piece-smooth boundary [5-7]. They also can be constructed for the boundary-value problems with variable coefficients admitting the separation of variables [8].

In the general case of the boundary-value problems with variable coefficients, their region of formulation of the boundary-value problem is divided by a mesh for using the block-element method. The mesh should be so dense that it could be possible to consider the coefficients in a division cell as constant [1-4, 9].

A certain practice of applying the block elements shows that their use simplifies the formulation of a number of boundary-value problems and also the construction of their solutions. For example, the block-element method makes it possible to solve the boundary-value problems for homogeneous and inhomogeneous sets of partial differential equations in a similar way [7]. For its use, it is unnecessary to construct individually the general solutions of homogeneous differential equations and the partial solutions of inhomogeneous equations with the subsequent fulfillment of boundary conditions. In the unsteady boundary-value problems, the block-element method raises both the edge boundary conditions for sets of partial differential equations and the initial conditions of a boundary-value problem [10] to the rank of boundary conditions; i.e., the initial conditions in the block-element method become the boundary conditions. The block-element method makes it possible to consider the same boundary-value problems in the bounded, semi-bounded, and unbounded regions.

The block elements enable us to simplify the derivation of certain important characteristics of the solution.

For example, the block elements describe the state function and the wave function of an elementary particle in the problems of quantum mechanics [11]. The normalized square of the modulus of its Fourier transform, which requires no calculation, gives the probability of keeping a particle in the block-element-carrier zone. Varying the shape of the block-element carrier, it is possible to obtain quantum-mechanical objects, which are more complicated than the quantum wells, wires and dots [11]. The pseudo-differential equations arising in these problems involve all cases of the particle energy state in the same way.

In the problems of continuum mechanics, the functions on the boundaries of the block-element carrier, which either require a determination or are set and included in the pseudo-differential equations, are the particular physical characteristics of the solution of the boundary-value problem under consideration.

For example, in the problems of elasticity theory, these are the displacements or stresses on the block-element boundary; in the problems of the theory of plates, they are the displacements, angles of rotation, or shear and normal forces and moments. In the boundary-value problems of electrodynamics, it is the electric potential, the electric charge, the tangential component of the electric-field vector, and the normal component of the electric induction.

5. By the example of a particular boundary-value problem, we present certain general properties of the block elements revealing their features and admitting the generalization on the general case. Here and below, considering the construction of solutions of a boundary-value problem by the block-element method, we mean that the solution of the corresponding pseudo-differential equations was constructed.

Let an unambiguously solvable boundary-value problem for the set of the partial differential equations of a finite order with constant coefficients be considered in the convex singly connected polyhedral region Ω with the boundary $\partial\Omega$ [7]. The block elements of such a boundary-value problem represent the vectors, the components of which are block elements similar to the scalar ones in the case of the boundary-value problem for a single differential equation. Further, we do not distinguish these two concepts calling them block elements in both cases.

Let us consider various divisions of the region Ω by the mesh, the boundaries of which represent various planes. As a result, the region Ω is divided into n polyhedral convex regions $\Omega_k(n), \quad k=1,2,...,n$. Rejecting certain boundaries in the division mesh, we obtain a new division of the region Ω containing a smaller number of larger regions $\Omega_k(p), \quad k=1,2,...,p, \quad p<n$, each of which can be a combination of several regions $\Omega_k(n)$). Continuing the process of elimination of boundaries of the division mesh, we obtain the sequence of divisions $1<p_1<p_2<...<n$.

In the case of $p=1$, we obtain a single cell, which proves to the region Ω.

The number n can be either finite or tend to infinity.

For each division p_r, we designate the block element corresponding to it as $B_k(p_r)$. We introduce the concept of the combination

$$B_k(p_r)=B_l(p_s)\cup B_h(p_s), \quad r<s \tag{6}$$

for the block elements contacting over the general boundary, which consists in constructing the block element located on the combination of their carriers. The number of united elements can be arbitrarily finite.

After constructing the solutions φ of the boundary-value problem for each of the divisions by the block-element method, for doing which we convert the corresponding pseudo-differential equations [7], we obtain the representation of solutions for each *r* in one of the following forms:

$$\boldsymbol{\varphi} = \sum_k B_k(p_r), \quad r = 1,2,...,n, \quad p_1 = 1, \quad p_n = n \tag{7}$$

The cited formula displays the completeness of the block elements in $\mathbf{H_s}$ for each of the divisions. We consider in more detail (6), we obtain the representation of the function $\boldsymbol{\varphi}$, which is invariable in the left-hand side and expanded in terms of larger block elements.

Thus, for constructing the solution of the boundary-value problem by the block-element method, it is possible to diversify the choice of divisions of the region Ω in which the boundary-value problem is formulated by the mesh on the basis of the reasons of an optimum selection of the corresponding block elements and their conjugation by means of solving the pseudo-differential equations. The fulfillment of this requirement substantially depends on the shape of the region of formulation of the boundary-value problem and the type of the differential equations for which it is formulated.

The practice of application of the block-element method shows that for constructing the block elements, it is possible in certain cases also to use other methods, which enable us to implement more quickly their derivation alongside with the general approach based on using the automorphism of varieties [8].

Let us consider the block elements introduced previously in [1-4, 7, 9]. Obviously, each of them displays the right-hand sides of inhomogeneous differential equations and the boundary conditions of the carrier taken in certain spaces $\mathbf{H_s}$ as a function in the open internal region of the carrier. The following statement is valid.

Theorem 1. *The set of block elements of the boundary-value problem unambiguously solvable in certain space* $\mathbf{H_s}$ *and considered in the region* Ω *with the piece-smooth boundary* $\partial\Omega$ *represents a topological set with the topology having the structure of hat of the* Ω*-region.*

In the problems of continuum mechanics, the topology in the space containing the region Ω is induced by the Euclidean space.

These are the boundary-value problem formulated in a certain region Ω with the boundary $\partial\Omega$ that are responsible for the block-element origin and analytical properties. This theorem is related to the representation of solutions of the boundary-value problems in the form of an expansion in terms of the block elements capable of being united in the elements with larger carriers leaving invariable the solution $\boldsymbol{\varphi}$ of the boundary-value problem.

This theorem explains the possibility of choosing a rich arsenal of every possible region admitted by the accepted topological structure and a particular boundary-value problem as the carriers of block elements. Each block-element carrier can have its own local system of coordinates, the relation of which with the local systems of carriers of neighboring blocks is controlled by a map [12, 13].

6. The results displayed below show that the dependence of the block element on the boundary-value problem is not an invariable property.

Let us consider two boundary-value problems unambiguously solvable in $\mathbf{H_s}$, for the set of partial differential equations with constant coefficients of an identical order having unknown vector functions of an identical dimension in the region Ω with the piece-smooth boundary $\partial\Omega$ [7].

The following statement takes place.

Theorem 2. *The solution of one of the above boundary-value problems admits the representation in the form of the expansion in terms of block elements of another boundary-value problem considered in the region* Ω *with the boundary* $\partial\Omega$.

7. We consider the boundary-value problem in the region Ω with the boundary $\partial\Omega$ unambiguously solvable in Hs with respect to the vector function φ1 for the set of partial differential equations of a finite order with variable coefficients and without features.

We consider the previous boundary-value problem in the region Ω with the boundary $\partial\Omega$ with respect to the vector function of the same dimension for the set of the same partial differential equations in which the constant values are found instead of variable coefficients providing the unambiguous resolvability of the boundary-value problem in $\mathbf{H}_s$.

The following statement is valid.

Theorem 3. *The solution* φ_1 *of the boundary-value problem with variable coefficients admits the representation of the form of the expansion in terms of the block elements of the boundary-value problem with constant coefficients.*

8. The possibilities of using the block-element method are even more extended due to the property presented below.

Let us consider the boundary-value problem for the set of partial differential equations of finite order unambiguously solvable in $\mathbf{H}_s$ with the maximum derivative ν in the region Ω with the boundary $\partial\Omega$ [7].

Let us designate the space of functions, which are continuously differentiated λ times with respect to all variables including the mixed ones, as .

It is valid as follows.

Theorem 4. *An arbitrary vector function* φ *from* $C_\lambda(\Omega)$, $\lambda > \nu$ *, can be represented as the expansion in terms of the block elements unambiguously solvable in a certain space* $\mathbf{H}_s$ *of the boundary-value problem for the vector function of the same dimension considered in the region* Ω *with the piece-smooth boundary* $\partial\Omega$ *having the maximum derivative of the order* ν *in the differential equations.*

This theorem opens ample possibilities for the most different applications of block elements. Alongside with the results presented, these possibilities increase with using various forms of automorphism of varieties [14].

ACKNOWLEDGMENTS.

This work was supported in part by the Russian Foundation for Basic Research (project nos. (11-08-00381); the program “South of Russia” (project nos. (11-08-96502), (11-08-96503), (11-08-96506), (11-08-96504), (11-08-96522), (11-08-96505); the Presidential Program of Support for Leading Scientific Schools in Russia (project no. NSh-914.2012.1), by the state contract from September 1. 10 no 16 740.11.01.35 in the framework of the FTsP "Scientific and Pedagogical Personnel of Innovative Russia" for 2010-2013.

REFERENCES

[1] V.A. Babeshko, O.M. Babeshko, and O.V. Evdokimova, Dokl. Phys. 54 (8), 373 (2009).
[2] V.A. Babeshko, O.M. Babeshko, and O.V. Evdokimova, Dokl. Phys. 54 (7), 329 (2009).
[3] V.A. Babeshko, O.M. Babeshko, and O.V. Evdokimova, Dokl. Phys. 54 (9), 418 (2009).
[4] V.A. Babeshko, O.M. Babeshko, and O.V. Evdokimova, Dokl. Phys. 54 (12), 557 (2009).
[5] V.A. Babeshko, O.M. Babeshko, and O.V. Evdokimova, Dokl. Phys. 54 (6), 285 (2009).
[6] V.A. Babeshko, O.V. Evdokimova, and O.M. Babeshko, Dokl. Phys. 415 (5), 596 (2007).
[7] O.V. Evdokimova, O.M. Babeshko, and V.A. Babeshko, Dokl. Phys. 418 (3), 321 (2008).
[8] V.A. Babeshko, and A.Rittser, Dokl. Phys. 56 (3), 178 (2011).
[9] V.A. Babeshko, O.V. Evdokimova, and O.M. Babeshko, Dokl. Phys. 55 (6), 292 (2010).
[10] V.A. Babeshko, O.V. Evdokimova, and O.M. Babeshko, Dokl. Phys. 56 (6), 338 (2011).
[11] V.A. Babeshko, O.V. Evdokimova, and O.M. Babeshko, Dokl. Phys. 55 (11), 545 (2010).
[12] V.A. Babeshko, O.V. Evdokimova, and O.M. Babeshko, Dokl. Phys. 55 (10), 512 (2010).
[13] V.A. Babeshko, O.V. Evdokimova, and O.M. Babeshko, Dokl. Phys. 56 (10), 544 (2011).
[14] V.A. Babeshko, O.V. Evdokimova, and O.M. Babeshko, Dokl. Phys. 56 (6), 338 (2011).