Structures of fracture near a longitudinal shear main crack

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Abstract

An evolution of a fracture scenario in the vicinity of a main fault of longitudinal shear and an additional action of normal stresses was studied. It was shown by the performed experiments that a transverse compression along the normal to the shear plane leads to deviation of a fracture region from the shear plane. A model of step by step formation of an echelon structure feathering the main fault in conditions of predominate shear was suggested. During this process a system of small cracks of the primary echelon is replaces by an echelon of larger and more rare located cracks.

Introduction

Brittle fracture in vicinities of the front of a longitudinal shear leads to formation of a regular system (echelon) of small cracks. A review on such fracture mechanism mainly related to a combination of shear and tensile loads is given, e.g., in [1]. This type of fracture is a subject of an interest in geology, tectonophysics and fracture mechanics of structural materials [1-4], including metals [5]. It was shown [6,7] that the observed in experiments transition from a combination of longitudinal shear and tension in a vicinity of a linear stress concentrator to prevalence of shear is accompanied by loss of stability of the main fault front. As a result this front is separated on a set of parallel defects (facets) feathering the front of the stress concentrator without their coalescence on the initial fracture stage. Then, according to experiments [1,3-5], coalescence of the cracks of the initial system (of the facets echelon) occurs with their turning along the curvilinear trajectories. In the given paper we realize and analyze another scenario consisting in formation of a system of small tensile cracks in a vicinities of a front of a main crack (cut) of a longitudinal shear and transformation of the system in the fracture process, including the case of a additional compression along the normal to the shear plane.

1. A structure of fracture at a combination of the longitudinal shear and normal loads

To specify the fracture scenario a series of experiments was performed on a brittle model material (gypsum). The samples with cuts creating on their fronts the situation of longitudinal shear, including combinations of longitudinal shear and compression (or tension) under the compression of the sample on three area as it is shown in Fig. 1.

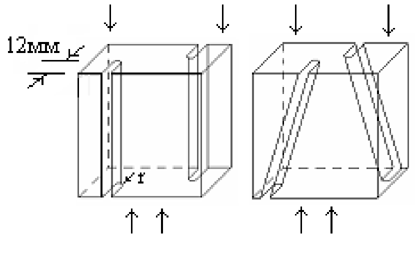


Fig.1. Variants of samples with cuts (sample sizes: 50x50x25mm; the curvature radius on the cut front r ~ 0.5mm)

The results of samples testing and preparation for samples in which the cuts of different inclinations provided normal loads proportional to shear are given in Fig. 2-8. Model experiments confirmed that a voluminous fracture region is formed in a vicinity of the initial linear concentrator at the development of longitudinal shear feathering in a brittle fracture regime. However, the shape and orientation of this region depend on the angle of inclination of the initial cut plane to the loading axis. At the inclination angles providing tension along the normal to the shear plane (Fig. 2, left) and up to 15^0 at compression in the same direction one observes exit of a series (an echelon) of tensile cracks inclined to the shear axis on the sample side opposite to the cut (Fig. 2, right). Turning of a region of multiple fracture around the cut front occurs at larger cut inclination angles. This effect was registered at the angle of cut inclination 28^0 (Fig. 3).

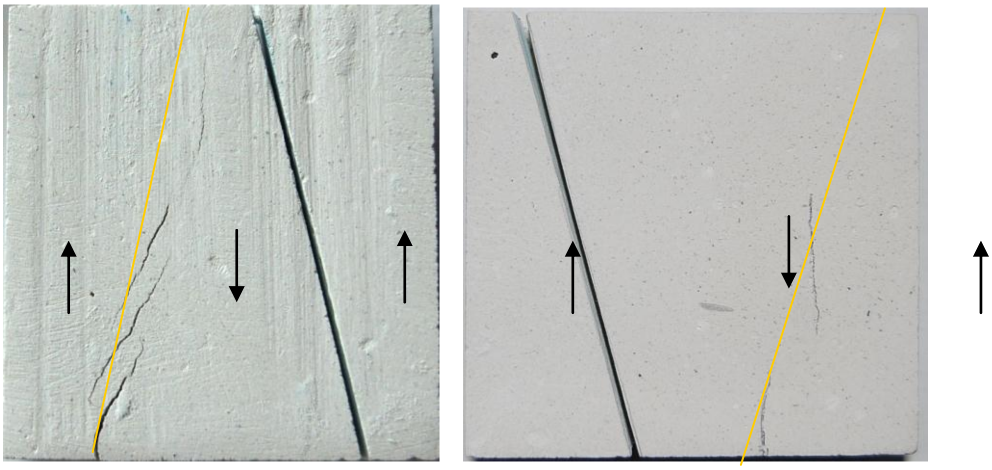


Fig. 2. Samples after testing. To the left – tension in the shear region (the angle of cuts inclination – 10^0). To the right – compression in this region (the angle – 15^0). The location of the cut axis and scheme of loads action are shown. It is seen the exit of feathering cracks on the opposite sample side

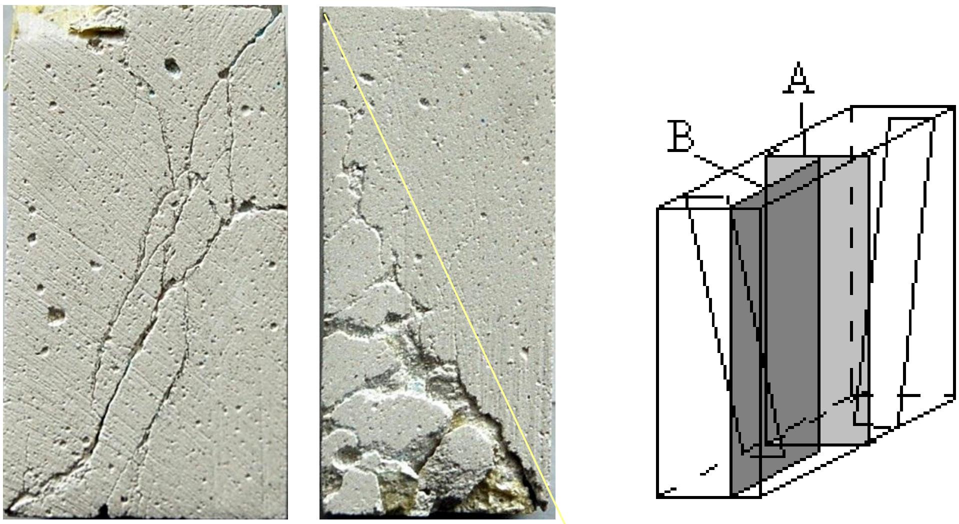


Fig. 3. The sample with the cut inclination angles – 28^0. To the left – section A, in the center – section B (location of the cut axis is shown). To the right – the scheme of the sample preparation

The turning effect is strengthened at further increase of the angle (up to 40^0 in experiment given in Fig. 4). The fracture picture is more and more influenced by the sample boundaries, its bending and other perturbations. Multiple ordered fracture is replaced by ruptures at bending deformations of a sample.

2. An analysis of the experimental results

The experiments showed that several characteristic scenarios of brittle fracture are realized in dependence on the value and sign of normal stresses in the region of the front of the longitudinal shear cut. In conditions of longitudinal shear prevalence the cracks feathering the initial cut form a regular structure ahead of the cut front. Within this structure equal initial cracks are located on fixed distances one after another on the axis of the cut front. This type of structure coincides with the known observations [1,3,5].

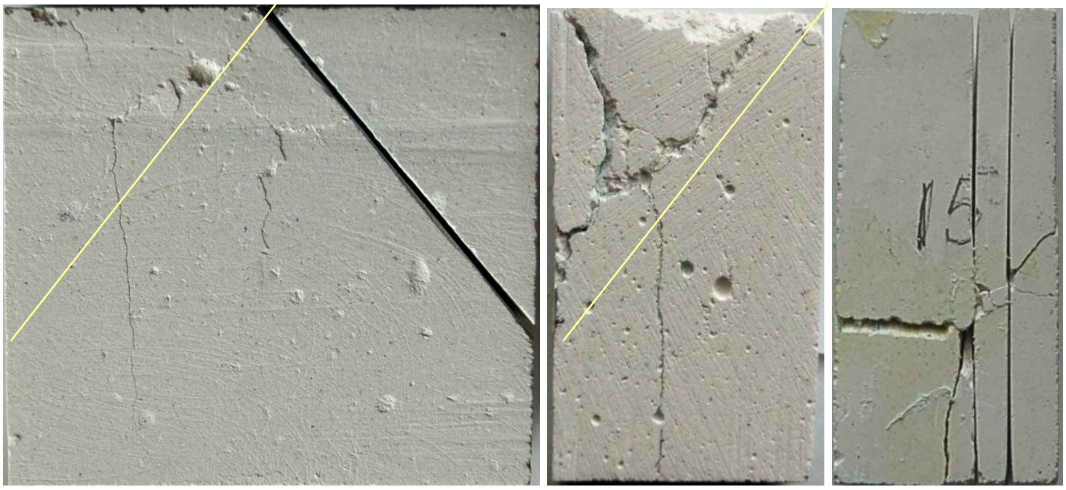


Fig. 4. The sample with the angle of the cuts inclination - 40^0. The sample view after testing. Location of the cut axis is shown. At the center – the sample section near the shear front. To the right – the sample side after preparation

A distance between the adjacent small cracks (facets) correlates to the crack sizes and width of the initial cut. Tensile component strengthening leads to formation of a main faults. Compression at large angles of the initial cut inclination (in experiments for angles 28^0 and larger) leads to turning of the fracture region in the vicinity of the cut front from the direction of the cut plane such that the feathering cracks do not exit on the opposite sample side (see, Fig. 3). The angle of the turning relative to the plane of the initial cut is close to 90^0.

Let us make some quantitative estimates. It is convenient to represent the results of the experiments in a traditional form for the cracks of combined shear and normal loading. For small vicinities of the cut front one can use an asymptotic approximation. This is justified for the moment of fracture initiation when the fracture processes are concentrated in a small vicinity of the cut. Let us make use of the following circumstance: in the main part of experiments limit loads are fixed in the moment of fracture initiation. To estimate the stress intensity factors of modes I and III, K_I and K_{III} we will use the approximate formulae usually being used for calculation of the fracture toughness on the basis of the experimental data [8]. The appropriate loading schemes are given in Fig. 5. According to [8]

$$K_I = 0.5\sigma_n\sqrt{\pi a}\cdot f_1(a/w);$$
$$f_1(a/w) \approx 1.12 - 0.23(a/w) + 10.55(a/w)^2 - 21{,}72(a/w)^3 + 30.4(a/w)^4 \qquad (1)$$

$$K_{III} = \tau\sqrt{\pi a}\cdot f_2(a/w); \qquad f_2(a/w) \approx \left[\frac{2w}{\pi a}\,\mathrm{tg}\,\frac{\pi a}{2w}\right]^{1/2} \qquad (2)$$

$$\sigma_n \approx \frac{F\sin\gamma\cos\gamma}{2Hd}; \qquad \tau \approx \frac{F\cos^2\gamma}{2Hd}$$

where σ_n, τ are the average normal and shear stresses on the cut plane in the fracture moment, F is the limit resulting traction, γ is the angle between the loading axis and the cut plane, H is the sample high, d is the distance from the cut front to the opposite sample side, w is the sample width.

The results of estimates are given in Fig. 6. One can see that the value of the stress intensity factor K_{III} which is necessary for fracture initiation increases with increasing of the normal compression ($K_I<0$) and decreases with increasing of the normal tension tending to the value of the fracture toughness K_{IC}. The value K_{IC} was determined in a special experiment.

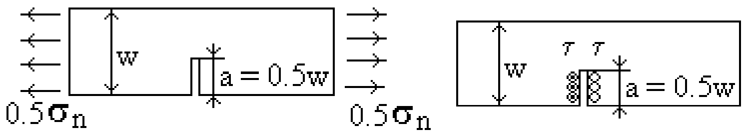


Fig. 5. Schemes of loading used for estimation of the stress intensity factors K_I (to the left and K_{III} (to the right) according to [8]

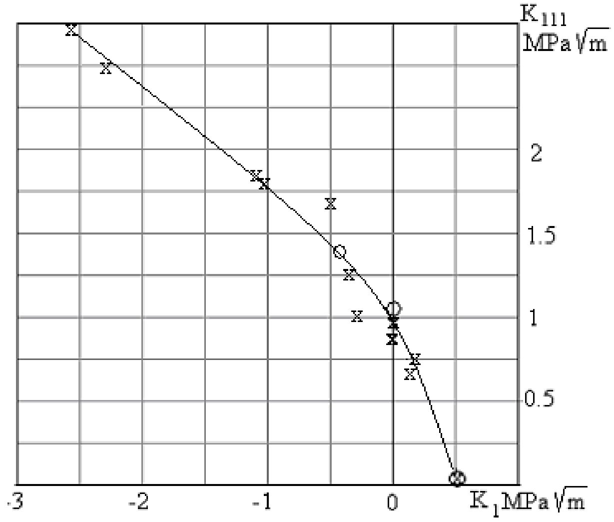


Fig.6. Limit values of the stress intensity factors at fracture initiation in the vicinity of a combined longitudinal shear and normal loading

The reason of deviation of the multiple fracture region from the direction of the cut plane can be determined by an analysis of the stress state in a vicinity of the combined shear in planes (x,z) and (y, z) (Fig. 7).

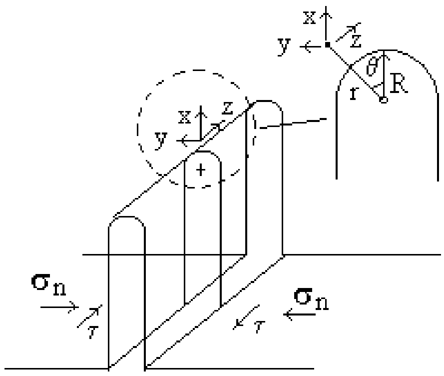


Fig. 7. To the determination of the principle stresses in a cut vicinity

By incorporating the known relations for the stresses asymptotics near the tips of mode I and mode III cracks [9] let us calculate the principle stresses in the selected planes. We have for the planes (y,z)

$$\sigma_{1,2} = \frac{K_{III} n}{2\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(\left(1 + \sin\frac{\theta}{2} \sin\frac{3\theta}{2} + 2\nu \right) \pm \sqrt{ \left(1 + \sin\frac{\theta}{2} \sin\frac{3\theta}{2} - 2\nu \right)^2 + \left(\frac{2}{n} \right)^2 } \right) \quad (3)$$

and for the planes (x, z)

$$\sigma_{1,2} = \frac{K_{III} n}{2\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(- \left(1 - \sin\frac{\theta}{2} \sin\frac{3\theta}{2} + 2\nu \right) \pm \sqrt{ \left(1 - \sin\frac{\theta}{2} \sin\frac{3\theta}{2} - 2\nu \right)^2 + \left(\frac{2}{n} \operatorname{tg}\frac{\theta}{2} \right)^2 } \right) \quad (4)$$

$$n = K_I / K_{III}$$

In estimation the stresses on the inner contour of the initial cut front at r = R we will account for that r = R at $|\theta| \le (\pi/2)$ while $r = R/\sin(\pi - \theta)$ at $(\pi/2) \le \pi$. The level of the principle stresses σ_1 in planes (y, z) and (x, z) is given in Fig. 8 for different values of parameter n.

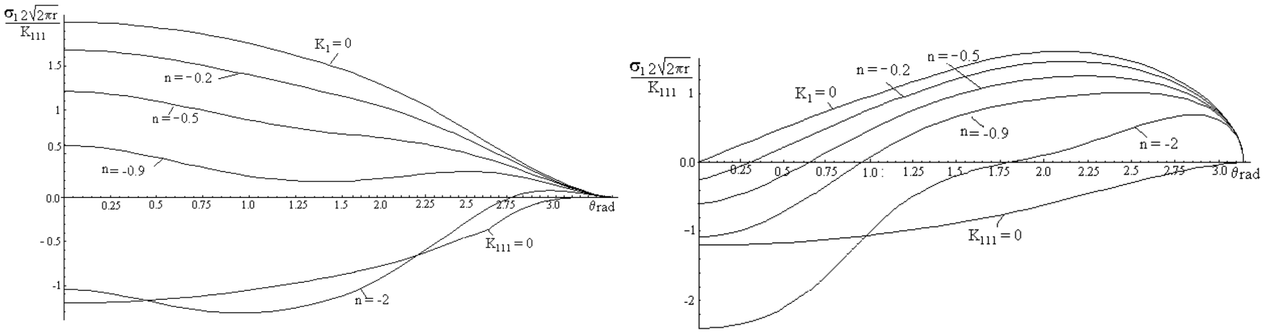


Fig. 8. Principle stresses σ_1 in planes (y, z) (to the left) and (x, z) (to the right), $n = K_I/K_{III}$

One can see that the principle stresses on the separated planes are changed by different manner at the variations of the planes orientation (values of the angle θ and distance r) and parameter n. Tensile stresses prevail in the plane (y, z) on the cut advance ($\theta \to 0$) at small $|n|$. The situation is changed if compression is strengthened. The maximal tension is shifted to the region of large values of the angle θ in the plane (x, z). A comparison of the maximal stress values in the competing planes is given in Fig. 9. Transition of the plane of the maximal principle stress location occurs at n ~ -0.55. This corresponds to experimental data for the angles of the initial cut orientation about $\gamma \sim 25^0$. Orientation $\gamma = 28^0$ for which the changing of the fracture region orientation was fixed in the experiments is related to the case n ~ - 0.6 and, hence, correlates to the results of the given calculations.

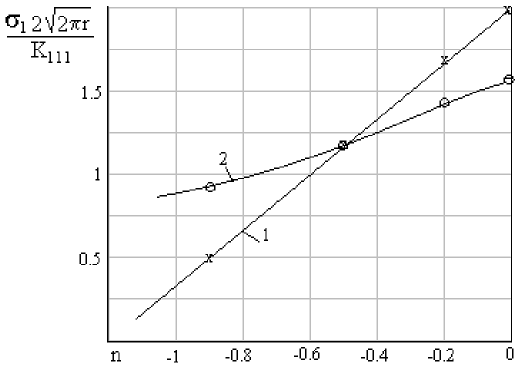


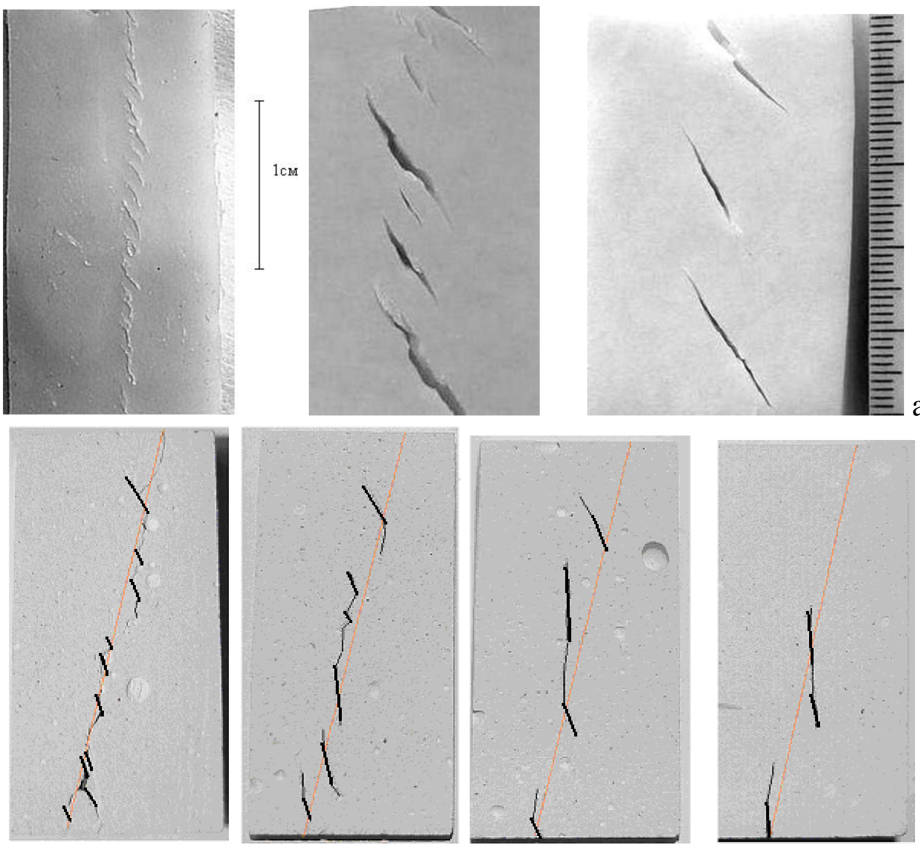
Fig. 9. Comparison of maximal principle stresses σ_1 in planes (y, z) (curve 1) and planes (x, z) (curve 2) for different values of the parameter n

Hence, the orientation of the tensile stresses maximum is changed relative to the front of the initial main cut leading to a shift of the orientation of the brittle fracture source if near the longitudinal shear front a concentration of normal compressive stresses takes place and the modulus of the stress intensity factor of these stresses exceeds a half of the stress intensity factor of the longitudinal shear stresses. Such situation can be realized, in particular, if the initial cut has a finite opening such that its surface do not contact under the action of normal compression. Note, that according to the experiments with $\gamma = 28^0$ (and $|n| \sim 0.6$) in spite of changing the orientation of the fracture region multiple fracture occurs within this region. The fracture region contains crack echelons as in case of smaller angles.

In case of experiments with $\gamma = 40^0$ and $n \sim 1$ (see, Fig. 4) the brittle fracture source also occurs at an angle to the cut plane, however, multiple fracture is less developed. Exit of single main cracks on the sample free surface is more characteristic. It seems that this phenomenon is related to an influence of sample sizes finiteness. As a result fracture by bending caused by eccentric compression of a sample with a cut becomes possible at a high level of the loads.

3. Remodel of a 3D-picture of fracture at longitudinal shear

According to what has been said above, if longitudinal shear prevails in the vicinity of the initial cut then a region of ordered fracture is formed such that the exit of this region on the opposite sample side has a view of a series (an echelon) of mode I cracks. At the same time the cracks nucleated directly on the cut surface have essentially less sizes and they are located more frequently as compared to the cracks observed on the opposite sample side. The sections of the samples from gypsum and low modulus material (cheese) are given in Fig. 10. One can see that fracture advance (development) is accompanied by formation of a series of echelon like crack subsystems having the elements of gradually increasing size which create a hierarchical system (Fig. 11)



a)

b)

Fig. 10. a) from left to right – sections of the sample on distances 0.5, 4, 8mm from the cut front in low modulus material; b) from left to right tracing of cracks on the sections surfaces for the gypsum sample on distances 1.5, 2.5, 7mm from the front of the initial cut and on the sample surface, respectively. The cut axis is also shown

Because of this circumstance one can not use the concept of the fracture process autonomity at the front of longitudinal shear. It should be noted that the complex character of local deformations and fracture at modeling effects of longitudinal shear in tectonophysics was discussed from another point of view in [10]. The shape of a section of the multiple fracture region was constructed on the basis of the samples preparation (Fig. 12). The shape looks like a sector with the tip angle $2\alpha \approx 80-90^0$ in the zone of the initial cut. The sector is bounded by an arc-type curve on its growing front which is formed by the tips (edges) of most large cracks. The results of experiments enable to follow the sequence of the events at the growth of a structure of fracture. First a system of small feathering cracks is formed on the initial cut axis. Then the systems of larger cracks are formed as far as the distance from the initial cut increases. Such regulation of the fracture scale can occur step by step several times up to fracture region exit on the opposite side of the sample.

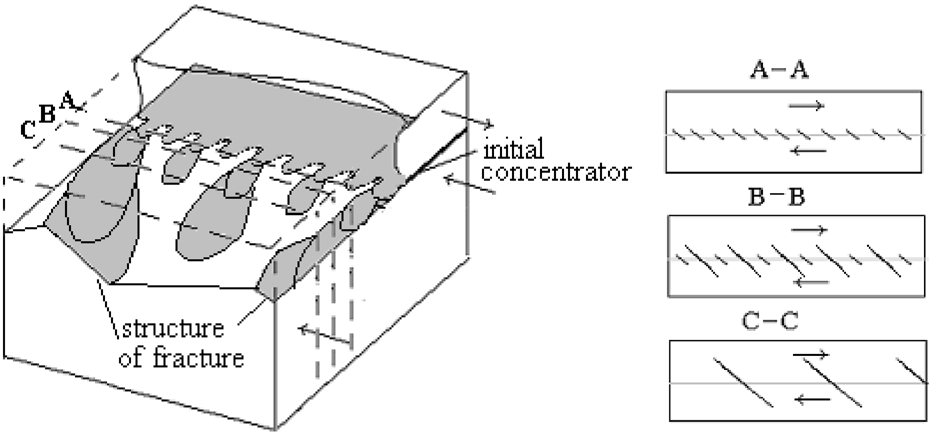


Fig.11. Scheme of interpretation of the observation results. Cracks surface is painted. The system of feathering cracks registered in the sample sections is shown to the right

It is convenient to consider a model situation in a section normal to the cut plane ahead of it. The principle stresses on the section plane are equal to [9]

$$\sigma_{1,2} = \pm \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \sqrt{\cos\theta} \qquad (5)$$

where r is a small distance from the cut front to the section plane.

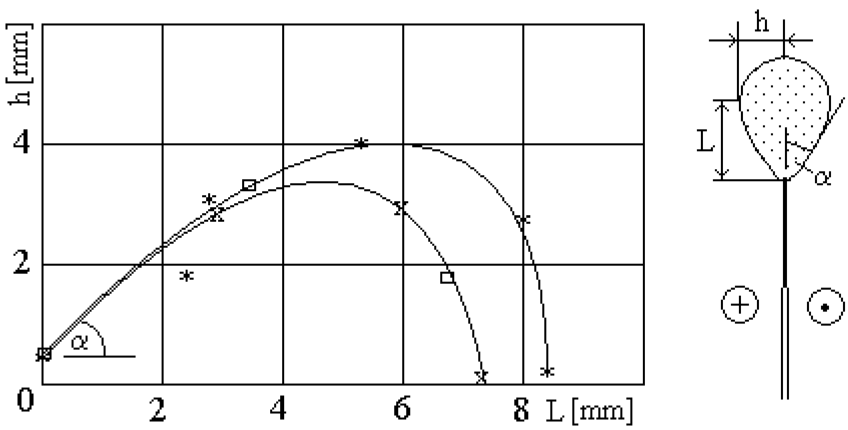


Fig. 12. Shape of the fracture region in a vicinity of the longitudinal shear front.

The curves are related to two different samples, L is the distance from the cut front to the plane of the fracture zone section, h is the half-width of the fracture zone

Assume [1] that the mode I cracks forming ahead of the main shear are oriented along the normal to the maximal principle stress. This stress can be written as follows in the coordinates related to the fault trajectory (x – axis is directed along this trajectory)

$$\sigma_y = \frac{K_{III}}{2\sqrt{\pi r}}\sqrt{\left(1+N^{-1/2}\right)N^{-1/2}}; \quad N = \left(\frac{x}{r}\right)^2 \sin^2\alpha + 1 \tag{6}$$

As a model of the process let us consider formation of an echelon of equal parallel mode I cracks in an elastic plane in the determined stress field (plane problem). Account for that the distance between the cracks is more important parameter than their length for the conditions of the crack limit equilibrium. This is characteristic for the echelons of equal parallel cracks at different loading conditions. The effect is related to the following circumstance: deformation of the material located between closely located cracks practically has no influence on the stresses in the end zones of the cracks. As a consequence the interrelation between the crack length and stress intensity factor disappears. The interdistance between the cracks and field of external stresses in the vicinities of the crack tips become the leading parameters.

Let us demonstrate this effect for an example of a model problem on the limit equilibrium of a rectilinear crack in a strip loaded symmetrically relative to both the crack line and transverse axis by arbitrary normal loads $\sigma_y(x)$. Assume that boundary conditions are related to contact of a strip with smooth rigid plates. This condition also corresponds to the problem on a periodic system of parallel cuts of equal length (ℓ) with the distance between the cuts (h) equals the strip width. To calculate the stress intensity factor we will use the compliance method [11]. The deformation energy for the strip with a crack equals

$$W = \int_0^{\ell} \frac{\sigma_y^2(x)h}{E}dx \tag{7}$$

The energy release rate will be equal to

$$J = dW/d\ell = \sigma_y^2(\ell)h/E \tag{8}$$

Hence, for the plane stress state

$$K_I = \sqrt{JE} = \sigma_y(\ell)\sqrt{h} \tag{9}$$

The stress intensity factor in a periodic cut system is proportional to the local stress in the zone of the cut tip in the body without cuts. The obtained formula coincides with a specific solution given in [8] for the periodic system of equal parallel cuts loaded by uniform pressure at $h < \ell$.

By incorporating (6) and (9) we obtain an estimate of the stress intensity factor

$$K_I = \frac{K_{III}}{2\sqrt{\pi r}}\sqrt{\left(1+N^{-1/2}\right)N^{-1/2}}\sqrt{h}; \qquad N = \left(\frac{x}{r}\right)^2 \sin^2\alpha + 1 \tag{10}$$

Let us consider two characteristic situation. At nucleation of an echelon of periodic cracks in front of the initial cut the distance between the cracks is fixed. It is evident that at increasing of the length of these cracks the distances between them remain unchanged. This situation corresponds to the condition h/r = const. Hence, the stress intensity factor decreases as the crack length increases. Note, that because of the inequality ($h < \ell$) in (9) the given statement is valid only for relatively large crack lengths. The initial stage of these cracks growth as isolated ones in the stress field (6) is characterized by increasing of the stress intensity factor K_I. Hence, the cracks in an echelon being unstable starting to grow become stable (and arrested) attaining the sizes exceeding the value of their interdistance.

If in this situation the distance between cracks can be changed then the stress intensity factor of an individual crack will be increased and its growth can be continued. This effect can be attained, for instance, by separation cracks in an echelon as it is observed in the described experiments (see, Figs 10,11). Only the cracks located over the intervals multiple to distances between the cracks in the formed echelon will continue to grow. These enlarged cracks form an echelon of the next scale level. In whole this process can be modeled by assuming that $h \approx n\ell$ in Eq. (10). The coefficient n, which determines the optimal relation between the crack size and distance between them in the moment of initiation of the echelon of the given scale level, has the value in the limits 1÷3 according to the data of the observations. The values of the ratio K_I/K_{III} for these n – values are given in Fig. 13. Hence, the scenario of the observed echelon structures development consists of a sequence of alternating events: initiation of equal cracks with equal interdistances; their arrest on some size; separation of a cracks subsystem for formation of a crack echelon of the next larger size level. The cracks in this new echelon are separated by intervals multiple to the intervals between the cracks in the echelon of the preceding level. An example of the scheme of the echelon structure development is given in Fig. 13 (to the right) for the variant when the optimal distance between the cracks on the each stage is close to the length of the cracks of this level. The plane view of the echelon structure related to the given scheme after four stages of its growth is given in Fig. 14.

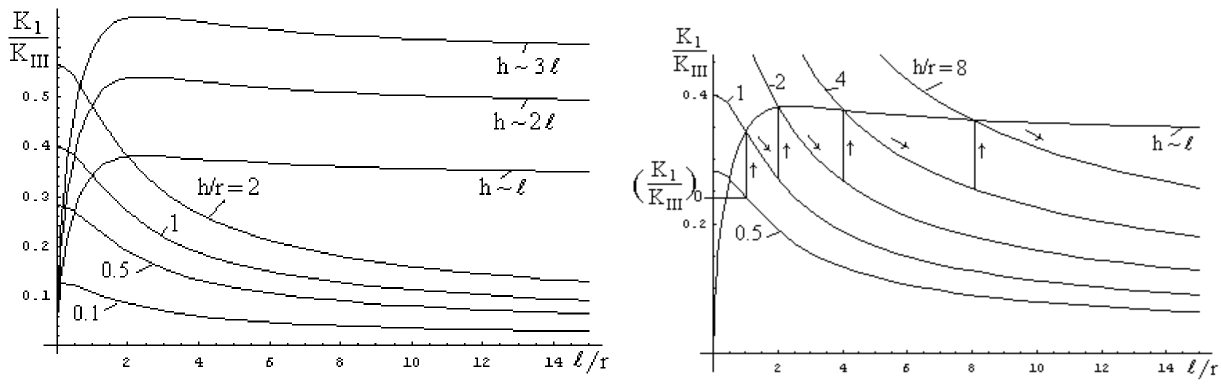


Fig. 13. Stress intensity factors at formation of echelons of cracks. The scheme of the echelon structure development is shown to the right

Development of multiscale echelon structures of cracks is not limited by the described scheme implying formation of the mode I crack system. It seems that this scheme can be an initial stage of more complex scenarios of fracture which can be realized at long loading history of large volumes of materials (media). As the examples of these scenarios one can mention the observed complex structures of fracture in the vicinities of tectonic faults in geological media [3,4,7].

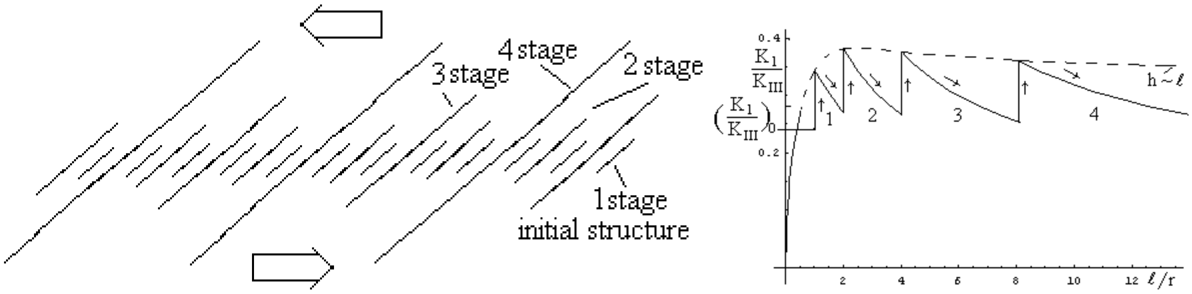


Fig.14. View of the echelon structure after fourth stage of its development. The appropriate stage-by-stage variation of the ratio K_I/K_{III} (according to Fig. 13)

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