Solution of Fracture Mechanics Problems by the Optical Method of Caustics - New Developments

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Abstract. The optical method of caustics constitutes a powerful tool in the hands of the experimentalist for the solution of fracture mechanics problems. The area in the vicinity of the crack tip is illuminated by a light beam and the reflected or transmitted light rays form an envelope in space. When this envelope is cut by a screen a highly illuminated curve, the so-called caustic, is formed. By measuring characteristic dimensions of the caustic the stress intensity factor is determined. The method has been based on the assumption that plane stress conditions dominate in the vicinity of the crack tip. However, experimental evidence has shown that the state of stress is not plane stress. It changes from plane strain near the crack tip to plane stress at a critical distance away from the tip through a three-dimensional region. In order the caustic to be generated from the light rays reflected or transmitted through the specimen from the plane stress region certain conditions among the dimensions of the optical arrangement, the specimen properties and thickness need to be satisfied. These conditions are fully investigated in this work. It is shown that special precautions should be taken when applying the method to determine stress intensity factors in crack problems. The applied loads, the specimen dimensions and the characteristic lengths of the optical arrangement should be properly selected for the correct application of the method. Otherwise, erroneous results may be obtained. A thorough investigation of the limits of validity of the method of caustics under conditions of plane stress was undertaken.
Furthermore, new procedures are developed that allow application of the method of caustics for the evaluation of stress intensity factors in crack problems when the initial curve lies in the region where the state of stress in three-dimensional. A triaxiality coefficient is introduced and is related to the stress optical coefficients. A linear relationship is established for the variation of stress-optical constants from plane strain to plane versus the triaxiality factor. This factor is determined experimentally and is subsequently used for the evaluation stress-optical constants and the stress intensity factors by the optical method of caustics. Furthermore, anisotropic materials are used. The double caustics obtained from the lights rays transmitted or reflected from the two directions of optical birefringence of the material are used to identify the state of stress in the region where the caustic is generated. When state of stress is identified region, the triaxiality factor is determined, and used for the evaluation of stress intensity factors.

Introduction

The optical method of caustics has extensively been used for the determination of stress intensity factors in crack problems [1-6]. The method is based on the assumption that the state of stress near the crack tip is plane stress. However, experimental and analytical solutions have shown that the state of stress changes from plane strain near the crack tip to plane stress away from the tip through an intermediate region where the stress state is three-dimensional. The changing state of stress results to changing values of stress-optical constants which enter in the equations for the determination of stress intensity factors. In the present work the method of caustics is critically reviewed, and its limits of applicability are studied. Furthermore, the use of optically anisotropic material is introduced for the determination of stress intensity factors.

The Optical Method of Caustics

In the optical method of caustics a specimen is illuminated by a light beam and the reflected or transmitted rays from the front or rear face of the specimen undergo a change of their optical path, due to the variation of the thickness and/or the refractive index dictated by the stress field (Fig. 1). At stress gradients resulting at crack tips, the reflected or transmitted rays generate a highly illuminated three-dimensional surface in space. When this surface is intersected by a reference screen, a bright curve, the so-called caustic curve, is formed. For transparent materials three caustics are formed by the light rays reflected from the front and rear surfaces and those transmitted through the specimen. For opaque materials, only one caustic is formed by the reflected light rays from the front surface of the specimen. The dimensions of the caustic are related to the state of stress near the crack tip. For the case of a mode-I through-the-thickness crack the stress intensity factor K_{exp} is given by [1]

$$K_{\text{exp}} = 0.0934 \frac{D^{5/2}}{z_0 c t m^{3/2}} \tag{1}$$

where z_0 is the distance between the specimen and the viewing screen where the caustic is formed, c is the stress optical constant of the specimen under conditions of plane stress, t is the specimen thickness, m is the magnification factor of the optical arrangement defined as the ratio of a length on the reference screen where the caustic is formed divided by the corresponding length on the specimen and D is the transverse diameter of the caustic at the crack tip. The above equation is valid when the state of stress in the vicinity of the crack tip is plane stress, so that the value of stress-optical constant under conditions of plane stress is used.

For optically isotropic materials, the caustic is created by the light rays reflected from the circumference of a circle, the so-called initial curve, which surrounds the crack tip. The radius of the initial curve is given by

$$r = 0.316\ D \tag{2}$$

Experimental

Specimens made of Plexiglas, of thickness d = 3.0, 4.5, 9.5 and 12.5 mm and width w = 42.4, 47.5, 51.5 and 63.5 mm, with an edge notch of length a = 15.5 mm were subjected to a progressively increasing tensile loading in an Instron testing machine. The specimens were illuminated by a convergent, divergent or parallel monochromatic light beam produced by a Ne-He laser. The caustic curves obtained from the light rays reflected from the front or rear faces of the specimen, or those transmitted through the specimen, were recorded on a viewing screen placed at a distance z_0 from the specimen. Caustics were obtained at different load levels for various values of the magnification factor of the optical arrangement, m, and the distance z_0. In this way, a host of caustics were obtained from different values, r, of the initial curve from the crack tip.

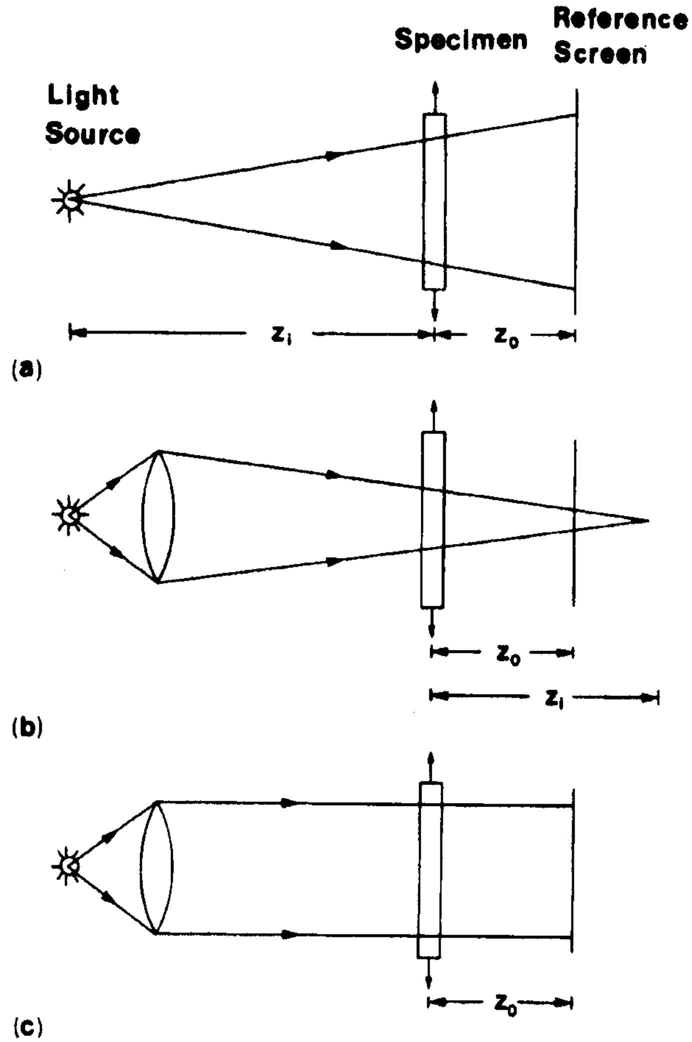

Fig. 1 Optical arrangement for divergent (a), convergent (b) and parallel (c) light

Experimental values of stress intensity factor, K_{exp}, were obtained. These values were compared with theoretical values of stress intensity factor K_{th} given by [7]

$$K_{\mathrm{th}} = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{w}\right) + 10.55\left(\frac{a}{w}\right)^2 - 21.72\left(\frac{a}{w}\right)^3 + 30.95\left(\frac{a}{w}\right)^4\right] \tag{3}$$

Note that the experimental values of stress intensity factor are obtained under the assumption that the initial curve of the caustic lies in the region near the crack tip where plane stress conditions dominate. Thus, if the values of K_{exp} and K_{th} coincide, this means that the initial curve of the caustic lies in the region where the state of stress is plane stress. In case the values of K_{exp} and K_{th} do not coincide, this implies that the initial curve lies in the region where the state of stress is three-dimensional

Fig. 2 present the variation of K_{exp}/ K_{th} versus r/d for a value of the specimen thickness d = 4.5, and different values of specimen width. Points in figure correspond to different values of the applied load, P, the magnification factor of the optical arrangement, m, the distance between the specimen and the viewing screen where the caustic is formed, z_0, and the specimen thickness, d. Note form figure that the ratio K_{exp}/ K_{th} increases with r/d and reaches a plateau value equal to one as the radius of the initial curve takes a limiting value r_c. At that value of $r = r_c$ the state of stress in the neighborhood of the crack tip becomes plane stress. For distances r smaller than r_c the state of stress is three-dimensional, while for values of r larger than r_c plane stress conditions dominate. It was obtained that the critical value of r for which the state of stress becomes plane stress depends not only on d, but also on the geometrical characteristics of the cracked plate, especially the ratio of the crack length to specimen thickness.

Limits of Applicability of the Method of Caustics

The condition that the initial curve of the caustic should lie at distances from the tip approximately greater than half the specimen thickness introduces limitations in the parameters (distance between the specimen and the viewing screen where the caustics is formed, the magnification factor of the optical arrangement, the specimen dimensions and thickness, and applied loads) entering in the determination of stress intensity factors. These factors should be properly selected so that the initial curve lies in the region where plane stress conditions dominate. In that case the value of stress-optical constant corresponding to plane stress should to be used.

In order to obtain caustics generated from the region of plane stress the radius of the initial curve of the caustic should be larger than a fraction of the specimen thickness. By taking this distance equal to half the specimen thickness we obtain

$$\left(\frac{3.385 z_0 c K}{m}\right)^{2/3} > d \tag{4}$$

Inequality (4) establishes a condition the quantities, z_0, c, K, m, d should satisfy in order to obtain caustics generated by an initial curve that lies in the plane stress region. Fig. 3 presents the variation of the critical (maximum) value of specimen thickness, d_c, versus K_I for a parallel light beam illuminating a notched Plexiglas specimen. The caustic is created by transmitted light rays ($c_t = 1.08x10^{-10} m^2 N^{-1}$). Fig. 4 presents the variation of r_0 versus stress intensity factor K_I for a Plexiglas specimen of thickness d = 10 mm illuminated by a parallel light beam. The caustic is created by transmitted light rays and the reference screen is placed at distances z_0 = 0.1,1 and 10 m from the specimen. K_I varies up to 1 MPa√m corresponding to the value of fracture toughness of Plexiglas. In the same figure the line $r_0 = d/2$ is drawn. Observe that r_0 increases as K_I and z_0 are also increased. Only for the part of curves above the line $r_0 = d/2$, does the radius of the initial curve lie in the region of plane stress. It is observed that the realm of validity of the method of caustics under conditions of plane stress increases with K_I and z_0. Finally, Fig. 5 presnts the limits of applicability of the method of caustics for a convergent light beam with z_i = 40 cm, d = 2 mm (a) and 10 mm (b). Note that as the specimen thickness increases the limits of applicability of the method of caustics decrease.

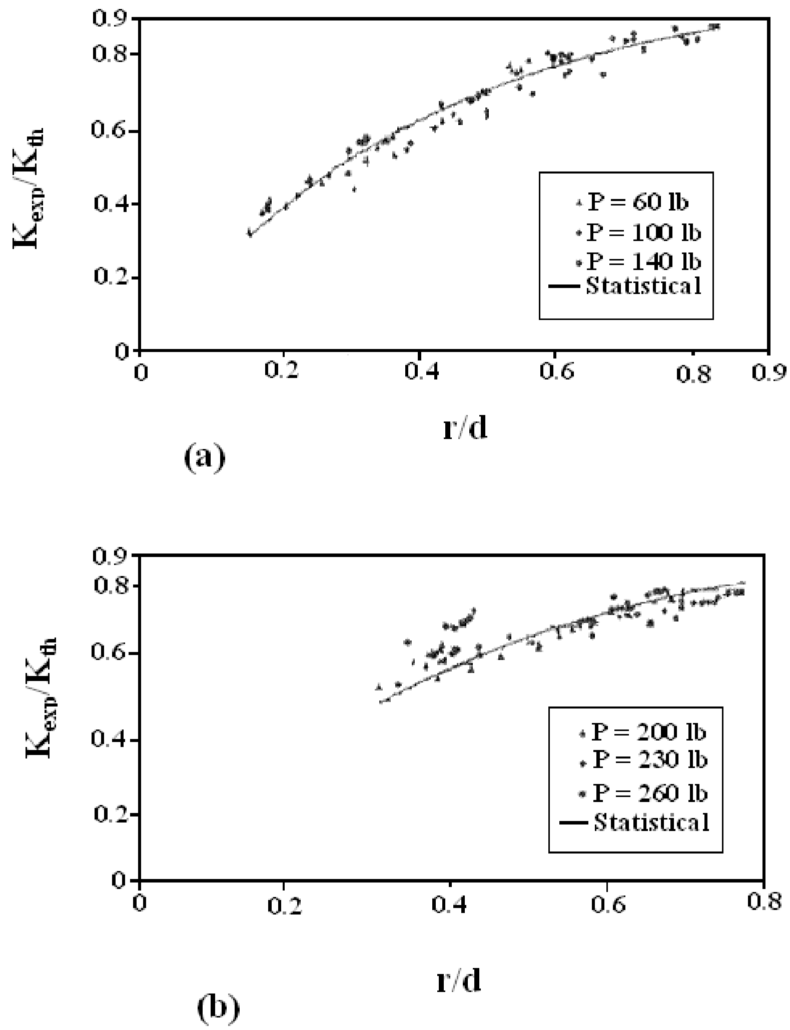

Fig. 2 Variation of K_{exp}/K_{th} versus r/d for a = 15.5 mm, d = 4.5 mm and w = 47.5 mm (a) and w = 63.5 mm (b)

Determination of Stress Intensity Factors

When the initial curve of the caustic lies at distances where three-dimensional effects dominate the proper value of the stress-optical constant, c, should be used. The value of the stress-optical constant changes from its plane strain value near the tip to its plane stress value at distances away from the tip approximately equal to half the specimen thickness. In order to characterize the three-dimensionality of the stress field near the crack tip an empirical triaxiality factor k is introduced, such that

$$\sigma_z = k\,\nu\,(\sigma_x + \sigma_y) \qquad (5)$$

where σ_z is the normal stress perpendicular to the plane of the specimen, and σ_x and σ_y are the in-plane stresses. k takes the values of 0 and 1 for plane stress ($\sigma_z = 0$) and plane strain [$\sigma_z = \nu (\sigma_x + \sigma_y)$], respectively.

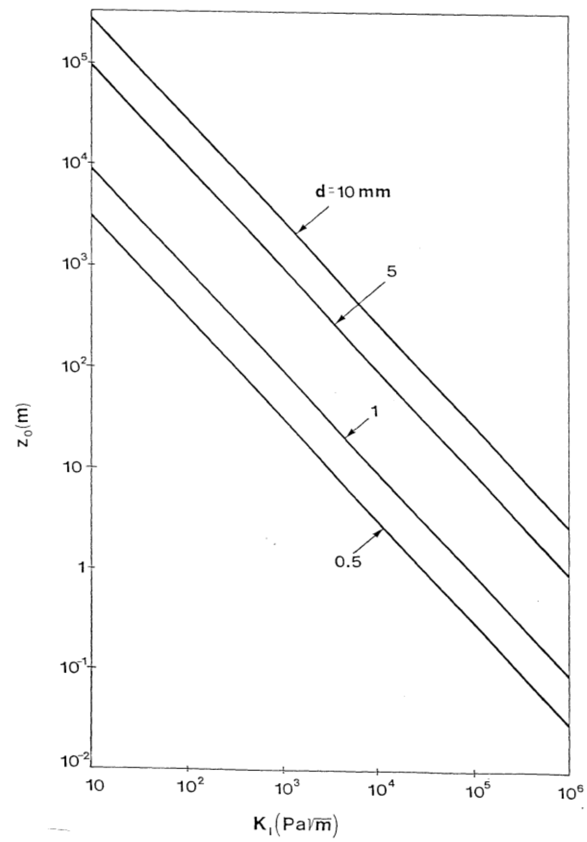

Fig. 3 Variation of maximum value of d (d_c) versus K_I for parallel light with d = 0.5, 1.0, 5.0 and 10.00 mm.

When the triaxiality factor is determined the corresponding value of the stress-optical constant, c, is calculated which subsequently is used for the determination of stress intensity factor. Fig. 6 presents the variation of the stress-optical constant c_t for transmitted light for Plexiglas (PMMA) versus the triaxiality coefficient k from its plane stress (k = 0) to its plane strain value (k = 1) for various values of the index of refraction n_0 of the surrounding medium. Note that c_t varies linearly with k. From Fig. 6 it is observed that c_t remains almost constant for $n_0 = 1.3$. This means that when the index of refraction of the medium surrounding the specimen is equal to $n_0 = 1.3$ the stress-optical constant c_t is independent of the state of stress near the crack tip. Under such circumstances Eq. (1) can be used for the correct determination of stress intensity factor K_I for any values of the parameters entering in Eq. (1).

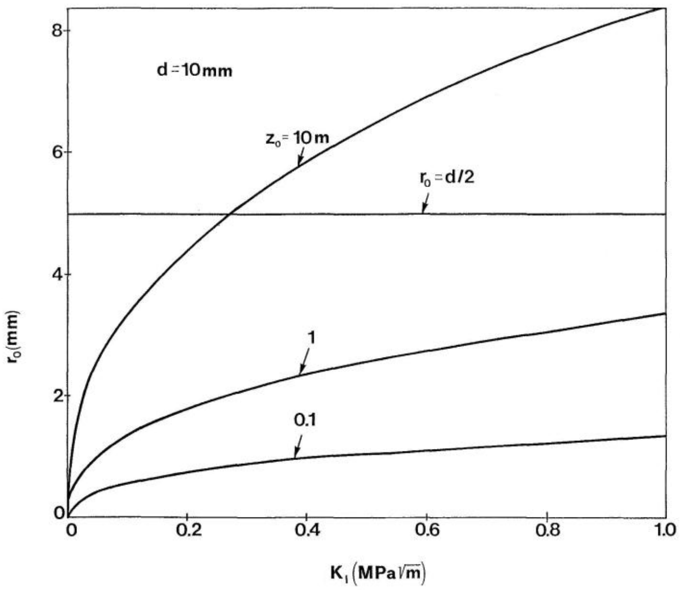

Fig. 4 Variation of r_0 versus K_I for parallel light. d = 10 mm, z_0 = 0.1, 1 and 10 m.

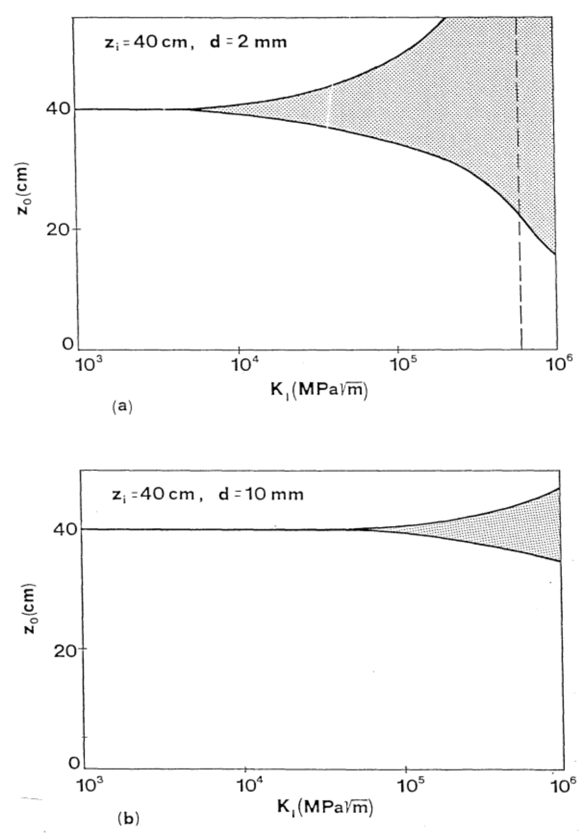

Fig. 5 Limits of applicability of the method of caustics for a convergent light beam with z_i = 40 cm, d = 2 mm (a) and 10 mm (b).

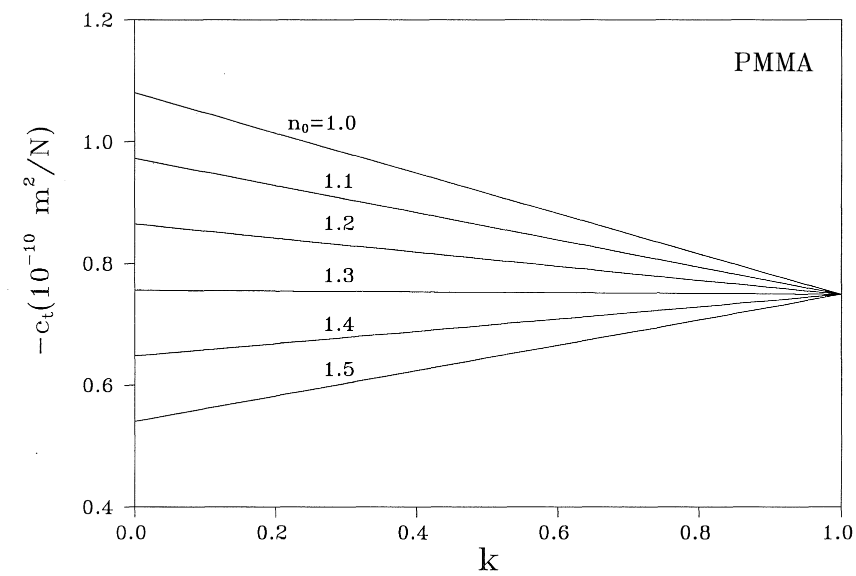

Fig. 6 Variation of stress-optical constant c_t versus triaxiality coefficient k for PMMA for various values of the index of refraction n_0 of the surrounding medium. k = 0 and 1 correspond to conditions of plane stress and plane strain, respectively.

Use of Optically Anisotropic Materials

In optically anisotropic materials the variation of the optical path of a light ray traversing the specimen along the two principal stress directions is given by:

$$\Delta s_{t1,2} = c_t \left[(\sigma_1 + \sigma_2) \pm \xi_{r,t} (\sigma_1 - \sigma_2) \right] d \tag{6}$$

where the coefficient $\xi_{r,t}$ characterizes the optical anisotropy of the material for light rays reflected from the rear face or traversing the specimen. The plus and minus signs in equation correspond to the values σ_1 and σ_2 of the principal stresses. Under such conditions the parametric equations of the caustic are given by [8]:

$$X_{r,t} = \left(\frac{3}{2} C_{r,t} \right)^{2/5} \left[A^{2/5} \cos\theta + \frac{2}{3} A^{-3/5} + \left\{ (\cos 3\theta/2) \pm \frac{3}{4} \xi_{r,t} \sin 2\theta \right\} \right] \tag{7a}$$

$$Y_{r,t} = \left(\frac{3}{2} C_{r,t} \right)^{2/5} \left[A^{2/5} \sin\theta + \frac{2}{3} A^{-3/5} + \left\{ (\sin 3\theta/2) \pm \frac{1}{4} \xi_{r,t} (1 + 3\cos 2\theta) \right\} \right] \tag{7b}$$

where:

$$A = \pm\frac{1}{4}\xi_{r,t}\sin\theta + \left[1 \pm \frac{1}{4}\xi_{r,t}\{(7\sin\theta/2 + (\sin 3\theta/2)\} + \frac{1}{32}\xi_{r,t}^2(25 + 9\cos 2\theta)\right]^{1/2} \quad (8)$$

$$C_{r,t} = \frac{\varepsilon z_0 d c_{r,t} K_I}{(2\pi)^{1/2}} \quad (9)$$

The equation of the initial curve is given by:

$$r = r_0 = \left\{\frac{3}{2}C_{r,t}A\right\}^{2/5} \quad (10)$$

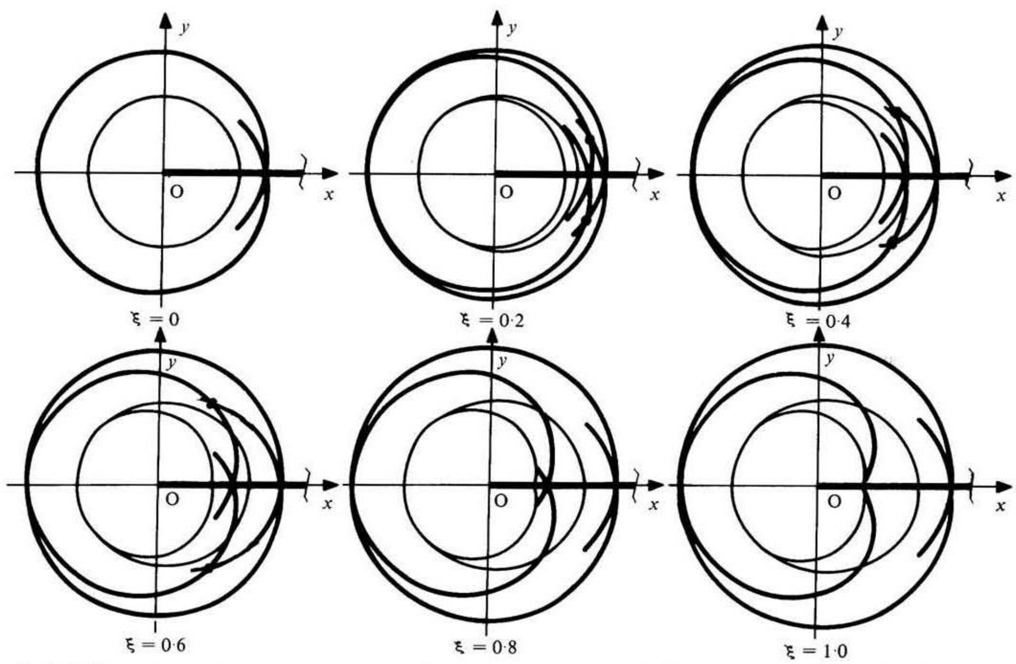

Fig. 7 Initial curves and respective caustics in a plate with crack subjected to tension made of birefringent materials with ξ= 0, 0.2, 0.4, 0.6, 0.8 and 1.0

Equations (7) express the equations of the caustic curve for optically anisotropic materials. Two caustics are obtained corresponding to the plus and minus signs in equations. These caustics are referred to the two principal stress directions. Note that for $\xi_{r,t}$ = 0 equations (7) and (9) reduce to the equations of the caustic for optically isotropic materials. Fig. 7 shows the initial curves and respective caustics in a plate with crack subjected to tension made of birefringent materials with ξ = 0, 0.2, 0.4, 0.6, 0.8 and 1.0 [8]. Observe that as ξ increases the shapes of the initial curves and caustics are progressively distorted. The distance between the two caustics increases as ξ also increases. The value of ξ depends on the state of stress, being plane strain, plane stress or three-dimensional. Thus the experimental caustics obtained can be used for the determination of the triaxiality factor k and the subsequent calculation of the stress-optical constant for the correct determination of stress intensity factors.

Conclusions

From the results of the present work the following conclusions may be drawn:

a. Direct application of the method of caustics without taking special precautions for the determination of stress intensity factors may lead to erroneous results.
b. The material, dimensions of the specimen, applied loads and geometrical dimensions of the optical arrangement should be properly selected to ensure that the initial curve lies in the plane stress region.
c. The above condition is satisfied for high applied loads, small specimen thicknesses, large distances between the specimen and the viewing screen and small magnification factors of the optical arrangement.
d. For specimens made of Plexiglas the stress-optical constant for transmitted light rays is independent of the state of stress around the crack tip for a value of the index of refraction of the medium surrounding the specimen approximately equal to 1.35. Under such condition the plane stress stress-optical constant of the material can be used for any location of the initial curve of the caustic.
e. Optically anisotropic materials can effectively be used for the determination of the state of stress around the initial curve of the caustic and the correct determination of stress intensity factors. For such materials the two caustics formed can be used for the determination of the triaxiality coefficient and the subsequent calculation of the corresponding stress-optical constant.

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