On modelling of fluid-driven fracture branching in jointed rocks

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Abstract. This paper investigates the parameters that controls branching and bifurcation of hydro fracture in rockmasses weakened by interfaces. The mechanism of generation of a network of the connected open-shear cracks is also discussed on the basis of formulation of the problem for the case when the delamination on the interface occurs over a part of the longer shear crack.

Introduction

Branching of discontinuities is a widespread phenomenon observed in both the structural materials or the earth's crust. In a brittle material, a dynamically propagating crack can deviate from its original straight trajectory and curve or split into two or more branches resembling a dendritic drainage pattern. In mathematical modelling of this process, the most frequently used criterion of crack propagation in 2D elastic brittle materials combines two hypotheses, namely, the Griffith's energy balance and the principle of local symmetry according to which the crack advances in such a way that in-plane shear stresses always vanish in the vicinity of the crack tip, e.g., [1]. Numerical simulations of the dynamic crack propagation behaviour have been difficult to develop and, to this date, a reliable method for simulating this complex problem has not been found in spite of considerable efforts in this direction (see details in [2]).

In the earth's crust, dynamic fracture branching is observed for complex earthquakes. The rupture zones of major earthquakes often involve geometric complexities including fault bends, branches and stopovers. Cracks' branching in brittle materials typically happens at acute angles relative to their propagation direction [3]. Off-fault secondary failures induced by a dynamic slip or slip pulse are theoretically investigated without appealing to strength anisotropy of the earth's crust induced by its divisibility at various scales. Nevertheless, the majority of fracture processes in the earth's crust, especially those driven by injected fluid, are related to inherent planes of weakness in the rockmasses. Examples include the development of magmatic structures, such as dikes and sills, which play an important role in the accretionary processes of the crust. The dikes intrude relatively vertical through a bedding plane, whereas the sills stemming from the dikes tends to intrude horizontally along the bedding plane. Discussions of physical mechanisms that may influence the dike propagation or arrest and the sill emplacement have been the focus of many studies; however, magma transport through fluid-filled fractures still represents a debated problem in geophysics.

At meso-scale, the most important and inherent feature of rocks is their jointing. This feature defines the orientation of larger fractures and lineaments in the crust. Structural fabrics such as joints are internal weaknesses of rocks possessing visual or statistically detectable regularity. A two family of mutually orthogonal planes equidistant from each other represent the simplest geometry of the joint systems. This geometry is typical; see for instance [4], for limestone (Fig. 1). The current study is aimed at modelling of the fluid-driven crack growth in the rockmass weakened by a network of natural joints.

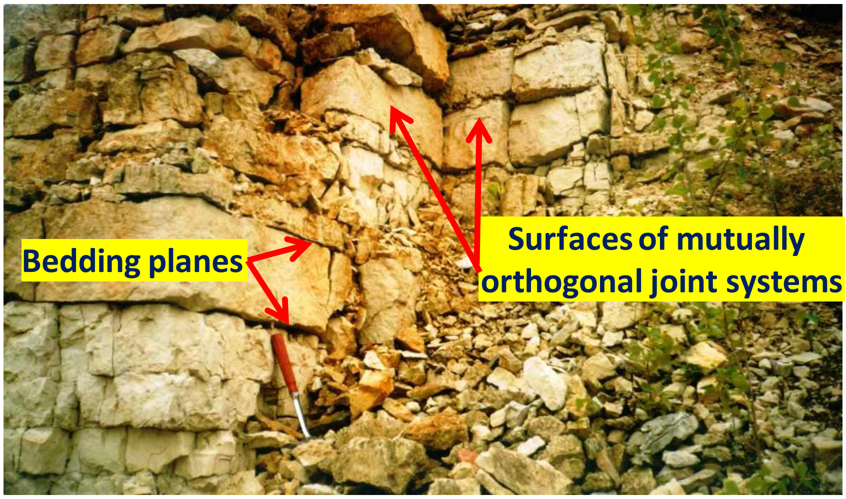

Fig. 1. Joint systems in Carboniferous limestones of the Moscow Syneclise (modified from [4]).

The paper also intends to provide an insight to explain the mechanism of so-called “compressional crossing” phenomenon [5] that occurs if compression perpendicular to the interface prevents slip while the tensile stresses acting parallel to the interface are sufficient to initiate a fracture (co-planar with the initial crack).

Quasistatic equilibrium of hydro-fracture.

We consider quasistatic propagation of a straight crack driven by fluid pressure towards a weak interface in rockmass. For convenience, let us assume that the crack occupies the interval (-*a*,*a*) of the *x*-axis, where *a* in the crack half-length. Since the direction of the crack propagation is usually associated with the direction of the maximum compressive principal stresses, $T_1<0$, acting in the rockmass, one can assume that the in-situ stress field (denoted by superscript 0 further on) can be modelled by the homogeneous stress tensor with the components

$$\sigma^0_{xx} = T_1, \quad \sigma^0_{yy} = T_2, \quad \sigma^0_{xy} = 0 \quad \left(\frac{T_1}{T_2} = \lambda > 1\right) \tag{1}$$

Here the magnitude of T_2 (minimum compressive principal stress) can be estimated as $|T_2|=\gamma h$ (γ is the specific weight of the overlying rocks and $h>0$ is the depth) in the case if the crack propagates horizontally; for vertical cracks $|T_2| < \gamma h$. For simplicity, we neglect variations of the in-situ stresses at the scale of crack length and assume that $a \ll h$. The latter condition allows one to consider the entire elastic plane. The crack is modelled by a mathematical cut, which assumes that the displacements suffer a jump when passing across the crack surfaces.

If the surfaces of a straight crack are loaded by arbitrary distributions of normal, σ_n and shear, σ_t, stresses, this creates the stress field described by two complex potentials $\Phi(z)$ and $\Psi(z)$, e.g. [6]

$$\begin{aligned}
\Phi^{cr}(z) &= \frac{1}{2\pi\sqrt{z^2-a^2}} \int_{-a}^{a} \frac{\sqrt{a^2-s^2}}{s-z}\left(\sigma_n(s)+i\sigma_t(s)\right)ds \\
\Psi^{cr}(z) &= \frac{1}{2\pi\sqrt{z^2-a^2}} \int_{-a}^{a} \frac{\sqrt{a^2-s^2}}{s-z}\left(\sigma_n(s)-i\sigma_t(s)\right)ds - z\Phi'(z) - \Phi(z)
\end{aligned} \tag{2}$$

The stress components are calculated by the Kolosov-Muskhelishvili formulas [6]; in particular, one obtains the following expression for the normal and shear stresses acting on the crack continuation along the x-axis caused by the crack presence

$$\sigma_{yy}^{cr}(x)+i\sigma_{xy}^{cr}(x)=\frac{1}{\pi\sqrt{x^2-a^2}}\int_{-a}^{a}\frac{\sqrt{a^2-s^2}}{s-x}\left(\sigma_n(s)-i\sigma_t(s)\right)ds, \quad |x|>a\ . \tag{3}$$

In the case of a hydro-fracturing, it makes sense to suggest that the shear stresses applied to the crack surfaces are much smaller that the normal stresses, which leads to the following equation of equilibrium on the crack line

$$\sigma_{yy}^{cr}(x)+p^{hf}(x)+\sigma_{yy}^{0}=0, \quad |x|<a \tag{4}$$

where $p^{hr}(x)>0$ is the pressure caused by the liquid acting inside the crack.
Therefore the complex potentials specified by Eq. 2 assume the form

$$\Phi^{cr}(z)=-\frac{\sigma_{yy}^{0}}{2}\left(1-\frac{z}{\sqrt{z^2-a^2}}\right)-\frac{1}{2\pi\sqrt{z^2-a^2}}\int_{-a}^{a}\frac{\sqrt{a^2-s^2}}{s-z}p^{hf}(s)\,ds, \quad \Psi^{cr}(z)=-z\Phi'(z)-2\Phi(z) \tag{5}$$

The potentials in Eq. 5 define the additional stress field created by an open crack in the rockmass, this field depends on in-situ stresses and on fluid pressure inside the crack. The latter can be found from the consideration of the problem of viscous flow through the channel with the cross-section defined by the crack opening displacements, which manifests a coupled problem and leads, in general, to non-uniform distribution of the liquid pressure that may also have certain lag, i.e. $p^{hf}(x)=0$ for $a-\delta<|x|<a$ ($0<\delta<a$). However for the analysis of the interaction between the hydro-fracture and natural pre-existing cracks in the rockmass (interfaces) the particularities of the pressure distribution are not so important, they do not affect much the mechanism of crack branching or its arrest. Therefore it is further accepted that the liquid pressure is constant, p_0^{hf}, over the whole crack length, which allows one to represent the complex potentials in an analytic form. Therefore

$$\Phi^{cr}(z)=-\frac{(\sigma_{yy}^{0}+p_0^{hf})}{2}\left(1-\frac{z}{\sqrt{z^2-a^2}}\right) \tag{6}$$

By using the Kolosov-Muskhelishvili formulas and superimposing the in-situ stresses in the intact rockmasses (Eq.1) one can express the stress components acting on an interface located perpendicular to the crack at a certain distance, d, from the crack tip. In particular, for the normal stress at any point lying on the crack continuation one finds

$$\sigma_{xx}(x,0)=-(\mathrm{T}_2+p_0^{hf})\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)+\lambda\mathrm{T}_2, \quad |x|>a, \quad (\lambda>1) \tag{7}$$

A normal crack may appear on the interface if $\sigma_{xx}(x,0)>0$. It gives an equation for the determination of the maximum distance, d_{max}, from the crack tip at which a small crack on the interface can be generated by the hydro-fracture propagating perpendicular to the interface

$$\frac{x}{\sqrt{x^2 - a^2}} = \mathrm{K}, \quad \mathrm{K} = 1 - \frac{\lambda \mathrm{T}_2}{\mathrm{T}_2 + p_0^{hf}}, \quad |x| > a \tag{8}$$

By introducing the dimensionless liquid pressure

$$\mu = -\frac{p_0^{hf}}{\mathrm{T}_2}, \quad \mu > 0 \tag{9}$$

one can express the maximum distance at which it is possible the formation of an open crack on the interface as a function of a single dimensionless parameter ρ combining two dimensionless parameters λ and μ as follows

$$d_{\max}(\rho) = \frac{1+\rho}{\sqrt{1+2\rho}} - 1, \quad \rho = \frac{\mu - 1}{\lambda}, \quad \rho > 0 \tag{10}$$

It is worth to emphasise that the parameter ρ characterises the degree of excess of the fluid pressure over the maximum compressive principal stress acting in-situ. In is evident that for large ρ the distance increases as a square root of this parameter, while for small ρ the distance is approximated by the expression $1/2\rho^2 - \rho^3$, which is accurate for $\rho<1/3$. The graph of the dependence in Eq. 10 is shown in Fig. 2.

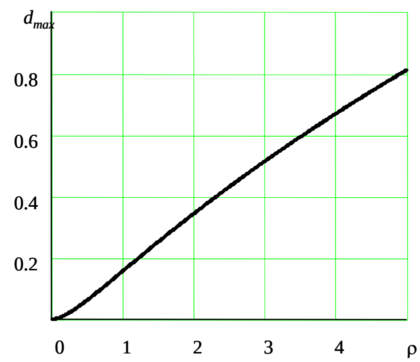

Fig 2. Maximum distance of crack generation as function of dimensionless parameter ρ.

The propagation of the hydro-fracture is described by means of the fracture toughness, K_{Ic}, bearing in mind that the mode I stress intensity factor for a straight crack of the length $2a$ under uniform load, σ, is given as $K_{\mathrm{I}}=\sigma\,(\pi a)^{1/2}$, one can find the following expression for the fracture toughness that has to be satisfied during the crack growth

$$K_{Ic} = -\mathrm{T}_2(\mu - 1)\sqrt{\pi a} \tag{11}$$

On the other hand, as evident from Eq 10 and Eq.11, the dimensionless parameter ρ can be linked with the fracture toughness as follows

$$\rho = -\frac{K_{Ic}}{\mathrm{T}_1\sqrt{\pi a}} \tag{12}$$

Therefore the dependence presented in Fig 2 can also be viewed as a function of fracture toughness and the major compressive in-situ stress. It can also be used for estimation of the fracture toughness from the observed initiation of a small open crack on the weak interface.

Crack development on the interface.

The stresses generated by the hydro-fracture on the interface can be obtained from complex potentials specified by Eq. 5 and Eq. 6. The profiles of normal and shear stresses acting on the interface are shown in Fig. 3. It is evident that the length of the zone of the tensile stresses decreases when the crack approaches the interface (the dimensionless distance δ in the figure is measured from the right crack tip to the interface, $\delta=d/a$). At the same time the intensity of the normal stresses (as well as shear stresses) increases. At the limiting case δ=0 they both become infinite near they crack tip. Therefore at certain distance, it is expected the appearance of a relatively small open crack located in the middle of the interface, i.e. on the intersection of the continuation of the hydro-fracture and the interface (the Origin in Fig 3). On the other hand one can also expect the appearance of the sliding zones of much bigger size (in the case if the shear strength of the interface is weaker than in surrounded rocks), which can be modelled by combined open-shear cracks. It is possible to estimate the lengths of both open and shear parts by analysing the crack equilibrium in the stress field created by the hydro-fracture (without taking into account the back influence of the interface cracks on the hydro-fracture).

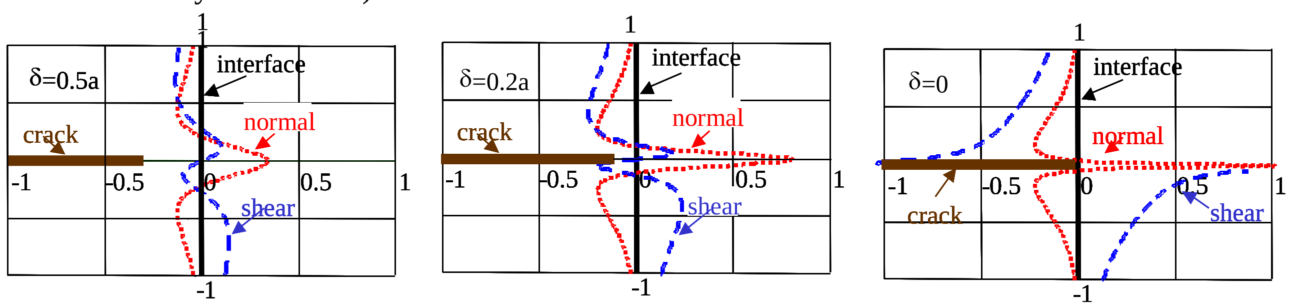

Fig. 3. Profiles of normal and shear stresses along the interface caused by hydro-fracture.

Let us consider a straight crack located on ($-L$,L) of the y-axis such that the open part is in the middle and its half-length, L_0, is less than the half-length of the shear part. Thus, on ($-L_0$,L_0) there is no resistance to shear, while on the rest of the crack pure sliding occurs without opening in accordance with the Mohr-Coulomb friction. The boundary conditions can be written in the form

$$\sigma_{xx}(0,y)=\sigma_{xy}(0,y)=0, \quad |y|<L_0 \tag{13}$$

$$\left|\sigma_{xy}(0,y)\right|=C-\sigma_{xx}(0,y)\tan\varphi=0, \quad \left(\sigma_{xx}(0,y)<0\right), \quad u_x^+(0,y)=u_x^-(0,y). \quad L_0<|y|<L \tag{14}$$

where C is cohesion and φ is a friction angle along the interface.

Since the normal load does not produce sliding and shear load does not contribute into crack opening the problem can be decoupled as follows.

At the first step the length of the open crack can be obtained from the analysis of the following singular integral equation (SIE),

$$\frac{1}{\pi}\int_{-L_0}^{L_0}\frac{g(s)}{s-y}ds+\sigma_{xx}^{hf}(y)+\lambda T_2=0, \quad |y|<L_0 \tag{15}$$

where the unknown real-valued function $g(x)$ is proportional to the density of the discontinuity of the normal displacements. The second term is taken from the solution for the normal stresses created by the hydro-fracture (see the previous subsection).

For uniqueness this SIE should be complemented by the condition of single valuedness of the normal displacements expressed in the form

$$\int_{-L_0}^{L_0} g(s)\,ds = 0 \tag{16}$$

The mode I stress intensity factor (SIF) at the end y=L_0 has the form

$$K_I = \frac{1}{\sqrt{\pi L_0}} \int_{-L_0}^{L_0} \sqrt{\frac{L_0+s}{L_0-s}} \left(\sigma_{xx}^{hf}(s) + \lambda \mathrm{T}_2\right) ds \tag{17}$$

The length of the open crack is found by assuming that K_I=0 vanishes. This provides the following condition (due to symmetry it is sufficient to analyse the SIF at one of the ends)

$$\int_{-L_0}^{L_0} \frac{\sigma_{xx}^{hf}(s) + \lambda \mathrm{T}_2}{\sqrt{L_0^2 - s^2}} ds = 0 \tag{18}$$

The condition in Eq. 18 also provides the boundedness of the sought function $g(x)$ at the ends. This function is found by using the inverse formula for SIE (Eq. 15) with the conditions given by Eq. 16 and Eq. 18. It can be presented in the form

$$g(s) = -\frac{1}{\pi}\sqrt{L_0^2 - s^2} \int_{-L_0}^{L_0} \frac{\sigma_{xx}^{hf}(t) + \lambda \mathrm{T}_2}{\sqrt{L_0^2 - t^2}\,(t-s)} dt = 0, \quad |s| < L_0 \tag{19}$$

Then the normal stresses acting on the y-axis can be found by substitution of $g(s)$ from Eq. 18 into the expressions for the complex potentials followed by integration, which yields

$$\sigma_{xx}^{open}(0,y) = \left(\sigma_{xx}^{hf}(s) + \lambda \mathrm{T}_2\right) - \frac{2}{\pi}|y|\sqrt{y^2 - L_0^2} \int_{-L_0}^{L_0} \frac{\sigma_{xx}^{hf}(s) + \lambda \mathrm{T}_2}{\sqrt{L_0^2 - s^2}\left(s^2 - y^2\right)} ds = 0, \quad |y| > L_0 \tag{20}$$

Therefore the normal stresses on the continuation of the open crack (and within the shear part of the interface cracks) becomes known, which makes it possible to perform the second step and to obtained the SIE based on the boundary conditions of the problem in Eq. 14. It has the form

$$\left| \frac{1}{\pi} \int_{-L}^{L} \frac{h(s)}{s-y} ds + \sigma_{xy}^{hf}(0,y) \right| = C - \tan\varphi \begin{cases} 0, & |y| \le L_0 \\ \sigma_{xx}^{open}(0,y), & L_0 < |y| < L \end{cases} \tag{21}$$

Here the shear stress produced by the hydro-fracture is taken from the results of the previous subsection, $h(s)$ is a real valued function proportional to the density of the jump of the tangential displacements. This SIE should be complemented by the condition expressing the single valuedness of the tangential displacements over the whole contour ($-L$,L), i.e.

$$\int_{-L}^{L} h(s)\, ds = 0 \tag{22}$$

It should be noted that the jump of the tangential stresses can have zeroes on the interval (-L,L) apart from its ends, in particular at x=0.

Also, if the normal stresses tend to infinity at the ends of the interval (-L_0,L_0) then the Mohr-Coulomb criterion cannot be satisfied, thus it is reasonable to assume that the total normal stress is negative and bounded, which necessitates the condition K_I=0 used above for the determination of the length of the open crack. The length of the shear crack on the interface can be found from the condition of bounded shear stresses at the crack ends. This means that the mode II SIF vanishes at the ends, therefore the analysis applied to the open cracks, as above, is also applicable to the determination of the sliding zone.

For the case when the pressure in the hydro-fracture is homogeneous over the whole length of the crack it is possible to perform evaluation of the integrals in a closed form. Thus, the estimation of the open crack length can be obtained by means of the following integrals

$$G_2(L_0,d) = \frac{2}{\pi}\int_0^{L_0} \frac{G_1(s,d)}{\sqrt{L_0^2 - s^2}}\, ds - 1, \quad G_1(s,d) = \mathrm{Re}\left[\frac{1}{\sqrt{2+d+iy}\sqrt{d+iy}}\left(1+d+iy-\frac{iy}{(2+d+iy)(d+iy)}\right)\right] \tag{23}$$

The dependences of $G_2(L_0,d)$ as a function of L_0 for different d are show in Fig. 4.

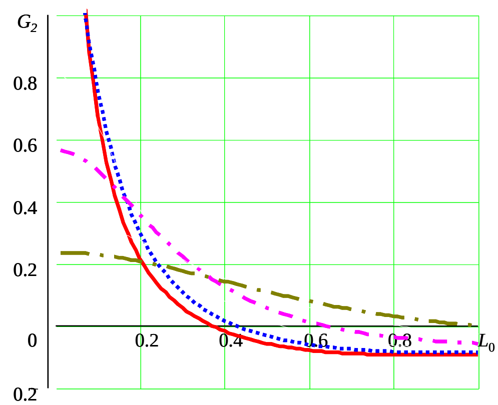

Fig 4. The dependences of $G_2(L_0,d)$ for different $\delta=d/a$: δ=0.05 – solid line; δ=0.1 – dot line, δ=0.3 – short dash-dot line, δ=0.7 – long dash-dot line.

The dimensionless length of the open crack (a=1) is determined from the solution of the equation

$$\rho G_2(L_0,d) = 1 \tag{24}$$

It can be seen from the figure that this length cannot exceed app. 0.4a for d<0.1a, app. 0.65a for d<0.3a and a for d=0.7a even if the parameter ρ is large; for intermediate values of ρ the length is much less the length of the hydro-fracture length.

Discussion and conclusions.

The length of the sliding zone depends on the length of the open zone of the interface crack and the other parameters involved. However it is evident the L should be greater than L_0 and essentially greater for the interfaces with the weak shear strength. This means, that regardless of the presence of the open zone, sliding can propagate far enough and reach the nearest interface perpendicular to the one along which it propagates. Fig 5 illustrates the normal and shear stresses that will be created by the mode II crack on the interface perpendicular to it.

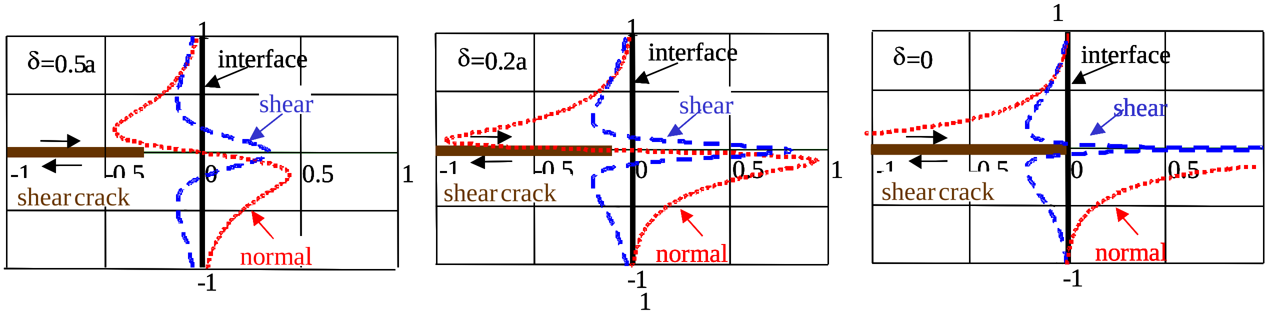

Fig 5. Profiles of normal and shear stresses on an interface located perpendicular to the shear crack of the length 2a for different distances from the shear crack tip.

It is evident from the figure that the sliding zone over one interface is capable to initiate an open crack on the perpendicular interface due to tensile stresses initiated by the shear crack. Therefore one can view the crack propagation as consecutive zigzag steps shown in Fig 6. Also, at every step the crack may not only branch but generate two open cracks (at least in symmetrical situations as considered above). The number of such cracks theoretically can be doubled after each step that can produce a network of multiple connected open-shear cracks. The liquid from the initial cracks can possibly open all the sliding part of the combined cracks, which may dramatically increase permeability of the rockmass.
The present study provides estimation of the geometrical and geomechanical parameters for the process of delamination of the interface caused by the approaching hydro-fracture. It is shown that the possibility of delamination is controlled by a single parameter specified by Eq. 10 that can be related to the fracture toughness in Eq. 12 and the length of the delaminating zone, Eq. 24. We also present the formulation of the combined open-shear crack equilibrium for the case when delamination occurs along a part of such crack. The formulation allows to decouple the considered crack problem and to justify the approach for description of possible ways of crack branching and bifurcation.

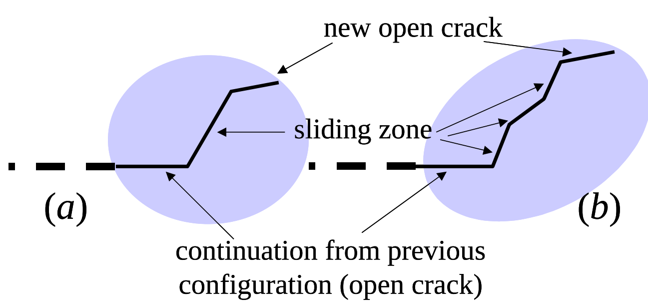

Fig 6. Zigzag steps in crack propagation: 3-leg step – (a), multiple-leg step – (b).

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