Numerical Prediction of Crack Path in Pre-Cracked Rocks under Uniaxial Compression Using Bonded Particle Model

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Abstract. *Sometimes an engineering structure seems to be stable and safe according to elasticity and material mechanics theories but due to pre-existing cracks or induced cracks during loading, it yields in lower level of stress and this sudden failure can lead to heavy losses. To prevent these unexpected failures, crack propagation mechanism should be studied carefully. Cracks propagate in rocks and rock-model materials in two forms of wing cracks and secondary cracks. Extensive theoretical, experimental and numerical studies on the failure process of intact rocks have been done. Goal of this research is to study numerically the mechanism of crack propagation in the rock under compressive load to predict crack path. For this purpose, three pre-cracked rock specimens with a flaw in the centers with different angles under uniaxial load were modeled in* PFC^{2D} *(a bonded particle model) and crack propagation processes were monitored. Results show firstly tensile cracks (wing cracks) starts at the ends of the initial and secondary cracks are produced coplanar or quasi-coplanar to the flaw later. Also, it has been concluded that due to increase of the angle between orientation of flaw and axial load direction, the axial fractures detain and more shear fractures develop and coalesce to form shear faults. The resulted crack paths have a very good agreement with crack paths which are observed in laboratory tests.*

Introduction

Fracture mechanics is the field of mechanics concerned with the study of the formation of cracks in materials. Previous research has shown that Griffith's brittle fracture theory can be modified to account for the effects of crack closure in compression [1, 2]. McClintock and Walsh (1962) have shown a useful basis for the study of the fracture of hard rocks based on the modified Griffith theory [3]. Brace (1964), in discussing the nature of the pre-existing cracks in rock, suggests that the grain boundaries act as or contain microcracks while joints and faults can be regarded as macro-cracks [1]. An analysis of the stress distribution around a crack indicates the points of fracture initiation as well as the initial direction of crack propagation [4-6]. As a result of the change in stress distribution associated with fracture propagation it is, however, impossible to predict the final path of the propagating crack. Consequently, a serious limitation of the Griffith theory lies in the fact that it can only be used to predict fracture initiation.

Recently, several researches have shown that the Mode II fracture toughness of rock material is usually higher than the Mode I fracture toughness, especially when the confining pressure increases [5, 6]. In this paper, the crack propagation mechanism of rocks under compressive loading conditions has been investigated. The Bonded Particle Method (BPM) which is a newly developed method for analysis of rock-like materials has been used for estimating the crack initiation angle and its propagation path [7, 8]. It has been concluded that due to increase of the angle between orientation of flaw and axial load direction, the axial fractures detain and more shear fractures develop and coalesce to form shear faults.

Analytical solution and theoretical condition for fracture initiation

Erdogan and Sih have analyzed crack propagation under the two dimensional general stress system [4]. In particular their work includes the case of a plate under a uniaxial uniform tension whit a central crack of length *2a* inclined at angle *β* to the direction of the stress as shown in Fig. 1 .after resolution of the stress, the crack may be regarded as subject to stress system of P_y normal to the crack, p_x parallel and a shear stress of P_{xy} having the values:

$$P_y = PSin\beta,\ P_x = PCos\beta \text{ and } P_{xy} = PSin\beta Cos\beta \qquad (1)$$

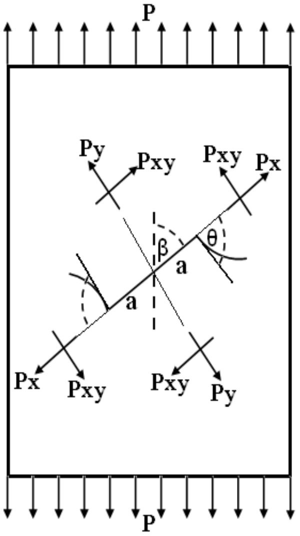


Fig.1. Wing crack formation at the tips of a center crack under uniaxial tensile stress

The analysis examines the stress distribution around the crack tip and postulates that the crack will run in a direction dictated by the maximum value of the stress P_θ normal to a radial line from the crack tip as shown in Fig.1. The result of their analysis is a curve of *β* versus *θ* (the angle *θ* is negative because the crack grows downwards). Propagation of the crack normal to the applied stress P_y would be given by the straight line $\beta + (-\theta) = \pi/2$.

The Erdogan-Sih theoretical solution therefore predicts that if β> 30 ° the growth should be below the horizontal, whereas for β< 30 ° above the horizontal (Fig. 2) [4].

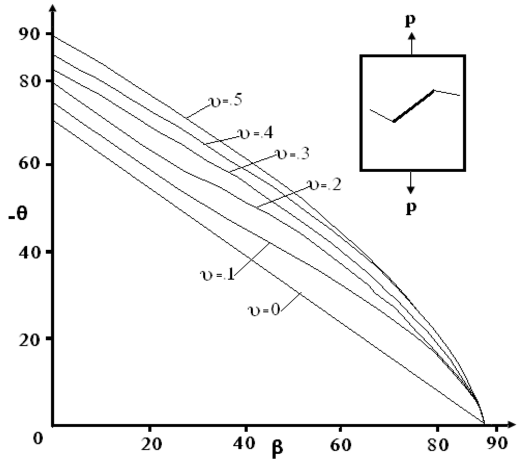


Fig.2.Variation of crack extension angle θc versus crack inclination angle β

A common mode of failure of rock specimens under uniaxial compression is splitting where the fracture surface is approximately parallel to the direction of applied loading [9]. Splitting is generally observed in cylindrical solid specimens and has been reported in many rock types. This type of splitting failure normally involves a sequence of progressive micro-fracturing. These initiated cracks will propagate with increased loading (Fig. 3).

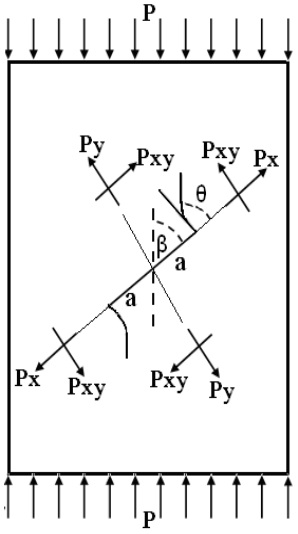


Fig.3. Wing crack formation at the tips of a center crack under uniaxial compressive stress

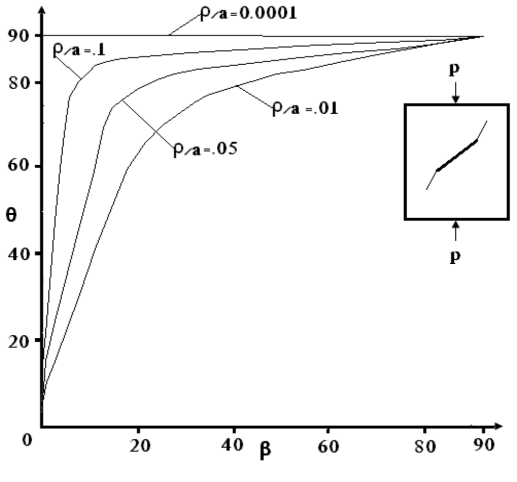


Fig.4. Variation of initiation angle for compressive loading based on the maximum stress criterion

Before the work of Erdogan and Sih (1963) much of the work on fracture mechanics was based on the assumption that a crack would be initiated in a self-similar manner but the question is that of what is the exact fracture toughness envelope as a criterion for mixed mode I-II cracking.

In order to generalize the analysis of mixed mode I-II fracture problems, a typical loading configuration is so chosen that angled discrete crack is subject to a uniform remote stress *P*, as shown in Fig.3 in which the inclined crack oriented at an angle with respected *P*. The quantity *P* may be tensile ($P \geq 0$) or compressive ($P \leq 0$), but in Fig 4 it is initiated to compressive case.

Among the classic fracture criteria for crack analysis, the maximum circumferential stress theory states that: when the circumferential stress at crack tip reaches critical value, the wing crack will occur and begin to spread along the direction of the maximum circumferential stress from the tip.

Numerical simulation

In the numerical simulation by PFC^{2D}, rock material is represented by an assembly of rigid circular disks bonded together at their contact points. PFC contains two bonding models: a contact-bond model and a parallel-bond model. The parallel-bond has a finite size that acts over either circular or rectangular cross section between the particles, whereas the contact-bond acts only at the contact point due to its vanishingly small size, which can be embodied with the parallel-bond of radius zero. Therefore, the contact-bond can only resist the force acting at the contact, while the parallel-bond can resist both the force and moment. The parallel-bonds are activated with five parameters, such as normal and shear bond strength, normal and shear bond stiffness, and the bond radius, among which the bond stiffness and bond radius are not as signed in the contact-bond model. The contact/parallel-bonds are broken if the applied stresses are larger than the bond strengths [10]. In the contact-bond model, bond breakage may not affect the macro stiffness significantly provided the particles remain in contact. However, in the parallel-bond model, bond breakage induces an immediate decrease in macro stiffness because the stiffness is contributed by both contact stiffness and bond stiffness. Therefore, the parallel-bond model can be more realistic for rock-like material modeling in which the bonds may break in either tension or shearing with an associated reduction in stiffness [11]. So, parallel-bond model is chosen for chosen for rock simulation in this investigation. A parallel-bond model is defined by five micro parameters consist of: normal and shear stiffness (k_n) and (k_s) [stress/displacement]; normal and shear strength (σ_c) and (τ_c) [stress]; and bond radius (R). These micro parameters should be adjusted to reproduce the macro properties of the real specimen under uniaxial compression such as Young’s modulus, UCS, and Poisson’s ratio which this adjustment process is done by calibration process. The calibration process is a "trial and error" process which by changing micro parameters in simulated biaxial or Brazilian tests, laboratory values of macro parameters (Young’s modulus, UCS, and Poisson’s ratio) will be produced. When simulated results and laboratory results are the same, the applied micro parameters will be set as input for numerical simulation. Also there are some quantitative techniques for selection of micro parameters [12]. In this study, calibration process is done by simulating biaxial test.

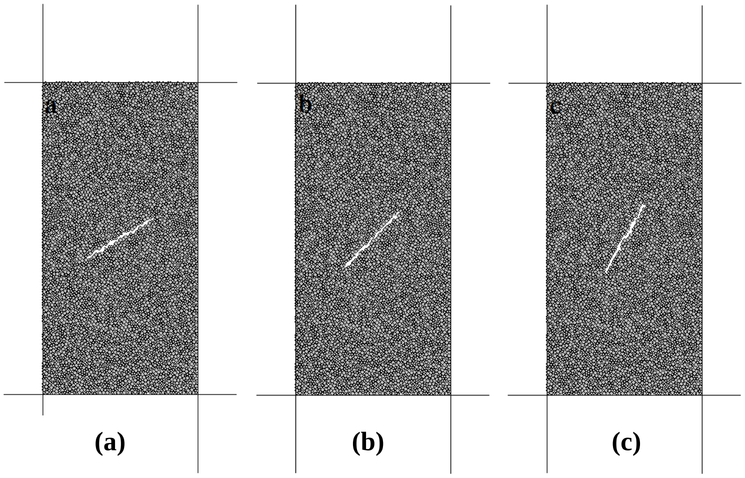
The uniaxial compressive test is going to be simulated to study mechanism of crack propagation from a single flaw in rock under compression. Physical and mechanical properties of simulated rock specimen are summarized in Table1. Also Table2 represents the selected micro parameters for numerical simulation by PFC^{2D}. The specimen has the height of H=12cm and width of W=6cm. The PFC^{2D} model of the uniaxial test is given in Fig. 3. After reproduction of specimen, the flaw is generated by deleting the particles in the appropriate region. Another method to generate the flaw is to set strength of particles between adjacent DEM elements in the crack surface equal to zero. Nohut modeled the notch in the single-edge-notched beam (SENB) test with both methods and observed same results [13]. It is reasonable because the radius of the notch tip is not determined by the radius of the particle which is deleted or whose bond strength is assigned to zero but it is determined by the radius, location and the arrangement way of the particle pair which stay just above the notch. Therefore it does not have any effect on the crack sharpness. The flaw has a length of 3.0 cm (w/a=4) and opening of the flaw is 1.5 mm. Four walls of model plays role of platens and confinement pressure. For that, lower wall is fixed, upper wall moves down with the velocity of V_p to simulate loading platen and lateral stresses are imposed by right and left walls. In next step, several simulation of uniaxial test on rock specimen with different central angled flaw of 30˚, 45˚ and 60˚ will be performed.

Table.1. Physical and mechanical properties of simulated rock

Properties	**Value**
Density (kg/m3)	2700
Young Modulus (GPa)	55.5
Uniaxial compressive strength (MPa)	145
Tensile strength (MPa)	47.60
Poisson ratio	0.27

Table.2. Microparameters used for the PFC^{2D} model

Microparameters	**Value**
Particle mean radius (mm)	0.4
Particle radius ratio, R_{max}/R_{min}	1.66
Particle density (kg/m^3)	2700
Particle contact modulus, Ec (GPa)	44.5×10^9
Particle stiffness ratio, k_n/k_s	1.0
Parallel-bond radius multiplier	1.0
Parallel-bond modulus, E_c (GPa)	44.5×10^9
Parallel-bond stiffness ratio, k_n/k_s	1.0
Particle friction coefficient	0.50
Parallel-bond normal strength, mean (MPa)	177.8×10^6
Parallel-bond normal strength, std. dev. (MPa)	45×10^6
Parallel-bond shear strength, mean (MPa)	177.8×10^6
Parallel-bond shear strength, std. dev. (MPa)	45×10^6



(a) **(b)** **(c)**

Fig.5. PFC^{2D} models of rock specimen with a single flaw in compressive test with the angle of: a) 30˚, b) 45˚ and c) 60˚

Results

To study crack propagation mechanism, several numerical simulations are carried out on the specimens. The specimens are in with different central angled flaw of 30˚, 45˚ and 60˚. Table 3 shows the values of axial stress at different stages of simulation (75% peak stress, 90% peak stress, peak stress, 90% post-peak stress and 80% post-peak). The stress-strain curves and the failure developments for specimens under compressive loads are illustrated in Figs 6 and 7, respectively.

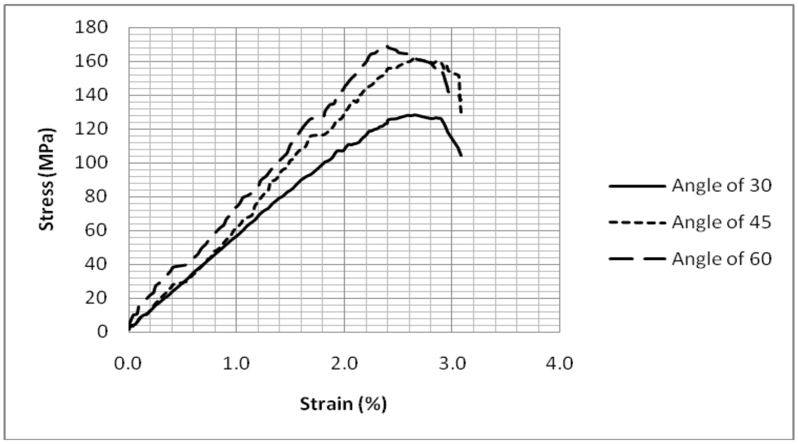


Figure.6. the stress-strain curves of specimens

Table.3. Values of axial and confinement stresses at different stages of simulation

Central Flaw Angle	Axial Stress (MPa)				
	75% Peak	90% Peak	Peak	90% Post-Peak	80% Post-Peak
30˚	95.8	115	127.8	115	106.2
45˚	119.2	143.1	158.9	143.1	125.8
60˚	125.5	150.6	167.3	150.6	135.3

Conclusion

In this paper, mechanism of crack propagation under uniaxial compression has been investigated. Several simulations of uniaxial compressive test have been executed on specimens with different central angled flaw of 30˚, 45˚ and 60˚.

Some common characteristics of failure development are resulted from numerical simulations which are same to laboratory observations. These characteristics consist of: a) initial micro cracks form in the pre-failure stage that due to existence of the flaw, their development are highly affected by stress concentration around the flaw, b) fractures are formed near peak stresses and c) fracture planes or faults appear in the post-failure stage and cause the specimen to fail.

Results have shown firstly tensile cracks (wing cracks) starts at the ends of the initial and secondary cracks are produced coplanar or quasi-coplanar to the flaw later. Also, it has been concluded that due to increase of the angle between orientation of flaw and axial load direction, the axial fractures detain and more shear fractures develop and coalesce to form shear faults.

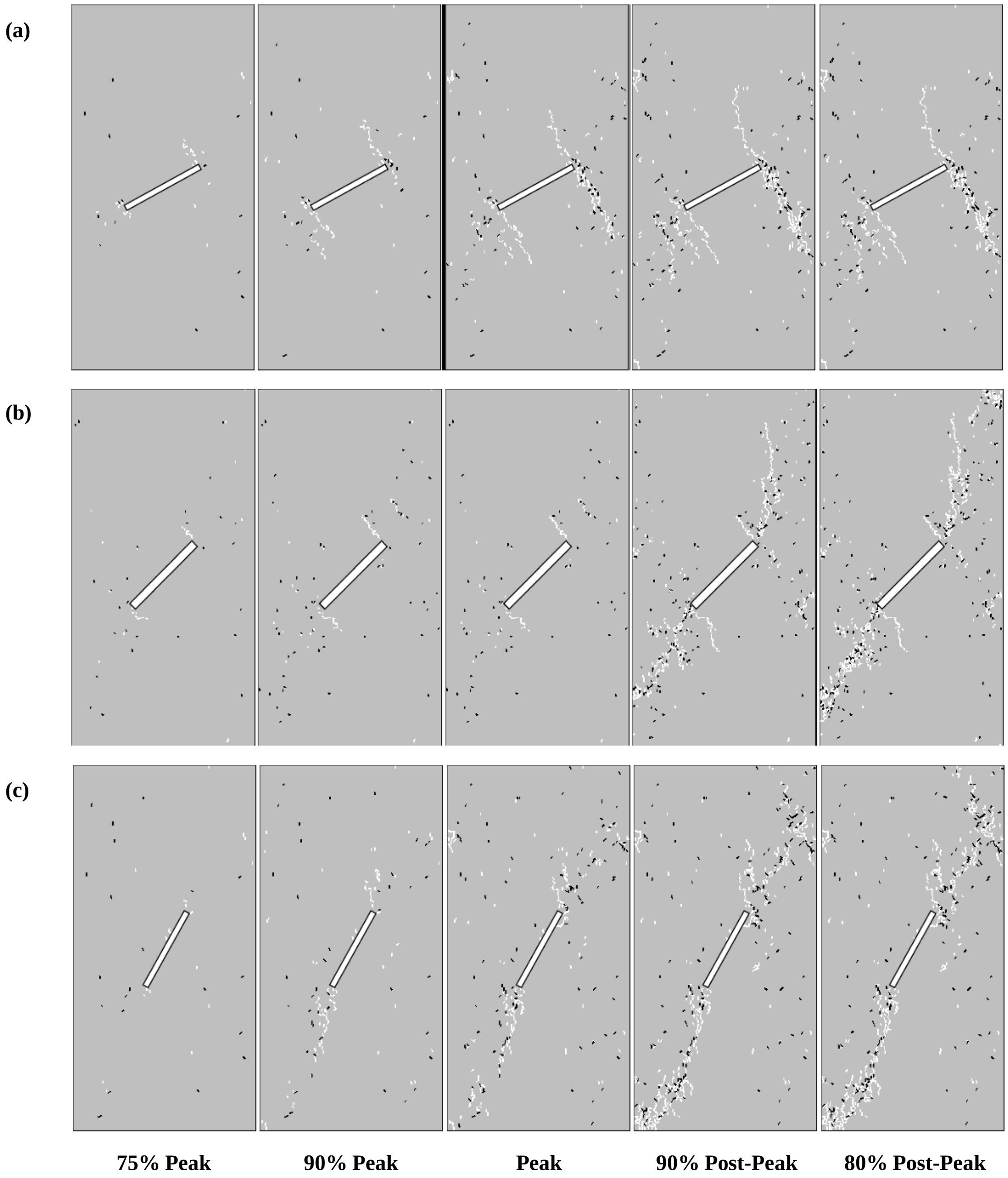


Fig.7. Crack propagation in single-flawed specimens with the angle of: a) 30˚, b) 45˚ and c) 60˚ from PFC^{2D} simulation

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