Notch Fracture Mechanics Approaches in an Analysis of Notch-like Defects

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Abstract. The concept of notch fracture mechanics has been developed for describing the notch failure assessment diagram as well as the J-integral for U- and V-notches under Mode I loading and materials obeying a power hardening law. Effects of constraint were incorporated into the basic equations which describe the constraint-dependent fracture toughness and failure assessment diagrams for various structural elements with a crack/notch and various types of loading. It is shown that a crack can be considered as a special case of a notch. The load separation method has been employed to measure the notch fracture toughness $J_{\rho,c}$ using non-standard specimens with notches. Structural integrity assessment of the components damaged by notch-like defects is discussed from viewpoint of the notch failure assessment diagram. Acceptable state of the damaged component with a notch-like defect is determined by introducing safety factors against fracture and plastic collapse in the fracture criterion describing the notch failure assessment diagram.

Introduction

Crack-like defect assessment methods can be based on two different philosophies, namely, failure assessment diagram (FAD) and crack driving force (CDF). At the present time, fracture mechanics principles are applied to study stress distribution in the vicinity of the notch tip and for describing failure of the components with notch-like defects. For example, the basic failure assessment diagram has been modified using the concept of the notch stress intensity factor. In this case, so-called a notch failure assessment diagram (NFAD) is written in terms of K_{notch}/K_{Nmat} and σ_C/σ_Y for a notch-like defect taking into account a finite notch tip radius ρ [1-6] and can be used for structural integrity assessment of a component. Here, K_{Nmat} is the notch fracture toughness, σ_C is the failure stress, σ_Y is the yield strength. The stress intensity factor K at the notch tip is denoted as K_{notch}. The notch fracture toughness, which is applied to the NFAD, should be calculated or measured for a structural component. Moreover, the FITNET assessment of a structural component by the standard and advanced J-integral based options also requires notch fracture toughness data in terms of the J-integral derived from tests of notched specimens.

The aim of this paper is to give a brief survey of some aspects of notch fracture mechanics applied to a component with a notch-like defect.

Notch Failure Assessment Diagram

Basic Criterion Equation. The methodology of the criterion of average stress in the facture process zone ahead of the notch tip is employed to develop failure assessment diagrams for a solid with a finite U-notch under mode I loading. The normal stress distribution at the notch tip is similar to that crack under uniform remote tensile stress but shifted from the notch tip to a point of abscissa to $\rho/2$, i.e. $r \geq \rho/2$, where ρ is the notch tip radius. The stress distribution is simplified considerably on the continuation of the U-notch [7]. Averaging these stresses over the fracture process zone ahead of the notch tip, the fracture criterion leads to the NFAD for a notch-like defect in the form [5, 6]

$$K_{notch} = K_{Nmat}\sqrt{1-\left(\frac{\sigma_C}{\sigma_0}\right)^2}\ . \tag{1}$$

The notch fracture toughness K_{Nmat} can be written as a function of the fracture toughness K_{mat}, the elastic stress concentration factor K_t, the local strength σ_0 within the fracture process zone and the applied failure stress σ_C as follows

$$K_{Nmat} = K_{mat}\left[1-\left(\frac{\sigma_0}{\sigma_C}\right)^2\frac{1}{K_t^2}\right]^{-1/2}\ . \tag{2}$$

Equation 2, describing the notch fracture toughness, suggests that the loss of constraint due to a notch (K_t) is independent on the loss of constraint due to the non-singular T-stress in the Williams series solution which is introduced into the local strength σ_0 to quantify constraint in different geometries and type of loading. In this case, the right-hand side of Eq. 1 could be considered as the notch constraint-dependent fracture toughness K_{Nmat}^c

$$K_{Nmat}^c = K_{mat}\sqrt{1-\left(\frac{\sigma_C}{\sigma_0}\right)^2}\left[1-\left(\frac{\sigma_0}{\sigma_C}\right)^2\frac{1}{K_t^2}\right]^{-1/2}, \tag{3}$$

which is transferred in the constraint-dependent fracture toughness K_{mat}^c for a crack at $K_t \to \infty$. Consideration of a crack as a special case of a notch ($K_t \to \infty$) changes the notch fracture toughness K_{Nmat} (Eq. 2) into the fracture toughness K_{mat} (for a crack) and Eq. 1 leads for the crack

$$K = K_{mat}\sqrt{1-\left(\frac{\sigma_C}{\sigma_0}\right)^2}\ , \tag{4}$$

It means that the proposed NFAD (Eq. 1 and Eq. 4) is the unified failure assessment diagram for a notch as well as a crack (Fig. 1). Difference of these two cases is just connected with calculation of the fracture toughness (K_{mat} or K_{Nmat}) and a position of the corresponding points for the notch and the crack of same length on the failure assessment curve.

The Local Strength. The local strength σ_0 within the fracture process zone is treated according to von Mises yield criterion as a property of both the yield stress and the non-singular T-stress which is introduced into the criterion to quantify constraint in different geometries and type of loading. The local strength σ_0 can be expressed as follows [5, 6]

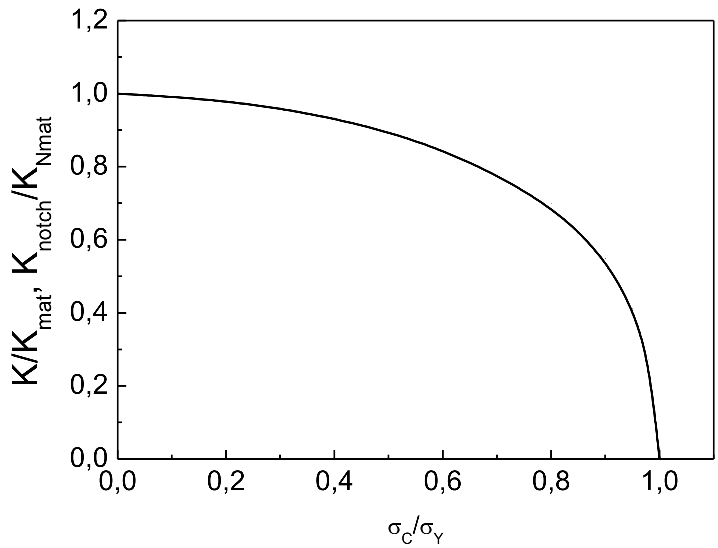

Fig. 1. The unified failure assessment diagram for a notch or a crack.

$$\sigma_0 = -\frac{T}{2} + \sigma_Y \sqrt{\frac{1}{4}\left(\frac{T}{\sigma_Y}\right)^2 - \frac{(1+\nu^2-\nu)(T/\sigma_Y)^2 - 1}{(1-2\nu)^2}}, \qquad (5)$$

for plane strain and

$$\sigma_0 = -\frac{T}{2} + \sigma_Y \sqrt{1 - \frac{3}{4}\left(\frac{T}{\sigma_Y}\right)^2}, \qquad (6)$$

for plane stress. Here, σ_Y is the yield strength. The local strength for finite geometries can be rewritten as a function of the applied failure stress and the notch (or crack) tip constraint characterized by a dimensionless parameter $\beta(a/W) = T/\sigma_C$ (so-called biaxiality ratio) which depends on geometry and loading mode. Values of β can be considered as a normalized measure of the crack-tip constraint and it is assumed to be the same for both the crack and the notch. The biaxiality ratio has been tabulated for various geometries in literature.

Validation of the Proposed FAD

The validation study is made on through-cracked plates made of different materials at different temperatures. The fracture toughness K_{mat} has been evaluated from Eq. 4 employing the fracture data and the calculated local strength. The failure assessment diagram has been constructed using these parameters. The experimental constraint-dependent fracture toughness K_{mat}^c was calculated for these cracked plates subjected to a uniform failure tensile stress σ_C using the well-known equation for the stress intensity factor K.

Comparison between the variation of the predicted constraint-dependent fracture toughness K_{mat}^c with the failure stress and the experimental results for crack growth initiation in aluminium alloy centre-cracked tensile specimens at 293 K shows good agreement for $0.8 \le \sigma_C/\sigma_Y$ (Fig. 2) [6].

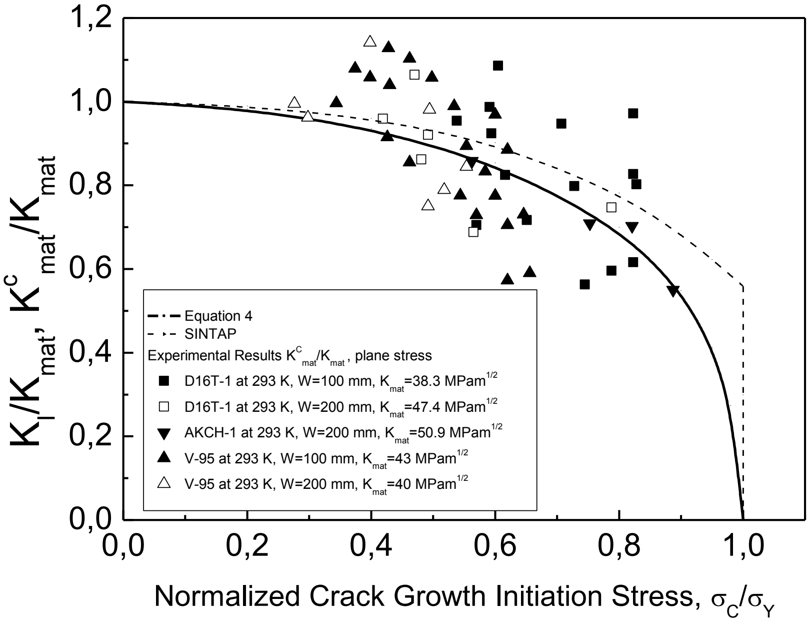

Fig. 2. Comparison of predicted and experimental results of the FAD.

The Concept of the J-integral for Notches

J-integral Evaluation for U- and V-blunt Notches. For the J-integral evaluation in the case of a plate weakened by lateral and central U- and V-blunt notches under mode I loading, the integration path has been assumed to be coincident with the semi-circular arc of the notch, which is traction free. The expression of the J-integral as a function of the strain energy over the notch edge, considering only the notch arc contribution of a blunt V-notch (and excluding the contribution of the rectilinear flanks), can be written in the case of a generic opening angle 2α (different from zero) as follows

$$J = \int_{-\pi/2+\alpha}^{\pi/2-\alpha} W(\theta)\,\rho\cos(\theta)d\theta\,. \qquad (7)$$

For a numerical investigation of the strain energy density distribution on the notch edge the equation

$$W(\theta) = W_{\max}\cos^{\delta}(\theta) \qquad (8)$$

has been assumed, where δ has been determined from finite element analyses [8]. A multi-parametric analysis has been carried out considering a large variability of the notch acuity $4 \le a/\rho \le 400$ and the opening angle $0 \le 2\alpha \le 3\pi/4$ taking into account both a linear elastic and nonlinear elastic material, the latter being modelled according a power hardening law. In order to analyze the effect of different load intensities on the strain energy density and J-integral formulations, different stress levels σ/σ_Y were applied to the plates. The results from the finite element analysis have demonstrated that the exponent δ depends on the notch opening angle and the notch acuity a/ρ. It does not depend on the stress level and the strain hardening exponent. The basic equations for the exponent δ have been summarized in a tabular form [8]. The predicted results of the J-integral are consistent with those directly obtained from finite element analyses (Fig. 3) [8].

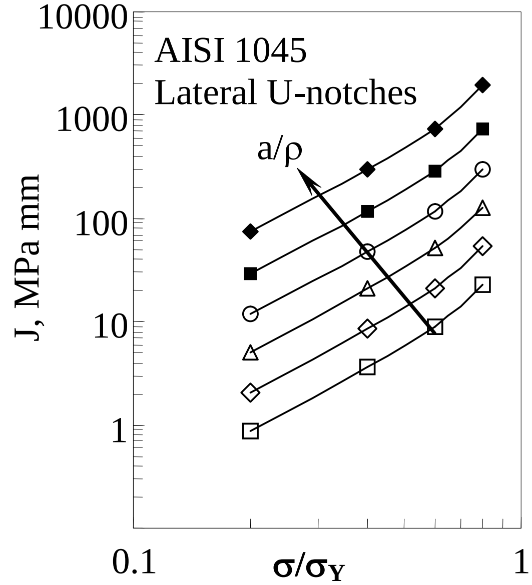

Fig. 3. Comparison between the predicted J-integral and FE results (a/ρ = 4, 10, 25, 60, 150, 400).

The Notch Fracture Toughness Based on the J-integral. The notch fracture toughness can be estimated in terms of the J-integral. To measure the notch fracture toughness $J_{\rho,c}$ and calculation of the *J*-integral updated for the crack growth in the case of different materials, the test procedure based on the load separation method is attracted to determine the η_{pl}- and η-factor of the non-standard curved notched specimen, namely, the CT specimen and arched specimen (so called “Roman tile”) under three-point bending. In this case, the original representation of the J-integral as total energy release rate and the tests records, namely, load versus total load-line displacement have been used taking into account an aquality between the η_{pl}- and η-factors. It is shown that the proposed procedure allows avoiding calculation of the stress intensity factor for non-standard specimens to determine the notch fracture toughness $J_{\rho,c}$. The η-factor (η_{pl}-factor) should be estimated by testing at least 3 specimens with different notch aspect ratio. Moreover, the load separation method allows predicting the growing crack length and constructing the *J-R* curve for these non-standard specimens. The details of the experimental procedures and results are given in Refs. [9, 10]. It turns out that the load separation method is very friutfull to determine the mixed mode plastic η_{pl}- and η_{pl}^{COD}-factors for the tension plate with an inclined centre through-thickness crack for power law hardening materials [11].

Structural Integrity Assessment

Failure Assessment Diagram and Acceptable Defects. The notch failure assessment diagram (Eq. 1) can be adopted for a component with a notch-like defect (or blunt crack with a finite tip radius) taking into account Eq. 2. To determine an acceptable (safe) region, it should be reasonable to introduce safety factors (e.g. [12, 13]) in the failure criterion. The following condition should be fulfilled if detected or assumed notch-like defect of a certain size should be assessed as acceptable

$$K_{notch} \le \frac{K_{mat}}{SF_K}\sqrt{1-\left(\frac{\sigma}{\sigma_0}\right)^2}\left[1-\left(\frac{\sigma_0}{\sigma}\right)^2\frac{1}{K_t^2}\right]^{-1/2} \qquad (9)$$

where SF_K is safety factor against fracture. The acceptable applied stress σ is suggested to be not more than σ_Y / SF_Y, i.e.

$$\sigma \le \frac{\sigma_Y}{SF_Y}, \qquad (10)$$

where SF_Y is safety factor against plastic collapse.

The safety factor SF_K can be calculated by making an assumption that the applied acceptable stress should be not less than the yield stress of material for an engineering component with a notch-like defect of the acceptable size. In this case, the safety factor against fracture of notched component is written as follows [5]

$$SF_K = SF_Y \frac{\sqrt{1-\left(\frac{\sigma_Y / SF_Y}{\sigma_0}\right)^2}\sqrt{1-\left(\frac{\sigma_0}{\sigma_Y}\right)^2\frac{1}{K_t^2}}}{\sqrt{1-\left(\frac{\sigma_Y}{\sigma_0}\right)^2}\sqrt{1-\left(\frac{\sigma_0}{\sigma_Y / SF_Y}\right)^2\frac{1}{K_t^2}}} \qquad (11)$$

It can be seen from Eq. 11 that the safety factor against fracture is a function of the yield stress as well as the safety factor against plastic collapse.

Thus, the right-hand side of Eq. 9 defines the acceptable region in the notch failure assessment diagram. If the assessment point falls within this region, the component with a notch-like defect is acceptable, i.e. it fulfils the required safety demands. For the special case of a crack ($K_t \to \infty$) the notch failure assessment diagrams are transferred to the failure assessment diagram for component with a sharp crack, and the safety factor (11) becomes the safety factor against fracture of cracked component.

The local strength σ_0 (Eq. 5 and Eq. 6) ahead of the crack tip can be rewritten taking into account the safety factor against collapse as follows [5]

$$\frac{\sigma_0}{\sigma_Y} = -\frac{\beta}{2SF_Y} + \sqrt{\frac{1}{4}\left(\frac{\beta}{SF_Y}\right)^2 - \frac{\left(1+\nu^2-\nu\right)\left(\beta / SF_Y\right)^2 - 1}{\left(1-2\nu\right)^2}} \qquad (12)$$

for plane strain and

$$\frac{\sigma_0}{\sigma_Y} = -\frac{\beta}{2SF_Y} + \sqrt{1-\frac{3}{4}\left(\frac{\beta}{SF_Y}\right)^2} \qquad (13)$$

for plane stress.

Acceptable Surface Longitudinal Notch-like Defects in a Pressure Vessel. An assessment of the acceptable surface longitudinal notch-like defects in a pressure vessel is based on the notch failure assessment diagram described by Eq. 9. It can be shown that the acceptable elastic stress concentration factor $[K_t]$ can be written as follows [5]

$$[K_t] = \sqrt{\frac{4}{\pi\rho}\frac{K_{mat}^2}{\sigma_Y^2}\left[1-\left(\frac{\sigma_Y}{\sigma_0}\right)^2\right]+\left(\frac{\sigma_0}{\sigma_Y}\right)^2} \quad . \tag{14}$$

Thus, acceptable state of the damaged pressure vessel has been presented by the following criterion

$$K_t \le [K_t], \tag{15}$$

where K_t is the elastic stress concentration factor for the surface notch-like defect under consideration.

The pressure vessel/defect geometry is described by the wall thickness *t*, vessel outer diameter *D*, defect depth *l*, and defect tip radius ρ. In the analysis presented below a wall thickness of 30 mm and a diameter of 1200 mm are applied. The notch-like defect length is assumed to be an infinite value.

To determine the elastic stress concentration factor for the surface external defect, the 2D finite element simulations of steel pressure vessels are carried out using the ANSYS code. It is shown [5] that the elastic stress concentration factor can be given by the following equation (Fig. 4)

$$K_t = 2\sqrt{\frac{l}{\rho}}Y\left(\frac{l}{t}\right), \tag{16}$$

where $Y(l/t)$ is a geometrical correction factor for the stress intensity factor in the case of the SENT specimen.

The following mechanical properties of the steel and the safety factor against plastic collapse for plane strain were used: $K_{1C} = 100\,\mathrm{MPa}\sqrt{\mathrm{m}}$, $\sigma_Y = 285\,\mathrm{MPa}$, $\nu = 0.3$, $SF_Y = 1.5$. In this case, the acceptable depth of a surface notch-like defect in the pressure vessel amounts to 10.23 mm.

Summery

A brief survey of some aspects of notch fracture mechanics applied to a solid with a notch-like defect has been given. A crack is considered as a special case of a notch. Notch fracture mechanics approaches have been employed to describe the notch failure assessment diagram and J-integral. Effects of constraint are incorporated into the basic equations which allow evaluating the constraint-dependent fracture toughness and failure assessment diagrams for various structural elements with a crack/notch and various types of loading. The load separation method is recommended to determine the notch fracture toughness in terms of the J-integral. Structural integrity assessment of the components damaged by notch-like defects is discussed from viewpoint of the notch failure assessment diagram.

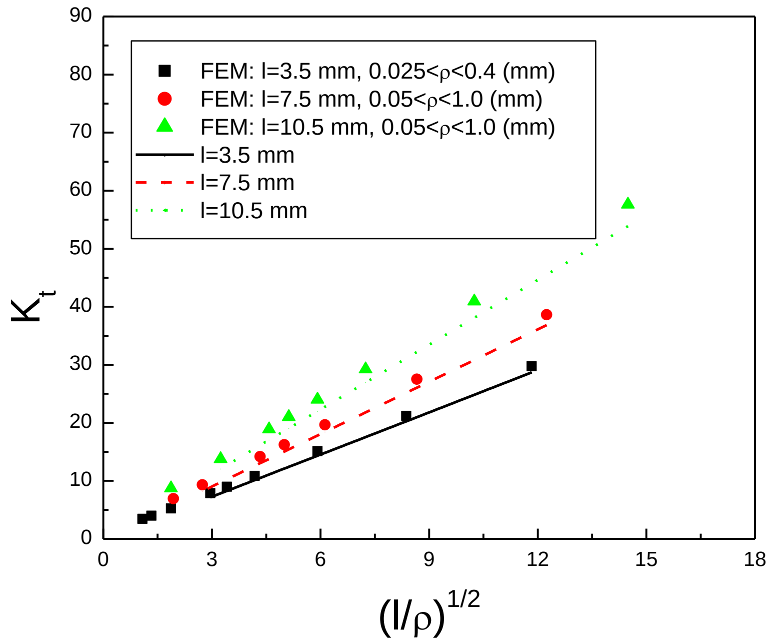

Fig. 4. The results of calculation of the elastic stress concentration factor for surface longitudinal notch-like defects by the FEM and Eq. 16.

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